Language and Logic in Wittgenstein's *Tractatus*, or Why "an Elementary Proposition Really Contains All Logical Operations in Itself" (TLP 5.47)

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1. Pictures and Truth-Functions: The 'Twofold Account' Reading

The following is a natural reading of Wittgenstein's account of elementary and molecular propositions, and thus of language and logic, in the *Tractatus*: first we have the so-called picture theory, the account of the representational nature of language, according to which propositions are logical likenesses of what they represent, because they share logical form with states of affairs. The claim that propositions are pictures, however, directly applies only to "the simplest kind of proposition" (Wittgenstein 1961, 4.21), namely to what Wittgenstein calls elementary propositions.

Wittgenstein accounts for complex (molecular) propositions by considering them as truth-functions of elementary propositions (cf. 5), the result of the application of truth-operations to elementary propositions. Truth-functionality also grounds Wittgenstein's account of logic; propositions of logic are tautologies (and contradictions), the two "extreme cases" (Wittgenstein 1961, 4.46) of the truth-functional construction of propositions out of elementary ones. In this sense, propositions of logic are simply a particular case, a sub-set, of truth-functional molecular propositions.

According to this reading of the relation between language and logic in the *Tractatus*, which I call the 'twofold account reading', therefore, Wittgenstein provides two *different* and *distinct* accounts of language (the pictorial and the truth-functional) and bases his understanding of the tautological nature of logic on a prior understanding of the nature of linguistic representation (in fact, in order to give an account of propositions in terms of truth-functions of elementary ones, one seems compelled to give a prior account of elementary propositions, and this is the purpose of the picture theory).

2. The Need for a Unified Account

The one sketched above, however, although indeed natural, cannot be a correct interpretation of Wittgenstein's conception of language and logic in the *Tractatus*. There is evidence, in fact, that Wittgenstein rejected the 'twofold account' reading.

As early as 1912, Wittgenstein wrote to Russell that the problems of logical constants and apparent variables will be solved as soon as a correct understanding of the nature of atomic (or elementary) propositions is reached (cf. Wittgenstein 1979, 121). In a 1915 entry from the *Notebooks*, Wittgenstein claims that "the problems of negation, of disjunction, of true and false, are only reflections of the one great problem" (Wittgenstein 1979, 40), where the latter amounts to "explaining the nature of the proposition" (Wittgenstein 1979, 39). In both passages Wittgenstein suggests that understanding the nature of a (atomic) proposition will put one in a position to understand the nature of logic (of logical constants) as well, for the problems of propositional and logical complexity are only by-product of the problem of providing an account of sentential complexity. In the *Tractatus* this view, although not discussed in such general terms, is maintained; Wittgenstein gives it expression by claiming that all logical operations/constants are present in an elementary proposition:

> An elementary proposition really contains all logical operations in itself. […] Wherever there is compositeness, argument and function are present, and where these are present, we already have all the logical constants. (Wittgenstein 1961, 5.47)

All of this seems in overt opposition to the 'twofold account' reading. Wittgenstein does not seem to hold that an account of elementary propositions should differ from an account of molecular propositions; indeed, he seems to be saying that the latter is contained in the former, and that an understanding of the former problem will therefore imply an understanding of the latter as well.

Secondly, the 'double account' reading, as noted, relies on the idea that the nature of logic is to be explained by means of a prior understanding of linguistic sense (by a prior account of the sense of elementary propositions); but, since Wittgenstein claims that all logical constants are already given by an elementary proposition, then understanding the nature of the (elementary) proposition (which the *Tractatus* discusses in terms of pictorial character) will be tantamount to understanding the nature of logic, for everything that is needed for an account of logic is already implied in the workings of elementary propositions. What Wittgenstein seems to be upholding, thus, is an account of linguistic representation that is by itself able to explain the nature of logical relations between propositions.

It is by no means easy, however, to assess Wittgenstein's general idea that the nature of logic is to be made clear by a correct understanding of the nature of the proposition; in particular, the main difficulty seems to be that of providing a plausible account of what Wittgenstein really meant with his claim that all logical constants are already to be found in an elementary proposition. In the remainder of this paper I propose to outline such an account by relying on Wittgenstein's conceptions of sense and a-b function in the *Notes on Logic* and of a picture and a truth-function in the *Tractatus*.

3. Sense, Truth and Logical Operations

Although in the *Notes on Logic* Wittgenstein does not claim that all that is needed for an understanding of logic is already contained in the nature of the proposition, his discussion of the interlocked notions of sense, bipolarity and truth-function seems to provide an account of language and logic that implements that general idea.

According to the *Notes on Logic*, a proposition has a sense – and therefore is bipolar (namely essentially true or false) – because it has a form (besides names) among its components. Wittgenstein conceives of the form of a pro-

position as operating a *discrimination* between facts in the world; for this reason propositions can be metaphorically be described as being like *arrows*:

> Names are points, propositions arrows – they have *sense*. The sense of a proposition is determined by the two poles *true* and *false*. The form of a proposition is like a straight line, which divides all the points of a plane into right and left. The line does this automatically, the form of a proposition only by convention. (Wittgenstein 1979, 101-102)

How does the form of a proposition make it effect a discrimination (or division) between facts? This Wittgenstein discusses in a famous (albeit rather obscure) passage from the *Notes*, where he considers the way in which the form of a proposition symbolises.

> Let us consider symbols of the form '*x*R*y*'; to these correspond primarily pairs of objects, of which one has the name '*x*', the other the name '*y*'. […] I now determine the sense of '*x*R*y*' by laying down: when the facts behave in regard to '*x*R*y*' so that the meaning of '*x*' stands in the relation *R* to the meaning of '*y*', then I say that they [the facts] are of 'like sense' with the proposition '*x*R*y*': otherwise, 'of opposite sense'; I correlate the facts to the symbol '*x*R*y*' by thus dividing them into those of like sense and those of opposite sense. […] Thus I understand the form '*x*R*y*' when I know that it discriminates the behaviour of *x* and *y* according as these stand in the relation *R* or not. In this way I extract from all possible relations the relation *R*, as by a name, I extract its meaning from all possible things" (Wittgenstein 1979, 104).

As I read this passage, a proposition is given a sense by its form (which, as said, is one of its components) that discriminates between two classes of facts, of *like* and *opposite* sense. The form *x*R*y* discriminates couples of things related by the relation *R* from couple of things that are not so related, and thus distinguishes facts of like sense from facts of opposite sense. The form of a proposition thus gives it the *possibility* of being true or false, by means of the discrimination between facts it operates, and is thus responsible for its *bipolarity*. In order to stress that truth and falsity are intrinsic to its sense, Wittgenstein writes a proposition, *p* for instance, as a-*p*-b, where *a* and *b* are the true/false poles, and he goes on equating a proposition's true/false poles with its sense.

> Every proposition is essentially true-false: to understand it, we must know both what must be the case if it is true, and what must be the case if it is false. Thus a proposition has two *poles*, corresponding to the case of its truth and the case of its falsehood. We call this the *sense* of a proposition. (Wittgenstein 1979, 98-99)

Now, this account of sense is crucial for understanding Wittgenstein's notion of propositional and logical articulation. In the *Notes on Logic* molecular propositions are called a-b functions (the *Tractatus* will call them truth-functions); a-b functions, as well as elementary propositions, have a-b poles (are essentially true/false), and therefore effect discriminations between classes of facts.

> The a-b functions use the discriminations of facts, which their arguments bring forth, in order to generate new discriminations. (Wittgenstein 1979, 105)

The link between the notion of an elementary and a molecular proposition is provided by the notion of discrimination between facts, above analysed. An elementary proposition is true or false because its form discriminates between two classes of facts, of like and opposite sense. a-b functions (complex propositions) simply exploit the dis-

criminations made by the (forms of) elementary propositions occurring as truth-arguments in them. A proposition's having a-b poles, truth-conditions, is thus everything that is needed in order to account for propositional and logical articulation, because a-b functions simply operate upon elementary propositions' a-b poles to generate propositions with different a-b poles, with different truth-conditions.

In the *Tractatus*, I argue, this conception is maintained. Of course the notation and the terminology is different there. Propositions are no more said to have a-b poles, but T-F (truth-false) ones, that is, the truth-possibilities. Consequently, a-b functions become truth-functions. But Wittgenstein's general position does not change significantly on this issue. Consistently with the *Notes on Logic*, the *Tractatus* claims that it is a proposition's *sense* that makes it intrinsically related to truth and falsity; unlike the old account though, for the *Tractatus* the sense of a proposition is not given by the peculiar nature of one of its components (its form) but by its being a *picture* of a possible situation: "A proposition states something only in so far as it is a picture"(Wittgenstein 1961, 4.03). So a proposition's being a picture makes it true or false: As the *Tractatus* states: "A proposition can be true or false only in virtue of being a picture of reality (Wittgenstein 1961, 4.06).

Why is it only pictures that can be true or false? For Wittgenstein a proposition has sense, and therefore is a picture, only in virtue of being logically articulated (cf. Wittgenstein 1961, 4.032), thus in virtue of being a *structured fact*. This is consistent with the account of sense given in the *Notes on Logic*, where the interplay of names and form determines a proposition to have a (determinate) *structure*. In the *Tractatus* a proposition is more explicitly held to be a *representational* (or *pictorial*) structured fact, that is, a fact representing elements in reality to be combined in the same way as its elements are combined. Its being a representation (a picture) of reality makes therefore the proposition intrinsically true or false; if things in reality are combined as it shows them to be, then the proposition is true, and otherwise false.

In order to express its intrinsic relation to truth and falsity, Wittgenstein sometimes expresses a proposition together with its true-false poles, the proposition *p*, for instance, as T-*p*-F or (TF)(*p*), not differently from what he had done in the *Notes on Logic*. Besides, the account of logical articulation there is consistent with the old one. Complex propositions, truth-functions, do not introduce new elements, but simply, as the *Tractatus* has it, agree or disagree with the possibilities of truth and falsehood of elementary propositions (cf. Wittgenstein 1961, 4.4). Again, everything that is needed in order to account for logical articulation is already in place as soon as a proposition is assigned T-F poles, that is, as soon as a proposition has a *sense* (or, is a picture); such a proposition provides everything that is necessary (T-F poles) for logical operations to be carried out.

Elementary propositions already ensure the possibility of all logical operations, because the latter operate upon a proposition's true-false poles, and get other true-false poles as a result; truth-poles, besides, are given by, and in an important sense *coincide* to, a proposition's having sense. This is the reason why, then, Wittgenstein can claim that an account of the nature of the proposition (its having sense, and thus true-false poles) will by itself be an account of the nature of logical articulation (of logical constants and operations). Nothing more than the former is needed in order to provide an explanation of the latter; in more specific terms, then, this amounts to saying that

propositions, by their having sense and true-false poles, are already given the possibility of having all sorts of logical relations with each other, and thus contain all logical constants in themselves. As Wittgenstein sums this up:

> The logical constants of the proposition are the conditions of its truth. (Wittgenstein 1979, 36)

4. Conclusion

The general conception that emerges from Wittgenstein's claim that all logical operations/constants are given by an elementary proposition sees the relation between language and logic as being, as it were, *internal*; the nature of logic is already made clear by a correct understanding of the nature of the proposition, that is, by a correct account of linguistic representation. Logic is internal to language in the sense that it is expressed in language's own capacity to convey thoughts about the world, thoughts that are true or false. Logical relations between propositions are given by propositions' expressing the sense they do, for those relations are already implied by propositions' own nature; this is the reason why the whole of logic is, for Wittgenstein, given at the level of elementary propositions, that is to say, is given as soon as propositions saying something about reality are given.

Literature

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