

# “Close to the Speed of Light”: Dispersing Various Twin Paradox Related Confusions

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...I noticed that mathematical clarity had in itself no virtue for Bohr. He feared that the formal mathematical structure would obscure the physical core of the problem, and in any case, he was convinced that a complete physical explanation should absolutely precede the mathematical formulation.

(Heisenberg 1967, p.98)

## 1. Introduction

One century after the appearance of the first of Einstein's several articles about what was later called Special Relativity Theory (SRT), one still cannot say that SRT has been sufficiently conceptually clarified to render concern about the meaning of its key concepts, or about the way in which it explains phenomena, a matter of mere ignorance. Some recent articles can be taken as evidence for this. Let me quote the diagnosis given in one of them, which concerns what Langevin called “The Twin Paradox” (Langevin 1911, p.31), referring to Einstein's original two clocks thought experiment described in “Die Relativitätstheorie” (Einstein 1911, pp.12ff.): “...Students often inquire as to ‘why’ the accelerated twin ages less and ‘when’ the extra aging of the home twin occurs. These questions are not well defined in the scientific sense but have promoted a variety of analyses (many can be found on the pages of this journal [*American Journal of Physics*]) which for most part have been useful additions to the pedagogy of special relativity theory” (Boughn 1989, p.792).

I join Boughn's appreciation of the analyses prompted by the two cited students' questions (see, for instance, Romer 1959, Perrin 1970, Unruh 1981, Good 1982), but, contrary to him, I hold the students' questions themselves perfectly well formulated, directly answerable, and more than vindicated due to the great diversity of answers (or at least of the ways in which “the

right answer” is formulated), including those given by some great physicists (among them Einstein himself).

In dealing with the two students’ questions, we shall come across more explanations of the Twin Paradox that are at least misleadingly formulated if not straightforwardly wrong, and the attempt to disperse the conceptual confusion they cause, or can cause, is the main task of this paper.

## 2. A purely phoronomic re-description of the paradox

Nearly all discussions of the Twin Paradox begin with the problem of symmetry: how can one of the twins turn out to be younger if *becoming further from* and *becoming closer to* are *symmetrical processes*?

Bergson solved the problem of symmetry in the way Alexander the Great had solved the problem of the Gordian knot: there will be no age difference at the end of the trip because the difference in aging during twins’ getting apart from each other is compensated by the inverse process during their approaching each other (see Bergson 1976, pp.434ff.).

The standard reaction to Bergson’s “solution” is that there *is* an asymmetry since one of the twins has to slow down and speed up in order to meet his brother. The problem with this answer is that deceleration and acceleration are neither necessary nor sufficient for the explanation of the phenomenon, as we shall see in what follows.

Let us describe the situation purely phoronomically, by stipulating that **A**’s and **B**’s “becoming closer to each other” and “becoming further from each other” have no commitments of “approaching” and “getting apart” in the sense that it is neither implied that it is only **A** or only **B** that is moving, nor that **A** and **B** are moving in opposite directions, nor that **A** is chasing **B**, nor that **B** is chasing **A**. In this way, it is also left indeterminate whether it is only *not said* (for whatever reason) what the case *de re* is or *it makes no sense* to differentiate *de re* among the possible cases mentioned because the distinction is not observer-invariant.

Let us suppose that **A** and **B** were initially becoming closer to each other at a uniform relative speed, and then, at the meeting point, synchronized their clocks. When later **B** met **C** after supposedly becoming closer to him at a speed that was uniform but greater than the speed of **A**’s becoming closer to **B**, **C** synchronized his clock with **B**’s. After some time, **A** and **C** must have met. Were their clocks synchronized at the meeting point or not, and if not, is it **C**’s clock or **B**’s clock that was slow relative to the other one?

### 3. Overcoming the underdetermination of the phoronomic re-description in SRT

The *underdetermination* of the situation described purely phoronomically follows from the *indeterminateness* of the directions of motion of **B** and **C**. Is this indeterminateness overcome in SRT even under the assumption that all motion has been *uniform since ever*?

Let **A** and **B** be light sources that, by passing each other, emit two light rays in both directions of their relative motion. Unexpectedly for the Newtonian physicist, each two of the four rays that move in the same direction will *simultaneously* reach any screen put orthogonally to their propagation. The fact that light behaves in this way has been known at least since 1849, when Fizeau performed the famous experiments that Einstein liked to cite (see Einstein 1977, pp.39–41). The three well-known theories explain this behavior of light in the following three ways.

According to the Maxwell-Lorentz’s theory, every lightwave propagates in any direction at the same speed *with respect to the ether*, which means that all the four rays propagate at the same speed with respect to the same point in absolute space from which they were emitted.

Since the ether theory is incompatible with the deeply rooted idea of classical physics that all inertial states of motion are equivalent, Einstein had tried, sometime before 1905, to *modify electrodynamics* by supposing that the speed of the light source is to be *added* to the propagating effect, which was later elaborated in Ritz’s 1908 emission theory (see Norton 2004, pp.58ff.). According to the *emission theory*, one cannot refer to a point left behind the moving sources but may say *only* that each two of the four rays that propagate in the same direction propagate differently *with respect to A* and *with respect to B*.

Re-considering constantly the thought experiment he had been allegedly obsessed with since he was sixteen, which concerns the possibility of chasing a beam of light till the point at which it becomes frozen for the chaser, Einstein finally gave up the emission theory for the three reasons (see Norton 2004, Sections 5–6), one of them being that it does not only allow for the possibility that the light beam be seen as frozen but makes something like this likely to happen, which, however, does not happen. In SRT, *this is impossible to happen by definition*: light is postulated to propagate, *with respect to any source*, at the *same speed in all directions*, which implies, automatically, that it is not possible to catch up with a light beam by chasing it.

If light propagates at the same speed in all directions, what about the case in which the light sources are moving relative to each other? The only way to avoid contradiction is to accept that the spacetime metric, and so also space-like and time-like metrics, are different *in the referential systems* of the two sources. This consequence is crucial for overcoming the *underdetermination of the phoronomic description* of the Twin Paradox, but it is not obvious *how* it is to be used.

Let us call a property Shakespearean (recalling the famous verse on the rose), be it non-relational or relational, if and only if its true ascription to something is observer-invariant (see Geach 1972, p.139). For instance, if John is shorter than Peter, he will *be* shorter *even if* he *looks* taller (for whatever reason) to *an* observer, and he will *remain* shorter even if we start to praise him as taller than Peter. The properties *being shorter than* and *being taller than* are relational, but *Shakespearean* relational properties.

Is *direction of motion* a Shakespearean property? According to Descartes' metaphysics, motion is *always only* relative because God, by permanently bringing the world into being, is also the ultimate cause of any change in the world (cf. Descartes 1986, Part II § 36, 25, 27). So, as in a movie, if the position between two bodies is changed, it is *not* changed *because* one of them or both *moved* but because God put them in a *different spatial relation*. Consequently, though different *world stages* are *real* and not just an appearance, *motion as such* is just an *appearance*. This is *why* for the Cartesians *the direction of motion* can be, and in fact is, a *non-Shakespearean* relational property.

In SRT, the light propagation is absolute not only in view of the assumed constancy of its speed, which is independent of the motion of light sources, but also because its *direction* is a *Shakespearean* property according to the main postulate of SRT: it propagates *in the direction in which* it is *impossible to catch up with* it by chasing it. If there were two bodies positioned at the line along which a light beam propagates but at the opposite sides of the light beam, the beam would propagate *necessarily in the direction* in which it could *come across* one of the two bodies, and *necessarily not* in the other direction. But what about the direction of motion of the bodies themselves (as potential or actual light sources)?

The main problem in answering this question follows from the assumption that light propagates *always* and in *all directions* at the *same speed* relative to *any* light source, so that *each* light source should be considered to be *at rest* relative to *any* ray emitted, while, at the same time, the light sources

can move, as they often do, *relative to each other*. Is it then *possible to say*, or perhaps even *necessary to conclude*, that the *direction of motion* of two light sources moving relative to each other is yet a *non-Shakespearean* property, in spite of the fact that the direction of the light propagation itself is a Shakespearean property?

For the negative answer to the last question it would be sufficient to show that, in some cases at least, the *electromagnetic effects* will *differ* according to *what* we suppose to be the *directions of motion* of the two light sources involved. Now, the Twin Paradox shows just this, which is one of its two main roles that has *never been explicitly* mentioned as such. Namely, the situation described in the Twin Paradox is the *exemplum crucis* for showing that in SRT the direction of motion of the light sources themselves must be a *Shakespearean property*, which is a good reason for agreeing with Michael Redhead that “the terminology ‘relativity’ for Einstein’s theory is arguably misleading” (Redhead 1993, p.120). The second important role of the Twin Paradox, obvious and recognized, is that it exemplifies *the essential difference between SRT* and the other two *rival theories* through some *striking consequences* that are in principle testable (and have already been tested indirectly—see, for instance, Hafele and Keating 1970).

(a) If **B** and **C** (in the above phoronomic description) are supposed to move in *opposite* directions, the average speed of the composite motion of **B** and **C** must be greater than the speed of **A** *independently of* whether **A** and **B** move in the same or in opposite directions. Namely, if **A** and **B** move in the same direction, then **B** is obviously faster than **A**, and **B** and **C** are supposed to move at a relative speed that is greater than the relative speed at which **A** and **B** move. So, the path traversed by **B** and **C** will be longer than that of **A**, and so their average speed greater than that of **A**. If **A** and **B** move in opposite directions, then, however small the relative speed of **B** may be, **C** will more than balance it by chasing and supposedly reaching **A**. So, the average speed of the composite motion of **B** and **C** will be greater again. This means that, because **B** and **C** were faster than **A** in the absolute sense, their average speed was closer to the speed of light when compared with the speed of **A**. This finally means (by taking into account how the contradiction threatening from the assumption that the speed of light is the same in all directions relative to any source has been avoided) that between the meeting of **A** and **B** and the meeting of **A** and **C** the composite time of **B** and **C** must have been *metrically shorter* than the time of **A**, which means in effect that, if **A**, **B**, and **C** emitted light signals periodically according to identical clocks

synchronized in the described way, the number of signals emitted by **B** and **C** must have been smaller than the number of signals emitted by **A**.

(*b*) It is easy to show, by applying the analogous reasoning, that the outcome will be *inverse* if we suppose that **B** and **C** move in the *same* direction: the time of **A** will be shorter than the composite time of **B** and **C**, and the number of signals emitted by **A** will be smaller than the number of signals emitted by **B** and **C**.

It follows from (*a*) and (*b*) that there will be differences concerning the *electrodynamic effects* depending solely on whether **B** and **C** move in *opposite* directions or in the *same* direction, which means that, according to SRT, the *direction of motion of the light sources* must be a *Shakespearean property* (Q.E.D.).

#### 4. Einstein's original description of the Clock Paradox and the role of the turning point

In his description of the Two Clocks Thought Experiment Einstein mentioned a “speed close to the speed of light” twice, the *first* time when *assuming* that one of the clocks “moves at a high speed (close to the speed  $c$ ) at a uniform motion”, the *second* time by *deriving the general conclusion* that, in the end, it will turn out that “the position of the hand of the clock almost didn't change during the entire journey” and that, in the analogous case of an organism, “the long journey lasted for just a moment because it moved at a speed close to the speed of light” (Einstein 1911, pp.12–13 [*my translation*]).

The natural question about Einstein's description is *why* we need a *round-trip* for *concluding* that the clock that moves back and forth will be slow, given that the conclusion is *derived* from nothing else but the closeness of its speed to the speed of light and that we have *assumed at the very beginning* that it is *this* clock that moves at a speed close to the speed of light.

One could be tempted to say that the *round-trip* is introduced with the sole intention to *shock* by its strange result, and this is the *first* reason why it is better to omit Einstein's assumption about the initial speed of any of the two clocks.

The *second* reason for omitting the assumption that one of the clocks (or twins) moves at a speed close to the speed of light already by becoming further from the other clock (twin) concerns the *generality* of the situation described. There is an *infinite* number of ways in which the round-trip can

take place with necessarily the *same* outcome concerning the question of *how* the clocks are desynchronized at the end. Recalling our phoronomic description, the *only additional* assumption *necessary and sufficient* for the conclusion that C's clock will turn out to be slow is that B and C move in *opposite* directions. So, Einstein's description is just one very special case.

The *third* reason for omitting any assumption concerning the initial speed of any of the clocks relative to the speed of light concerns the *impossibility* of *testing* any such assumption either during the clocks' receding further from each other *alone* or during their getting closer to each other *alone*. As we shall see below, it is only the *change in frequency* of the received signals that can enable the twins (or A and C, with the help of B, in our re-description) to calculate what will be the ratio of the number of the sent signals to the number of the received signals, which is necessary for concluding how each traveler has been moving and aging.

In view of the last point, however, it is important to note that the fact that it is *not testable how* the aging processes are developing while the twins recede *further from* each other *alone* and get *closer to* each other *alone* does *not* mean that the *final age difference* is not the *result* of the *difference in aging*. In spite of the fact that Einstein's assumption about the initial speed of a clock is unnecessary, it is certainly *not wrong* in the sense that it *makes no sense* to assume something like this. The suggestion that it cannot be said "when" the difference in aging is coming into being, based on the *necessity of a turning point* for saying anything about the age difference, led to the "mysterious" explanation (we shall come across below) that the age difference takes place instantaneously.

## 5. Invoking dynamical factors

All misleadingly formulated, incomplete or directly wrong solutions to the Twin Paradox are based on a misinterpretation of the role of the turning point on the round-trip.

Lecturing about the Twin Paradox, Feynman said that those who think it is a real paradox do so because they "believe that the principle of relativity means that *all motion* is relative" (Feynman 1963. p.16–3). What he actually wanted to say is not that they think just that all motion is relative but rather that the *direction* of motion is an *observer-dependent, non-Shakespearean* property. Feynman's answer to this Cartesian challenge is that "*the man who has felt the acceleration*, who has seen things fall against the walls, and so on,

is the one who would be younger” (loc. cit.). Since Feynman, after having indicated an asymmetry between the twins, didn’t explain *in which way exactly* it is relevant for the difference in aging, his explanation is incomplete. But it is also misleading because deceleration and acceleration are *neither necessary nor sufficient* for the difference in aging.

Deceleration and acceleration are *not necessary* because the *direction of motion* is a *Shakespearean* relational property according to the *main postulate* of the SRT, which is sufficient, as we have seen, for the explanation of the difference in aging even in the case in which all motions involved in the experiment are supposed to be uniform.

Deceleration and acceleration are *not sufficient* for the occurrence of the phenomenon because a twin that has *changed direction* can still age *faster*, namely, *if the relative speed after* he has changed his direction of motion is *not sufficient to reach* his brother.

In brief, possible *dynamical factors* related to the turning point are as such *not causally* responsible for the difference in aging, being at best an *indication* of *which twin* will be younger *given* that he moves at a speed that enables him to reach his brother.

However, arguing against “the orthodox relativists”, Whitehead claimed that “acceleration and deceleration (as distinct from uniform velocity) express an essential fact of the life of any body” (Whitehead 1923, p.41), and that it is just “the diverse history” of the body of the chronologer on the earth and of the traveler’s body, that is, “the real diversity of relations of their bodies to the universe”, that “is the cause of their discordance in time-reckoning” (ibid., p.34).

Similarly, by trying to explain why it is the traveler’s clock  $U'$  that will be slow relative to the earth clock  $U$  and not *vice versa*, Reichenbach invoked “the theory of gravitation”, which “shows that the special theory of relativity is applicable only because the distant masses of the fixed stars determine a particular metric field”, so that a retardation of  $U'$  is “the effect of the moving fixed stars, which produce a gravitational field at the instant of the reversal of the motion” (Reichenbach 1956, p. 193). To refute this “solution”, it is sufficient to imagine two journeys of the same person, with the same clock, moving at the same speed and turning himself around in the same way at the same cosmic point, with the only difference being that one of them lasts longer (see Mermin 1968, p. 144). The two journeys bring about *different age differences*, since the age difference depends on the length of the journey, and not on anything that happened at the turning point.

Finally, Richard Tolman, the most famous of Feynman’s predecessors at the California Institute of Technology, claimed that due to the possibility of the symmetrical analyses the paradox “can arise when the behavior of clocks is treated in accordance with the principles of special relativity without making due allowance for the principles of the general theory” (Tolman 1962, p.194), while it is “readily solved with the help of the general theory of relativity, if we do not neglect the actual lack of symmetry between the treatment given to the clock *A* which was at no time subjected to any force, and that given to the clock *B* which was subjected to the successive forces  $F_1$ ,  $F_2$  and  $F_3$  when the relative motion of the clocks was changed” (ibid., p.195).

## 6. The light signals exchange and the geometric representation of the thought experiment

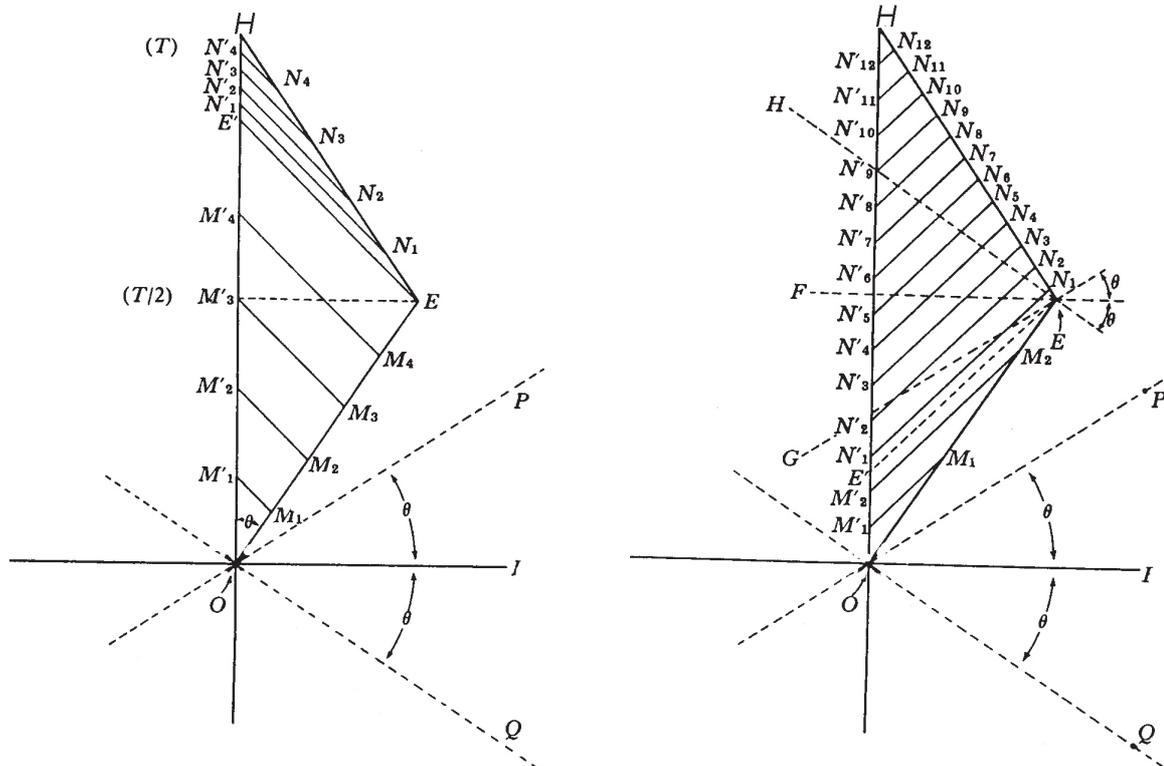
As shown in Section 3, the Twin Paradox exemplifies the situation in which we are vindicated in saying that one of the twins (or **B** and **C** in our re-description of the situation) moved at an average speed that was closer to the speed of light than was the speed of the other twin (**A** in our re-description). *Ontologically* speaking, this fact is *ratio essendi* for claiming that the number of light signals emitted by the back and forth traveling twin (**B** and **C** in our re-description) must have been smaller than the number of the signals emitted by the other twin (**A** in our description) in spite of the fact that the signals were supposedly emitted after the same time intervals had elapsed according to the identically made and synchronized clocks of the twins (of **A**, **B**, and **C** in our re-description). But then, the *ratio* of the total numbers of the signals emitted and received must be *ratio cognoscendi* for establishing, by the twins (**A**, **B**, and **C**) themselves, *how* their trip actually looked in view of the questions of the *direction* of their motion and of *when* and *how close* their motion was to *the speed of light*.

Let us suppose that, at the time they meet, **A** and **B** have arranged to send light signals to each other every year. Let them arrange, in addition, that if **B** meets or reaches a cosmic wanderer **C** moving at a uniform speed, it will be **C** who will instead of **B** continue to receive and send signals in the arranged way (according to **B**’s clock). Now, due to the Doppler shift, both **A** and **B** will receive signals from each other with a frequency that is less than the frequency of the signals they send. But, the frequency with which the signals are received along with **A**’s and **B**’s knowledge of the relative speed

at which they recede further from each other will not enable them to say anything about *whose* speed, if anyone's, is closer to the speed of light. *Mutatis mutandis*, the same holds for A and C.

However, the fact that neither A and B nor A and C can get any information about their speed relative to the speed of light on the basis of what was happening during the time A and B were receding further from each other *alone*, and during the time A and C are getting closer to each other *alone*, it does not follow that A and C (the latter with the help of B) cannot *infer* what was and what is going on in the *absolute* sense by *calculating*, on the base of their frequency, *the total number of signals that will be sent and the total number of those that will be received, and finally by comparing the two*. Namely, if B and C *de facto* moved in *opposite* directions (independently of whether there is any dynamical indication of that or not), then, by the time A and C meet, the total number of signals that A received must be *less* than the total number of the signals he sent, while the total number of the signals received first by B and then by C must be *greater* than the total number of the signals they sent. If, on the other hand, B and C *de facto* moved in the *same* direction, the situation concerning the total numbers of sent and received signals will be the inverse.

The just said can be illustrated through the following two Minkowskian diagrams (borrowed from Bohm 1996, pp. 169, 171).



It is important to note that, by drawing these diagrams, Bohm followed Einstein's original description of the thought experiment, according to which **A** (whose world-line is  $OH$ ) is taken to be at rest while **B** (whose world-line is  $OE$ ) and **C** (whose world-line is  $EH$ ) are taken to be moving in opposite directions at the same uniform speed relative to **A**. That's why  $OE$  is equal to  $EH$  and why, on the left diagram, the intervals between signals  $M_1, M_2, M_3$ , and  $M_4$ , sent by **B**, are equal to the intervals between signals  $N_1, N_2, N_3$ , and  $N_4$ , sent by **C**. But the point we are interested in would not be lost if we changed these special assumptions, obtaining a triangle of any other shape. The only necessary assumption, without which we wouldn't obtain *a triangle at all* as the representation of what is going on, is that the speed at which **B** and **C** move relative to each other is *greater* than the speed at which **A** and **B** do so. The additional assumption, not necessary for getting a triangle but necessary for getting  $E$  to be the turning point, is that **B** and **C** move in opposite directions, for the purely phoronomic description allows for the possibility that, by meeting **C**, **B** actually *overtakes* **C**. In this latter case, due to the fact that, after **B** and **C** meet, it is **C** whose time starts being compared with that of **A**, the turning point would occur at **A**'s world-line, and not at the point where **B** and **C** meet, for it is now **A** who (without doing and feeling anything new!) actually starts chasing **C**.

Now, it is clear from the left diagram that **A** will start receiving signals with a greater frequency (than the frequency of the signals he sends) only at  $E'$ , that is, relatively late, so that the total number of received signals will be *less* than the total number of sent signals. On the other hand, as it is clear from the right diagram, only a few signals sent by **A** will be received by **B** with a lesser frequency (than the frequency of the signals **B** sent), while the rest of signals, sent during a much longer time, will be received by **C** with a frequency that is *greater* than the frequency of the signals sent by **C**. So, the total number of the signals received by **B** and **C** will evidently be *greater* than the total number of the signals sent by them.

Is it at all possible that **A** and **C** meet finding their clocks synchronized? No, because there is only one turning point. But if we imagine **A** meeting some fourth person **D** under conditions that are completely symmetrical relative to those in which **B** meets **C**, then, when **C** and **D** met, their clocks would be synchronized. The Minkowskian diagram representing this situation would contain two broken world-lines symmetrical in view of the line connecting the meeting points of **A** and **B**, and **C** and **D**. So, the Bergsonian "solution" is true only in a special case, where there are *two* turning points.

## 7. When does the difference in aging come into being?

However nice a device the Minkowski diagram method is for representing geometrically what is going on physically, an understanding of what is going on physically must be the guide for “reading off the diagram”.

Since we have explained *in physical terms* in which way the final *age difference* of the twins is the result of the *difference in aging*, which *can* be but *needn't* be the same during the whole trip though it *must* always be in favor of the twin who traveled faster in the absolute sense, we may use Minkowski diagrams to show students *when* and *how exactly* the difference in aging came into being. For instance, according to the two diagrams above, which represent Einstein's original description, the *final age difference* is the result of the *difference in aging* that was coming into being *during the whole trip*. Namely (by using again our three persons **A**, **B**, and **C** instead of two twins), it can be directly read off from the left diagram that, if **A**, **B**, and **C** arranged to send signals every year, the composite motion of **B** and **C** lasted 10 years (these 10 years being  $OM_1, M_1M_2, M_2M_3, M_3M_4, M_4E, EN_1, N_1N_2, N_2N_3, N_3N_4, N_4H$ ), while, as can be read off from the right diagram, **A** lived 15 years (these 15 years being  $OM'_1, M'_1M'_2, M'_2N'_1, N'_1N'_2, N'_2N'_3, \dots, N'_{11}N'_{12}, N'_{12}H$ ).

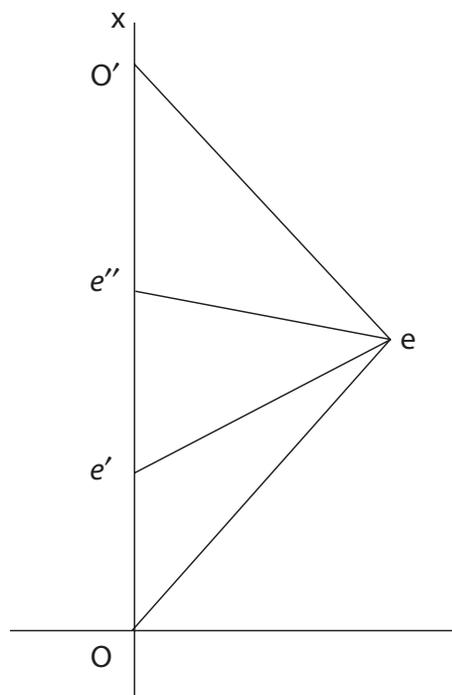
Let us turn to other, less specific triangles, but still with the meeting point of **B** and **C** as the turning point ( $E$  on the diagram), meaning that both **A** and **B** must be supposed to be moving, and moving in opposite directions. In this case (in contrast to the example of Einstein and Bohm), the length of **B**'s and **C**'s years must be *different*, because **C** has to *chase* and *catch up* with **A**. The *final age difference* must again be in favor of **B** and **C** *taken together*, but though it can still be true that it is in favor of both **B** and **C** relative to **A**, it can be also in favor of **A** over **B**, and **C** over **A**. The former would be the case if **B** were supposed to move, when compared to **A**, at a speed closer to the speed of light, while the latter would be the case if the speed of **A** were closer to the speed of light.

## 8. A “mysterious” geometric explanation of the age difference

Though one can find what has just been said evident and even trivial, the students' question of *when* the difference in aging takes place has been often suggested, as in Boughn's case, not to be well defined. But worse than that,

Minkowski diagrams are often used to "show" that the faster aging is based on the existence of some "extra time", which another twin saved "instantaneously", which is at best a very misleading *façon de parler*.

In his famous book *Space, Time, and Spacetime*, Lawrence Sklar explains the age difference using the following diagram:



He says: "The accelerated observer calculates that the inertial clock runs slow according to his from  $O$  to  $e'$ , and also from  $e''$  to  $O'$ ; but he sees himself as moving instantaneously from one inertial path to the other (at event  $e$ , his acceleration being 'almost instantaneous'), yet  $e$  is simultaneous with  $e'$  in his first inertial frame, and with  $e''$  in the second. It is the *life of the inertial clock from  $e'$  to  $e''$*  which makes the inertial clock read a greater time interval from  $O$  to  $O'$  than does the accelerated clock" (Sklar 1977, p.270 [*my italics*]).

The crucial thing is that Sklar says that the life of the inertial clock is *longer due to its life from  $e'$  to  $e''$*  (which is the interval between events  $N'_2$  and  $N'_9$ , on the right Bohm diagram above, where  $OI$ , and its parallels, are the simultaneity lines from  $A$ 's reference system, and  $OP$  and  $OQ$ , and their parallels, simultaneity lines from  $B$ 's and  $C$ 's referential systems respectively). Complementary to this, Sklar says that the accelerated clock saved time *at turning point  $e$* , which is "simultaneous with  $e'$  in his first inertial frame, and with  $e''$  in the second". Speaking about a similar diagram, Redhead says that it "shows  $\beta$ 's [the traveling twin's] clock running ahead of  $\alpha$ 's [the oth-

er one's] along sections  $Oe'$  and  $e''O$ , but mysteriously standing still along  $e'e''$  allowing the differential aging of  $\alpha$  to take place" (Redhead 1993, p.123 [*letters substituted to fit Sklar's diagram*]). I find the adverb "mysteriously" used by Redhead quite appropriate.

By dealing with the Sklarian-style explanation based on the Minkowskian flat spacetime geometry, we are faced with a situation that is strikingly similar to that we were already faced with in Section 5, when we were dealing with the explanations based on dynamical factors. In both cases, the *turning point* is *correctly* associated with the twin who will finally turn out to be younger, but some *effects* of the change of the inertial system, independently of whether accidental or necessary, are *wrongly* allowed to *figure in the explanation*, which should be based instead on the difference in *metrics alone*, as a consequence of the difference in (at least average) *speed relative to the speed of light*.

If we want to use *geometric representation* to explain what is going on *physically*, we have first to use *two* Minkowski diagrams, in the way in which Bohm did it, for it is only so that we can *directly read off*, and then *compare*, the time metric of the accelerated twin shown in terms of the total number of time units during the whole trip (the left Bohm diagram) and the time metric of the other twin shown through the total number of time units of the round-trip (the right diagram). Each of the two diagrams taken *per se* shows only *in which way one of the twins judges* the time of the other one.

It is of great importance to notice that the time-distance between  $e'$  and  $e''$ , which Sklar speaks of as the time saved by the change of the inertial system, depends on *both* speeds, the one between  $O$  and  $e$  and the other between  $e$  and  $O'$ , which determine the *angle* at  $e$  on the diagram. So, at  $e$ , the *age difference* that will show up at the end of the trip *is still to come into being* through the *difference in aging caused by the difference in metrics*, which itself is *caused by the difference in speed relative to the speed of light*. So, it is at least a *very misleading façon de parler* to say that the accelerated twin saved some time (represented on the diagram as  $e'e''$ ) by the very act of changing the inertial system.

"The discrepancy between the simultaneity relations" that Wesley Salmon takes to be "the key to the whole problem" (Salmon 1980, p.98) is something that itself should be, and can be, explained through the *spacetime metrics of the whole situation*. It would be mysterious indeed if the fact that "there is no moment between  $e'$  and  $e''$  that the traveling twin could find simultaneous with any moment of his trip" were to mean that the age difference at the

end of the trip is the result of an extra life that the home twin lived while his brother was (only for an instant!) at the turning point of his trip. But there is nothing mysterious in saying that due to the difference in spacetime metrics, and consequently in metrics of space-like and time-like intervals, the twins not only age differently but also cannot use any *simultaneity relation* that would be the *equivalence relation*—from which the “discrepancy” caused by the change of the referential system, which Salmon speaks about, can be derived as a *corollary*.

## 9. Conclusion and consequences: The Twin Paradox in the flat but closed spacetime

Once it is clearly explained *why* the twin who turned out younger did (because the metric difference between his and his brother’s world-lines favored him due to his speed, which was, on average at least, closer to the speed of light) and *when* the favoring age difference was coming into being (which can be either during the whole trip or during a part of it only, but in a more than balancing way), the question about the *necessity* of the *turning point* becomes answerable without confusion. In the standard representation, we need the turning point for *epistemological* and *operational* reasons. Namely, in the *open flat* Minkowski spacetime, it is *only the round-trip* that enables us to *prove* that one of the twins must have been faster in the absolute sense and so “closer to the speed of light”. At the same time, it is *only the change* of one of the twins’ inertial systems that brings about the change in the *frequency* of the received light signals, enabling the twins themselves to *calculate metrical differences* between their world-lines.

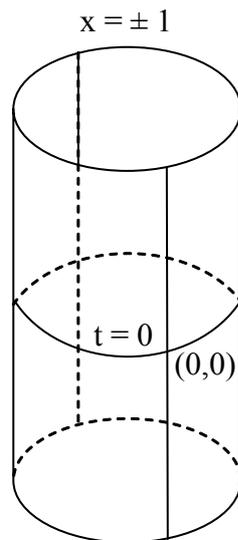
After dispersing all the confusions caused by the insufficiently clarified role of the turning point, we are free to turn to various *closed* spacetime models and describe the Twin Paradox situation *without* a turning point, as it is done, for instance, in (Brans and Stewart 1973), (Dray 1982), (Low 1990), (Uzan, Luminet, Lehoucq and Peter 2000). Let me give a nice example that shows what happens in one of these cases.

It is shown above why in the *open flat* Minkowski spacetime the existence of *one* turning point implies the asymmetry that is necessary and sufficient for the final difference in age between the twins, whereas, on the other hand, the existence of *two symmetric* turning points implies that there will be no difference in age between them. What will happen if the twins move at a uniform speed in spacetime that is topologically so structured that it is *flat*

but *closed*, so that it is unnecessary that there is a turning point before the twins meet?

According to the account given above, there can be an infinite number of cases lying between the two extremes. One of these extreme cases is that both twins move at a speed that is equally close to the speed of light. In this case, there will be no difference in aging and, consequently, no difference in the final difference in age between the twins. In the other extreme case, one of the twins is at rest. In this case, he will turn out older at the meeting point. In all other cases, the twin who is moving at a speed that is closer to the speed of light will finally turn out younger. How is the second extreme case to be represented?

The first thing to notice is that the topological structure is supposed to be *flat* in the sense that its Riemann tensor vanishes identically, so that we can pick a coordinate system in which the metric is constant. This means that we can have a case in which neither twin undergoes acceleration.



At the same time, however, the topological spacetime structure is supposed to be *closed* in the sense that there are pairs of space points—say, the point  $x = 1$  and the point  $x = -1$ —which are identical for all values of  $t$ . This means in effect that, as the geometric Minkowski spacetime representation, we get a cylinder (see the diagram above) with the  $t$ -axis as the vertical one.

Now, let us suppose that one of the twins remains at rest at  $x = 0$ , while the other starts off at  $(0, 0)$ , moving to the right with relative speed  $v$ , undergoing no acceleration at any time. This means that, between  $(0, 0)$  and  $(0, \pm 1)$ , the former will travel along the  $t$ -axis only, while the latter will travel along a *helix* winding round the cylinder with slope determined by  $v$ .

When  $t = 2/v$  (in the frame of the first twin), the two twins will meet at  $(0, \pm 1)$ .

It is clear that, in the given case, no resolution in Sklar’s style is possible (cf. Section 8 above), since there is no turning point at which the twin traveling along the helix could allegedly save some extra time. Moreover, the original paradox seems to re-emerge because one could argue that both twins can apply Lorentz transformations *in the same way during the whole trip* and get the *same result* at the end of the trip (see Low 1990, § 2). In other words, if one of them calculates the other’s measure of elapsed time by multiplying  $2/v$  by  $\sqrt{(1-v^2)}$ , so does the other.

Low says that “the entire technology of using Lorentz transformations to calculate is suspect in this context” because “the transformations make sense locally” but not “on the whole” of Minkowski spacetime (ibid., § 3). Namely, due to the fact that “spacetime in this instance, although flat, is not simply connected”, the twins’ receding from each other is, at the same time, their approaching each other. After that, Low claims that it is only the twin who remains at  $x = 0$  who is right in calculating the other twin’s measure of the *whole* elapsed time by multiplying  $2/v$  by  $\sqrt{(1-v^2)}$ , because by unwrapping the cylinder and finding various positions of the twins we see that in the frame of the twin who is at rest, for any value of  $t$ , the departure of the other twin from the corresponding position is always simultaneous, while it is not so in the frame of the twin who travels along the helix.

What Low says is, of course, correct from the point of view of the *space-time geometry*. But again, following Bohr’s requirement, cited by Heisenberg—that a physical explanation should absolutely precede the mathematical formulation—there is more that needs to be said in order to give a full explanation in *physical terms*.

Firstly, from the *physical point of view*, a part of the reason why only one of the twins is right in calculating the measure of the elapsed time of the other twin by multiplying  $2/v$  by  $\sqrt{(1-v^2)}$  is that it is *supposed*, in the given *extreme* case, that it is he who is at *absolute* rest relative to the light propagation, so that it is he who is in the *absolutely privileged position* to calculate *any* time saved by anybody moving relative to him. It is so, once again, only because in SRT it is not only true, as in the *emission theory*, that light propagates at the same speed in relation to any light source, but also that it propagates so in an *absolute sense*, independently of whether one can detect it or not.

Secondly, another part of the reason why one of the twins is right in calculating the measure of the elapsed time of the other twin by multiplying  $2/v$  by  $\sqrt{(1-v^2)}$  is that it is *supposed*, in the given *extreme* case, that light itself propagates in such a way that it will finally reach the same space point from which it started off and that the other twin is *chasing* such a light beam. In other words, the cylindrical spacetime representation, according to which one of the twins travels along a helix, is correct only because light is supposed to propagate in the described way.

Thirdly, the *supposed* fact that light propagates at the same speed *both* in an absolute sense *as well as* in relation to any light source, implies that there must be a difference in both space-like and time-like metrics, which means that traveling along the spacetime *helix* must be *longer* in view of space and *shorter* in view of time. In other words, the difference between the *simultaneity relations*, which Low draws our attention to, is only the *consequence* of the difference in *metrics* and not the other way round.

Finally, the most delicate question, which Low does not address in his article, is *how* the twins *themselves* can find out by exchange of light signals, if they can do it at all, what their clocks *will* show at the end of the trip. They *can* do that in *essentially the same way* in which it has been shown that they can do it by traveling in the *open* Minkowski spacetime. Namely, though in the closed Minkowski spacetime there is no turning point, so that the twins cannot use the difference in frequencies between the signals sent periodically before and after the turning point, they can send and receive signals sent periodically in *opposite* directions (because the spacetime is supposedly *closed*) and use the difference in their frequencies for calculating the time that has elapsed and the time that is to elapse before they meet. So, *both can* apply Lorentz transformations for calculating the *whole* elapsed time.

All in all, from the *physical* point of view, the resolution of the Twin Paradox formulated in the *closed flat* Minkowski spacetime is the *same* as the resolution of it when formulated in the *open flat* Minkowski spacetime.

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