Time and the Deep Structure of Dynamics

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Paul Davies was originally going to give this opening address, but he was unable to come, so he asked me if I would stand in. It is a great pleasure to be here. Ten years ago Paul published a book about time and in the preface he says: "You may well be even more confused about time after reading this book than you were before. That's all right; I was more confused myself after writing it" (Davies 1995, p. 10). I am sure that, if Paul had come here, he would have spoken about the notion of time that has dominated the last century, in physics at least. That is the notion of time that Einstein introduced.

I am going to come to that later. First, I am going to talk about what Leibniz and Ernst Mach said about time. This was radical and very different from what Einstein seems to be saying. But in fact you can put the Leibnizian/Machian idea into a theory of dynamics, into the way the universe works. And in the end it turns out to have a very interesting relationship to the way time is treated in Einstein's theory. And the bottom line is this: There is a great likelihood that time does not exist at all, that it is a redundant concept. Moreover, I think the confusion that Paul Davies was talking about arises from the mistake of thinking that time is a real thing. What we call time arises from something else; that at least is my view.

Everything in the discussion of time within dynamics and modern science has to begin with Newton. Now Newton got from Galileo and perhaps even more from Descartes the law of inertia, which Kant described as the most fundamental law and the basis of natural science. According to it, a body left to its own either remains at rest or travels at a uniform speed in a straight line forever unless it is acted upon by other forces and deflected from this rectilinear motion. Newton saw that this was a wonderful basic law, the first law of motion. Adopting it and just the notion of forces which deflect bodies from their inherent rectilinear inertial motion, he brought off the great coup: the explanation of the laws of planetary motion that Kepler had found some seventy, eighty years earlier.

So Newton felt it was essential to have a framework of dynamics in which the law of inertia can be formulated and makes sense. Moreover, like

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Descartes and Leibniz, he wanted to put the science of dynamics on an axiomatic foundation as crystal clear and secure as the axioms of Euclidian geometry. This was his ideal, and he thought that it would not be possible unless there was some huge fixed space in which objects move. Therefore he imagined a space that is really like the one created by the walls of this room. We can talk of straight lines in this room because we can see that they are straight relative to the walls. So that was the first thing he imagined, and he called it *absolute space*. He was so excited with the evidence that he thought he had found for the existence of this space that he called it the *sensorium of God*, the arena in which God perceives all that is and moves. He was that impressed with it, and rightly so.

In addition to absolute space, if he was going to say that a body moves *uniformly* in a straight line, he had to have some measure of time to confirm that fact. You can imagine Newton's time as some mysterious absolute clock hanging on the wall and ticking steadily away. Objects moving on their inertial lines in this room are then moving at a uniform speed as measured by the clock on the wall.

There is something mysterious about Newton's absolute space and time: they are both invisible, unlike the walls of the room. Newton was well aware of this, but he argued that, from the changing *relative separations* of things that you can observe, the existence of absolute space and time could be demonstrated. *The invisible could be deduced from the visible.*

Newton produced some promising arguments for his claim, but he never really established it in his work. Remarkably, nobody has ever taken the trouble to do properly what Newton said you should do: to show how you can recover the motions in absolute space and time from the relative motions you can observe. This was the problem that he posed at the end of his famous *Scolium* near the start of the *Principia*, where he says that he actually wrote his treatise to show how the problem is to be solved. ("For to this end it was that I composed it." (Newton 1934, p.12)) But he never returned to the topic in the body of his masterpiece! He left the problem in limbo.

Now for the famous reactions that any of you who have studied the philosophy of science will surely have encountered: The Leibniz-Clarke Correspondence and Ernst Mach's famous book on mechanics. Let me give you some quotations. Leibniz said: "I hold space to be something merely relative."(Alexander 1956, p. 25). His claim is ontological: *only relative things exist*.

I would like to make his point as vividly as possible. Here is a triangle. I want you to imagine that there are three material points, one at each vertex of the triangle. According to Newton, when those material objects move around, they move in absolute space, which is ontologically there. But let us consider more closely the way the particles can move. First, they can move relative to each other. However, this 'relative' change decomposes into two distinct kinds of motion. The ratios of the triangle sides can change. This changes the *shape* of the triangle. But the triangle can also get bigger and smaller, its *size* can change. Then there is a quite different kind of motion, which affects the triangle as a whole. This too decomposes into two distinct motions. First, the centre of mass of the triangle can move in three directions in space; second, its orientation can change—it can be rotated about three directions of space. And all of these changes are real according to Newton.

But Leibniz says, no, that is wrong. Only the relative changes are real. One must think of the triangle in itself, not relative to some imagined invisible space. He says space is the order of coexistences but initially is vague about what he means by *order*. Luckily for us, he was pressed later in the correspondence by Clarke (surely at Newton's instigation) to come clean about the meaning of order. Leibniz responded by saying that it is *the distances between the particles* that define the order that is space. Space is just a summary of all of these relations of distance between the objects in the universe. That is the viewpoint of Leibniz. Mach said very similar things many years later.

It has always been recognized that Leibniz made powerful criticisms of Newton's ideas at a philosophical level, but he made no attempt to set up an alternative dynamics based on his underlying ontology of relativity: that only relative things count. With the hindsight of history, you can see that it would have been impossible for him to do that. The technical means did not exist in his time to do that.

Such means do now exist, and I will come to them later, after we have considered what Mach said. First, we must consider Leibniz's view on time. According to Newton, one must suppose some uniformly flowing time or an absolute clock ticking away. Leibniz's view is very different. In a universe that consists of just our three particles the reality is that, at one instant, they form one triangle and then at another instant they form another, slightly different triangle. All you have is just a succession of different configurations of the three bodies, different triangles. And you must simply fix your mind on the succession, the one triangle coming after another. As he says explicitly: "I hold space to be an order of coexistences, as time is an order of successions." (Alexander 1956, p. 25/6) For our imagined three-particle universe, the distances between the three bodies define the order of the existences, and time is just the order in which they follow each other.

If we plot the three sides of the triangle with respect to three orthogonal Cartesian coordinates, each possible triangle is represented by a point in the space spanned by these three coordinate axes. There is a *space of possible triangles*. Let me call it *triangle land.* Each point in it represents one triangle. A succession of triangles that evolve smoothly from one into another is represented by a smooth curve in triangle land.

In the Newtonian view, one is to think of the history of the world happening at some 'speed in time'. It is as if there is some little red spot, marking the instantaneous *now*, moving along the curve from past to future at a speed that is sometimes faster, sometimes slower. History happens at a speed. The Leibnizian view is much more a timeless one. One must not think of a curve *and* a red spot moving along it. The curve is all there is. You can imagine each triangle defining an instant of time, but they are there all at once. History is not the path traversed at speed. *The path* is history.

However, we need to go further into the issue of time if we are to create a viable dynamical theory of the universe. One of Mach's most famous sayings gives us a hint of the direction we must go: "It is utterly beyond our power to measure the changes of things by time. Quite the contrary, time is an abstraction at which we arrive by means of the changes of things … ." (Mach 1960, p. 263) But what is this abstraction? We get it from the changes of things, from the moving parts of clocks. But why is it that we use one particular clock, say the rotation of the earth, and say it is a good clock and keeps good time, whereas some other measure of time is a bad one? Mach was one of the greatest experimentalists in the history of science; he very nearly got the Nobel Prize for his work on the shock waves. He was very aware that the sense of things ultimately always has an experimental basis. So he is already hinting that we need to understand how the measure of time that we use, the time that actually appears in both Newton's and Einstein's theory, emerges from change. We need a theory not just of the succession of instants, but of the way they relate to each other. How is it that we can meaningfully talk of seconds, hours and years? More mysteriously, what is the warrant for saying that a second today is the same as a second yesterday—or just after the Big Bang? Back then there was no earth circling the sun that could define a year.

Again, Mach gives us hints. The quotation above ends with the remark that the abstraction of time is "made because we are not restricted to any one measure, all being interconnected."(Mach 1960, p. 273) The interconnection of everything is emphasized in another often quoted saying of Mach: "Nature does not begin with elements, as we are obliged to begin with them. It is certainly fortunate that we can, from time to time, turn aside from the overpowering unity of the All. But we should not omit ultimately to complete and correct our views by a thorough consideration of the things which for the time being we left out of account." (Mach 1960, p. 287/8)

This emphasizes a feeling that you get very much from both Leibniz and Mach, namely, that the only way to understand the universe is to consider it in its totality. It is the opposite of the atomistic, reductionist viewpoint due to Descartes and Newton. That is based on absolute space and time. In the rigid framework that they form, each individual particle is forced to move in a straight line. The world is broken down into atoms. It is no accident that Leibniz and Mach were the two great opponents to Cartesian-Newtonian materialism based on atoms moving in space. Of course atoms go back to the ancient Greeks, to Democritus and others, but it was the success of Newtonian dynamics with its foundation in an absolute framework that gave substance to the idea. In contrast, whenever you read Leibniz and Mach, there is constant reference to the universe as a whole, the totality of the universe. And that I think is essential if we are going to develop a deep understanding of what time and motion are. Another essential element is the relational nature of position advocated by Leibniz and even more strongly by Mach, as in this well-known remark: "The universe is not twice given, with an earth at rest and an earth in motion; but only once, with its relative motions, alone determinable" (Mach 1960, p. 284).

Now, I am going to tell you how you can set up an alternative scheme of dynamics, which just uses the relative quantities that Leibniz and Mach said you should. In fact that it turns out that, in a very pleasing way, both Newton and Leibniz were right. But the Leibnizian/Machian element that comes in is very important and makes dynamics more precise and more strongly predictive than it was in Newton's hands. We will see how that happens.

The alternative scheme is based on eight principles, which are either taken directly from Leibniz and Mach or are strongly suggested by their writings. I shall present the principles one by one, with comments.

1. *The universe is a closed dynamical system*. I have already hinted at this. The basic idea is that, in a well-defined sense, you can meaningfully treat the universe as a completely self-contained closed thing. The attraction of this principle is that, in Einstein's words, it enables one to close the circle of cause and effect. You can find a cause for every effect. In the history of Western thought there have been two important and very suggestive models in which the universe has been closed and self-contained. The first was the spherical universe of Aristotle and the second, 2300 years later, the spherical universe of Einstein, the first cosmological model in the theory of relativity. From the latter, all of modern cosmological theory sprang. And it is not an accident that these two models played such important roles. It is almost a necessity of thought that, if you are striving to understand things as deeply as you can on the basis of a fully rational explanation of things, you have to postulate a self-contained closed universe.

However, I should emphasize that Einstein's theory does allow solutions in which the universe is not closed but infinite. The Machian/Leibnizian principles still play a vital role in the structure of such solutions, but they are not the whole story. There is always something out at infinity that you cannot get your hands on, and that is a bit unsatisfactory. Personally, I believe we have an indication here that Einstein's theory is still not quite "der wahre Jakob" and that it needs to be made even more Machian in order to eliminate solutions that are not closed. Certainly, the plethora of solutions allowed by general relativity, many of which seem very unphysical (by allowing time travel, as in Gödel's solution), has long been regarded as an embarrassment by several eminent cosmologists.

2. *The universe has geometrically defined instantaneous states*. The role of geometry in modern cosmology is not controversial. Indeed, Galileo said: "Trying to deal with physical problems without geometry is attempting the impossible" (Galileo 1953, p. 203) This is one of the great sayings of science. Dynamics, that marvellous achievement of the seventeenth century, grew out of the aspiration of Galileo, Descartes and above all Newton, as I have already mentioned, to erect the science of motion on an axiomatic basis as secure as that of Euclidean geometry. So the strategy here will be to take purely geometrical structures as building bricks from which dynamics will be constructed by additional purely dynamical principles. This is similar but not identical to what happens in general relativity, in which space and time are fused *ab initio* into a four-dimensional geometrical structure, to which purely geometrical principles are applied.

3. *The Identity of Indiscernibles.* As most of you are philosophers, I am sure that you will be familiar with Leibniz's 'two great principles'. One is the Identity of Indiscernibles. It says that if you imagine two things and say that they are different, but that *all* of their attributes are identical, then that is nonsense. If all of their attributes are the same, they are one and the same thing. This principle has immediate application to Newton's absolute space. Here, *in this room*, these situations of this triangle, here or here, or in these different orientations, or bigger or smaller but with the ratios of the sides remaining the same—these are different situations according to Newton, not only in this room but similarly in absolute space. Leibniz said the claim was simply false if the whole universe is considered. That is a key qualification. I will not have a chance to go into all the strengths of the Identity of Indiscernibles, but it is a crucial underlying philosophical standpoint.

4. *The instantaneous states of the universe are shapes.* To grasp the meaning of this principle, which has already been anticipated, you can again think in terms of just three particles. If the geometry that we presuppose is Euclid's, we are in the geometrical world that Newton inhabited. Then if you take the Leibnizian/Machian standpoint to its limit, you must say that all that counts is the triangle and not just the triangle as I hold it to you here, but any triangle which has the same shape as it but is either smaller or larger. If the triangle of the three particles represents the whole universe, the complete dynamical system, then all imagined places, orientations and sizes of the triangle are one and the same—or, rather, they do not exist at all.

So the space of states of the three-particle universe is all possible shapes of the triangle, nothing more, nothing less. And that is actually a two-dimensional space, the shape space for three particles in a Leibnizian world has two dimensions. It is the world in which things happen. I spoke of the history of the universe as curve in the space of its possible states. In the present case, the curve is in a two-dimensional space. Each point in it represents a shape of a triangle. We are going to talk about *a dynamics of pure shape*; apart from masses, only shape counts. (Inertial mass is, of course, an important concept in Newtonian mechanics, but there are several strong indications in modern physics that it is an emergent quantity.)

In arguing that only shape counts, I am going beyond the present interpretation of cosmology. In it, size is crucial—the universe is firmly believed to be expanding. In the standard cosmological model based on Einstein's general relativity, two main things have been happening in the universe since the Big Bang. The universe has been expanding and simultaneously changing its shape (by evolving from an extraordinarily uniform state to its present highly structured state). If only shape counts, the story of cosmology reduces to one of transformation from perfect uniformity to rich structure. I must admit that this conclusion is hard to tally with present observations. Nevertheless, the standard explanation of the Hubble red shift by expansion of the universe can in principle be explained by a gravitational red shift induced by the clumping of matter that has undoubtedly been happening throughout cosmological history. This is a long shot, but it is remarkable that general relativity is relational in its treatment of all aspects of motion except overall expansion. It is therefore of interest to explore what a theory in which absolute size is meaningless would be like. I therefore include relativity of size among the criteria of an ideal Leibnizian/Machian theory.

Thus, in a theory in which only shapes count, the history of the universe will be a continuous curve in the space of its possible shapes. This is a statement about what physicists call the kinematics of the situation. I have not talked about any law of dynamics; I have talked about geometry and about what is to be regarded as real. How do we get from a notion of the real to its law of change? We must invoke a further principle.

5. *The Principle of Sufficient Reason*. Leibniz also said that you must never introduce anything arbitrarily. There must always be a reason for anything that happens; there cannot be an effect without a cause. One of Leibniz's most outspoken philosophical objections to Newton's concept of atoms was vindicated centuries later by the discovery of quantum mechanics. If atoms were identical, as Newton (through Clarke) argued, one could arrange them in arbitrary orders. But then God would be in a quandary; wishing to place atoms in space, he would lack a reason for placing one here and another there rather than the other way round—the two choices would be identical in all real attributes and indiscernible. Leibniz therefore stated categorically that such atoms would not exist. He was vindicated when quantum mechanics showed that Maxwell-Boltzmann statistics, which should hold for such Newtonian atoms, does not hold in nature.

Leibniz was the supreme rationalist and said that not even God could escape the dictates of rationality. He must have a reason for everything. Leibniz's God was quite unlike Newton's, who was straight out of the Old Testament, ready and free to intervene in the world as he saw fit. His will was arbitrary. But Leibniz said God is not like that.

If for God we moderns substitute Nature and look to the results of modern physics to see how well Leibniz's two great principles stand the test of experimental verification, the answer is remarkably well. The idea I am just about to present is a striking example.

Leibniz said it is necessary to invoke something that goes beyond pure geometry in order to pass from geometry to dynamics. Geometry is about things that we can see, about shapes. So the question is, why does the realized curve in the shape space of the universe take the particular path it does rather than some other path? I will show how it can be done for a succession of shapes of triangles. The generalization to shapes formed by a greater number of particles, or even to fields and curved geometries, is straightforward. The trick is to take our geometrical building blocks, shapes of triangles in this case, and *build a higher-level geometrical structure on top of them*. With its help, we can solve our problem.

6. *The dynamical law of the universe is a geodesic principle on shape space*. Shape space is itself a higher-level geometrical structure. Each of its 'points' is an ordinary geometrical shape. In ordinary space, distances are defined. Because of this, we can find *shortest paths* between two given points. In Euclidean geometry, they are straight lines. However, in a curved space, such as the surface of the earth, they are 'straightest' paths and are called *geodesics*. A geodesic principle consists of two parts: 1) a rule that tells you the distance between neighbouring points; 2) the statement that realized paths are geodesics with respect to the distances defined under 1). Then provided that we can find a way to define 'distances' between shapes that are nearly the same, so that they are neighbours in shape space, we will have solved our problem. I mean, we will have a theory of the universe that we can test against observations.

In the particular case of triangle land, we will have a principle that will generate a curve of history between *any* two triangles, say the equilateral triangle and one in which one side of the triangle is much shorter than the other two. What we do is check out all conceivable paths that join the two points in triangle land corresponding to the two triangles and find the shortest of these paths. That will be the realized history between the two triangles.

What I am proposing is a form of one of the most fundamental principles in physics, *the principle of least action*. You can put a form of the principle into action here in Kirchberg. If you walk about the mountains here, between any two points of the curved surface of the mountains, you can find the shortest path between any two points. But to have a geodesic law, you have to know what is the distance between any two neighbouring points, any two infinitesimally separated points. Therefore, the task is this. Given any two triangles for which the angles between the lines joining the particles that form them are nearly the same, so that the shapes differ only slightly, find the 'distance' between them. We need a universal law for finding this 'distance'.

As of now it does not appear that a unique such universal law exists. However, I do believe that a certain basic form of such a law is predetermined. At the least, it is hard to see how any other serviceable universal law could be found. But some freedom still remains.

Before we consider the unique universal part of the law, let us briefly look at this residual freedom. It arises from the different kinds of constituents that could exist in the universe and from the manner in which they interact through forces. These forces can differ in kind. You all know the inverse square law of Newtonian gravity. The forces in Newtonian gravity are acceptable to Leibniz and Mach, because they depend only upon the relative separations. In fact, if we leave aside for the moment the question of scale, all forces that depend only on relative separations are acceptable. It turns out that in a scale-invariant theory the forces, or rather the potentials from which they are derived, must all have one fixed and definite dependence on the ratios of the relative separations. It also turns out that in sophisticated theories in which the constituents of the universe are fields rather than Newtonian particles, certain necessary conditions of consistency impose remarkably stringent restrictions on the possible fields and their interactions (Barbour et al. 2002). However, that is an issue we cannot consider here except to say that the conditions are respected by nature. It is time to consider the unique universal procedure that will enable us to determine a timeless Leibnizian/Machian dynamics.

7. *The geodesics are constructed by the principle of best matching*. In the case of a three-particle universe, our task is to find a 'distance' between two triangles that are nearly identical. This will enable us to pass from the geometry of our 'building bricks' to the higher-level geometry that we seek. Now ordinary geometry is based on the principle of congruence. Two geometrical figures are said to be identical, or congruent, if, by moving one around relative to the other, they can be brought to exact coincidence. This cannot be done if the figures are not identical. What we can do—and this is the basic idea of best matching—is bring the two figures as close as possible to exact coincidence and define a measure of the amount by which they fail to be exactly congruent. This will then give us the 'distance' between the two figures.

The procedure is not unique because one can choose different measures of the failure of the two figures to be congruent. There are different possible measures of 'incongruence'. However, these all turn out to be variations on a common theme. They yield broadly similar evolutions of the universe but subject to different laws of interaction between its constituents. Since the constituents and interactions can be observed experimentally, we have a method that will in principle fix the 'incongruence measure'. I will show how this is done for the simplest possible case: three particles of equal masses without any interaction between them. This will show how Newton's inertial motion, which he supposed took place in absolute space and time and thereby demonstrated their existence, follows very naturally from a Leibnizian/Machian scheme in which these absolute structures are not presupposed.

Call the particles *A, B, C*. In two slightly different configurations they will form two slightly differing triangles. In your mind's eye, imagine holding one of these triangles in any arbitrary position relative to the other. Then between the two positions of particle *A* there will be a distance *a* and similarly *b* and *c* for the other two particles. Now take the sum of the squares of *a*, *b* and *c* and call it *I*. This number can be called the *trial incongruence*. It is clearly arbitrary, since the relative placing of the triangles has been chosen arbitrarily. But we can find a number that is not arbitrary by exploring all possible relative positions of the two triangles and *seek the position in which the trial incongruence is minimized*. This is the essence of best matching. Let us call the minimum value I^*. It must exist because all the trial values *I* are positive. It is also easy to show that the relative position in which I^* is attained is unique. We may call I^* the *incongruence* of the two figures. We can find it for any two triangles, i.e., between any two neighbouring points in triangle land. Thus, we can find the geodesics representing possible histories of our three-particle universe between any two points in triangle land.

What has all this to do with Newtonian dynamics? The answer is a lot but with one or two significant differences. The evolutions of Newtonian dynamics are found through the principle of least action, which has a very similar form to the best matching that I have just described. In the case of Newtonian dynamics, there is an analogue of the trial incongruence *I*. It is the trial infinitesimal action *J* corresponding to motion of the particles through certain distances in absolute space in a certain interval of absolute time. The Newtonian rigid framework supplied by absolute space and time plays an important part in determining *J*. In contrast, *I* has no such dependence on an external structure.

We can easily see this by taking this room as a surrogate absolute space and the clock on the wall as the measure of absolute time. The procedure that I have described for finding the incongruence of the two triangles that I hold in my hand clearly does not depend on where I do the best matching. I can do it here or here; I can do it now or later. The result will be the same. The incongruence is something *intrinsic* that is determined by the two triangles and nothing else. What one can say is that the incongruence *I* is that part of the normal infinitesimal action that remains when the contribution of absolute space and time is subtracted.

What is the outcome of all this? Let us take an ordinary solution of Newton's equations for inertial motion of three particles in absolute space and time. We can then plot the succession of the triangle configurations that are realized in triangle land (at this stage I assume that size is meaningful). We obtain some curve. All possible Newtonian motions form a huge set of curves in triangle land. (There are, however, vastly more curves in triangle land that do not correspond to Newtonian motions.) If we now consider the curves that are geodesics with respect to the higher-level geometry defined by best matching, we find that they are curves that correspond exactly to Newtonian motions. However, they form only a very small subset of the Newtonian curves. They have a vanishing value of the important dynamical quantity known as the angular momentum.

This is very interesting. As I said earlier, both Newton and Leibniz were, in a sense, right. However, the way of arriving at a dynamical law of the universe suggested by the arguments of Leibniz and Mach leads to a more restrictive—and hence more predictive—dynamics. What is more, angular momentum is a measure of the rotation of the universe. It is hard to see how it can have any meaning if there is no absolute space. Thus, it seems eminently sensible that the Leibnizian/Machian dynamics exhibits no effect that one might attribute to rotation. Moreover, the universe in which we find ourselves reveals no trace of rotation.

It is important to note that if, more realistically, we consider universes formed by millions of particles, localized systems (like the solar system) can still behave exactly as in Newtonian mechanics and have a non-vanishing angular momentum (as the solar system does). It is merely required that the angular momenta of all the subsystems of the universe add to zero. What the Leibnizian/Machian approach does is give a complete explanation within a relational scheme for those features of local dynamics that Newton believed were evidence for absolute space and time (in modern terms, it explains the existence of inertial frames of reference). It also makes cosmological predictions that the Newtonian scheme cannot.

The broad scheme of best matching—minimization of a measure of incongruence—can easily be made to accommodate the familiar forces of Newtonian mechanics: gravity and electrostatics. One just modifies the actual expression that is minimized while retaining the best-matching procedure. As I already mentioned, this corresponds to changing the kinds of forces that act between the particles.

I will not go into the details, but the generalization of the best matching described above, in which overall size still has meaning (is absolute), to one in which only shapes count is very interesting. It leads to the epitome of Leibnizian principles, a *dynamics of pure shape* (Barbour 2003) which is *scale invariant*. The universe has no size, only a shape. In this case the solutions must not only have vanishing angular momentum but also vanishing energy. Moreover, whereas in Newtonian theory and best-matching theories with scale there is a wide freedom in the choice of interactions (forces), in the scale-invariant theories there is the strong restriction on the forces that I mentioned earlier. For example, one cannot have only gravitational and electrostatic forces. There must be a further force, which is very weak at short distances but at long distances acts like a piece of elastic tending to pull the particles towards each other. This is what ensures that the universe cannot give any appearance of expanding or contracting. For those who know about it, this new force mimics the effect of Einstein's cosmological constant.

One can consider a much more radical generalization. Instead of considering the Leibnizian/Machian dynamics of particles in Euclidean space, one can consider such dynamics for geometry that can vary. As most of you should know, in 1854 Riemann developed the famous geometry now deservedly named after him. In small regions, such a geometry is just like Euclidean geometry—Pythagoras's theorem holds. However, a Riemannian geometry is in principle curved, and its curvature is different at each point (as is the mountainous surface of the earth here in Kirchberg am Wechsel). Euclidean geometry represents the special case when there is no curvature anywhere.

Riemannian geometries, which can have any number of dimensions, can be closed up on themselves like the surface of the earth. Again, just as triangle land is the space of possible triangles, there is a space of all possible closed Riemannian geometries of some fixed dimension (say three, as we observe). It is called *superspace*. John Wheeler, who coined the expression 'black hole', initiated the study of *geometrodynamics* and showed that it gives one a very interesting alternative way of representing Einstein's general relativity.

Wheeler never thought to use geometrodynamics to go beyond Einstein's theory. However, the general scheme of timeless dynamics based on best-matching opens up possibilities that I at least find exciting. Let me explain. Just as the shape of a triangle is something of a quite different nature from its size—and clearly much more fundamental—there is a notion of the shape of a Riemannian geometry that is distinct from and more fundamental than the size. (Strictly speaking, we are talking about shape and size at each point—think of the shapes and sizes of mountains, which are different all over the Wechsel.)

The part of Riemannian geometry that concerns shapes alone is called its *conformal geometry*. There is a space of all conformal geometries, not surprisingly called *conformal superspace*. Some years ago, I conjectured that it ought to be possible to create a best-matching theory of timeless dynamics in conformal superspace and that this might describe the world. I started to work with Niall Ó Murchadha, Brendan Foster, Edward Anderson and Bryan Kelleher on this project. We fairly soon showed that such a theory can be constructed (we have called it *conformal gravity*) and that it has several remarkably interesting properties (Anderson et al., 2003). But does it describe the universe? This was our conjecture, and I must admit that, as of now, it seems to be false.

Nevertheless, the situation is very tantalizing. To explain why, I need to say something about general relativity. Its wonderful experimental successes are well known, but it has a very curious feature, brought to light especially clearly by our work (Anderson et al. 2005), concerning the extent to which it may be called a theory of pure shape. I said that a closed Riemannian space has a shape and size at each point. These are the *local* shapes and sizes, of which there are infinitely many. There is also one solitary further thing that characterizes a Riemannian geometry. That is its *total size*. You would think that the total size of the universe cannot have any meaning. What can it be measured against?

In my view, the most surprising thing about Einstein's theory is that it fails to be a theory of pure shape by a whisker. When you cast general relativity into the rather natural form in which the local shapes (each described by two numbers) are separated in their effect from the local sizes (each described by one number) (Anderson et al. 2005), you find that the dynamics is driven solely by the local shapes and not at all by the local sizes. They are what are, in modern terminology, called gauge variables. However, the total size does play a dynamical role. It interacts with the local shapes.

For centuries, if not millennia, thinkers have made the following comments: 1) If *all* sizes in the universe were doubled instantaneously, it would be impossible for anyone to observe the effect. Size is relative to the observer, so if the observer is magnified with the universe nothing can be observed. 2) Exactly the same argument has been applied to a doubling of the speeds of all motions in the universe. 3) It has also been applied to a displacement in space of all objects in the universe by the same amount. 4) Finally, rotation of everything together in the universe should be unobservable. Gut instincts like this, derived from succinct philosophical axioms by Leibniz in his two great principles, have always been the stimulus to the search for theories in which motion—and more generally change—are relational.

Let me now state the mystery about general relativity. Because Riemannian geometry is infinitely flexible—the surface of the earth can in principle have any topography—the arguments above can and should be made much more stringent. The demand that dynamics should be completely relational must be applied locally, to the local shapes, and not just globally (as I have done in the best matching applied to triangles, in which they are shifted rigidly relative to each other). Fortunately, the way in which best matching is to be done in this much more sophisticated situation is uniquely determined. There is also a uniquely determined manner in which the theory can be made timeless in a local way. Moreover, the two principles that I have outlined to you—timelessness and best matching—are, I assure you, sitting right in the heart of general relativity. You normally do not see them, because Minkowski first created the wonderful notion of space-time, a (rigid and flat) *four*-dimensional space, which Einstein then transformed into a curved spacetime. And in that language of a curved four-dimensional spacetime, you do not see these two principles at work nearly as well as in the geometrodynamical picture advocated by Wheeler.

The obvious question to ask is this—how well does general relativity meet the criteria 1–4 listed above (formulated, of course, in the stringent local form)? The answer that it does so outstandingly well for all criteria except for that single solitary role of overall size. In fact, it isn't even overall size that counts but *change* of overall size. This is not only a decidedly odd remnant of Newton's absolute space, it also spoils a certain inner harmony of Einstein's theory. The local shapes all interact with each other in a perfectly relational manner according to a common rule; the overall size could not be more different in kind, and it plays by a very different rule.

I think this is very challenging. The fact is that it is the changing overall size that allows the universe to expand and the Big Bang model to make sense within Einstein's theory. So what are we to make of this fact? The incredibly tiny little defect from the point of view of Leibniz and Machian philosophy is what makes all of modern relativistic cosmology so successful!

It is mysterious. I will risk my neck and say that I think Einstein's theory is wrong because it fails that one last Leibnizian test. It is something that I believe but cannot prove. The task is rather large. The conjecture is that the Big Bang theory of the expanding universe is quite wrong, that there is some other quite different explanation of the Hubble red shift and the many other observations (such as the helium abundance) that give such strong support to the standard model.

I do not think my idea of a dynamics of pure shape is entirely hopeless. According to standard cosmology, the universe emerged from an utterly mysterious 'Big Bang' in an extremely uniform state with everything very close to each other. Since then it has being expanding and simultaneously becoming much more variegated through the formation of atoms, galaxies, stars, planets and human beings. Structure formation is ongoing and undeniable. We literally see it happening. In contrast, expansion is a *deduction* based on both theory and observation. It is not impossible that the huge known growth of structure could, by a mechanism as yet unknown, explain the Hubble red shift, helium abundance, etc. My collaborators and I could show that in conformal gravity a *gravitational* red shift essentially identical to the one that exists in general relativity would be generated by increasing clustering of matter. However, the effect is small, and as yet we have made no progress concerning the other issues like helium abundance. So there the matter stands. Pure shape is an appealing idea but as yet is far from confirmation.

I am going to end by saying something about the quantum implications of the Leibnizian/Machian nature of general relativity that is well founded, i.e., everything except the absolute expansion. What I am going to say may seem startling, but it is not really controversial (though seldom expressed in the dynamical terms that I prefer to use). I have spelled out what I am going to say in much more detail in my popular-science book *The End of Time* (Barbour 1999).

The starting point for this is the notion of shape space (if we do assume that only shapes count, but essentially the same startling things emerge without that assumption). As I explained, each point in shape space is a possible configuration of the universe. All shape spaces have an interesting structure with a distinguished point corresponding to the most uniform state that is possible. In the case of a universe of three particles, this state is the equilateral triangle.

According to classical Newtonian physics with time, the history of the universe is a curve in shape space *traversed at a certain speed*. The difference between the Newtonian and Leibnizian viewpoints as regards time is relatively minor. One simply removes that red spot moving along the curve of history. The curve, which encapsulates *everything* that observers within the universe can observe, remains. The quantum implications of the idea that there is no external absolute time but only change, that we construct our notion of time out of change, are much more radical than the elimination of an imaginary red spot.

I am sure you have all heard of the wave functions that Schrödinger introduced. It is my belief that Schrödinger made the biggest revolution in physics by a long way, the Copernican revolution paling into insignificance compared with the introduction of wave functions.

Why was it so remarkable? First, of course, because it introduced probability in what seems to be an irreducible way. But perhaps even more surprising is what the probabilities represent. Normally textbooks discuss probabilities for individual particles. For example, the wave function will give the probabilities for finding the particle you are considering at different possible positions. One can easily get the impression that, in a system of particles, there is an associated wave function for each of them, each giving probabilities for the corresponding particle. Nothing could be further from the truth.

Suppose we consider the non-relativistic quantum theory of three particles in this room (free of outside disturbance). There are not three wave functions, one for each particle, but a single wave function that, if we are considering positions, gives the probabilities *for all possible configurations* of the three particles treated as a single entity. There are vastly more probabilities than you might imagine. There is one for each possible triangle of the three particles in all possible positions and with all possible orientations in the room. Mathematically, this is expressed by saying that the wave function is defined on the configuration space of the system.

Schrödinger was actually rather disturbed by the revolution he had introduced. The probabilities and quantum jumps were bad enough, but he was even more concerned by the configuration-space aspect. He devised his notorious cat paradox precisely to highlight this aspect of quantum mechanics. Let us now consider what might happen to probabilities on configuration space in the context of a putative quantum theory of the universe.

First, we must take away the Newtonian absolutes represented by the walls of the room. The Newtonian configuration space is whittled down to the shape space of the universe (assuming a scale-invariant theory, but as I already said the issues relating to time are not affected by that). We shall now have a very simple situation, assuming that the universe does have a wave function (a big assumption, but many people make it *faute de mieux*). The wave function will give probabilities for each possible shape of the universe. This sounds much the same as in ordinary quantum mechanics, but there is a big difference. To understand what it is, we must consider the *two* wave equations that Schrödinger found.

He first of all found an equation that is called *the time-independent Schrödinger equation* because time does not occur in it. His aim at this stage was to describe the stationary states of the hydrogen atom. In such a state, there would be a tremendous amount of activity, but overall the time-averaged wave function would not change its shape. That is what one means by a stationary state. Because the stationary state is unchanging, the equation that describes it does not contain the time.

This time-independent equation is the wonder of physics. It explains why all the atoms and molecules in the universe have the structures that they do. You put in the kinds of particles and the forces between them, and out come the shapes and sizes of the atoms or molecules they form. Amazing.

A few months after he had found that first equation, he found what is called *the time-dependent Schrödinger equation*. This equation describes how the wave function evolves in time. Now quantum mechanics was created using the old-fashioned Newtonian absolute notion of time. In fact, people doing quantum mechanics have not really learned how to live with a relational notion of time. However, in the quantum mechanics of the universe in a Leibnizian situation, it cannot make sense to talk about the evolution of a wave function in time, because there just is not any time in which it can evolve. That is not part of the picture.

Let me give you two quotations from Feynman. Physicists like to quote Feynman—he has such a reputation it must rub off on the speaker and lend plausibility to his or her claims. But I am going to give you two quotations from Feynman that I think are quite wrong. The first is a funny comment on time that I saw attributed to Feynman but cannot now locate the source. Time, said Feynman apparently, is "what happens when nothing else does". That is certainly not going to be accepted by Leibniz and Mach, for whom passage of 'time' without change is unthinkable. In this case I would rather have Leibniz on my side than Feynman.

The other quotation comes from the well-known book on quantum path integrals by Feynman and Hibbs (Feynman and Hibbs 1965). They comment (pp. 57/8) that if we know the wave function of a particle "at a particular time, then we can calculate everything that can happen to that particle after that time. All of history's effect on the future of the universe could be obtained from a single gigantic wave function." I have no quibble with the idea that the universe has a gigantic wave function, but the idea that it evolves in time is much more problematic.

Experiments in the laboratory unfold against the background of multifarious changes taking place throughout the entire universe. It is some average of all these changes that, in Leibnizian/Machian (classical) dynamics, should be identified with time. Let us call it the universal flux F. If we now pass from observation of a single particle evolving with respect to F to an attempted observation of the universal flux itself, we get into a manifest vicious circle. You cannot measure F with respect to F. We cannot contemplate evolution of a gigantic wave function of the universe with respect to the universe. Feynman's error arises from the tacit assumption that time is something distinct from the universal flux.

Therefore, if we want to retain the idea of a wave function of the universe—and I admit to a liking for the idea—its seems we cannot suppose that it evolves in accordance with a time-dependent Schrödinger equation. There isn't any time with respect to which it could do that. The only immediately apparent alternative is to conjecture that the putative wave function of the universe satisfies a time-independent wave function on the shape space of the universe. It will be *static* but still satisfy a definite law of variation along the various possible directions in shape space.

This suggestion is not at all so radical or controversial as you might suppose. Quantum mechanics was originally discovered by a process called quantization. One starts with some classical dynamical system and transforms it in accordance with rules that are more or less well established. There are two main approaches of this kind: the path-integral approach of Feynman and the canonical approach (favoured especially by Dirac). Roughly in the decade from 1955 to 1965, a determined effort was made by several eminent theoreticians (Dirac especially) to perform a canonical quantization of Einstein's general theory of relativity. Since Einstein's theory is the modern theory of gravitation, it was hoped to arrive in this way at a quantum theory of gravity.

All of this work was brought to a conclusion of sorts in 1967 by Bryce DeWitt, who found the basic form of the equation that the wave function of a self-contained (closed) universe should satisfy (DeWitt 1967). Because John Wheeler persistently prodded DeWitt into the finding of this equation, it has long been known as the Wheeler-DeWitt equation. (DeWitt is wont to say that he only found the equation "to get John Wheeler off my back".) Because general relativity is not fully scale invariant (as I explained earlier), the Wheeler-DeWitt equation is defined on superspace rather than on conformal superspace.

Many people were very surprised to learn that DeWitt's equation has the form of the time-independent Schrödinger equation. No variable with any resemblance to something one could call time appears in it. It seems to describe a static wave function. The probabilities associated with it are like a mist distributed once and for all over superspace. For me, this conclusion, which still makes many people uncomfortable (not to say distraught), is an inevitable consequence of the way in which profoundly Leibnizian/Machian principles lie hidden in the heart of general relativity. One could say that the Wheeler-DeWitt equation represents a triumph of Leibniz over Newton in the matter of philosophical principles.

This is an appropriate place to stop given the nature of my audience of philosophers. I would have liked to say how I at least try to make sense of the idea of a static wave function of the universe. There is a full account in my book *The End of Time* for those that are interested. I hope that at least in this talk I have managed to persuade you that Leibniz's philosophical principles are worthy of serious consideration. Perhaps I can end this written version of my talk with something controversial for supporters of Wittgenstein, who seems to have believed that he had reduced the role of philosophy to the study of the meaning of words. I believe that there is still a great role for philosophy as it was employed by the giants of the 17th century.

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