

Are the Laws of Nature Time Reversal Symmetric? The Arrow of Time, or Better: The Arrow of Directional Processes

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The paper discusses time reversibility (“arrow of time“) and shows that: (i) it is not satisfied on the microlevel; (ii) irreversibility should be replaced by very improbable recurrence; (iii) the “arrow” is in process not in time.

1. Introduction

Max Planck hoped that all statistical laws, especially the law of entropy could (and should) be ultimately reducible to dynamical laws:

I believe and hope that a strict mechanical significance can be found for the second law along this path, but the problem is obviously extremely difficult and requires time.¹

Many fundamental laws of physics (of Classical Mechanics, of Special Relativity, of Quantum Mechanics) are invariant w.r.t time reversal; i.e. in a differential equation like that of Newton’s second law of motion (for Classical Mechanics) or that of Schrödinger for Quantum Mechanics, one can replace the sign t (for time) by $-t$ without making the law invalid.² The underlying view about dynamical laws is illustrated by the famous quotation from Laplace:

We ought to regard the present state of the universe as the effect of its anterior state and as the cause of the one which is to follow. Given for one instant an intelligence which could comprehend all the forces by which nature is animated and the respective situation of the beings who compose it—an intelligence sufficiently vast to submit these data to analysis—it would embrace in the same formula the movements of the greatest

1 Planck in a letter to his friend Leo Graetz. Cited in Kuhn (1978), p.27.

2 For Quantum Mechanics, this was shown by Wigner (1932) and Dirac (1937).

bodies of the universe and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present to its eyes.³

The dynamical law describes the time development of a physical system S in such a way that the following condition D1 is satisfied:

D1 The state of the physical system S at any given time t_i is a definite function of its state at an earlier time t_{i-1} . A unique earlier state (corresponding to a unique solution of the differential equation) leads under the time evolution to a unique final state (again corresponding to a unique solution of the equation).

But D1 is not satisfied in statistical laws, like those of thermodynamics or those describing processes of radiation: A unique later state S_2 at t_2 is not a definite function of an earlier state S_1 at t_1 . The same initial state may lead to different successor states (branching).

According to Prigogine this is a sign that the laws of physics are still incomplete since many processes are irreversible in time.⁴

Feynman expresses his view quite directly:

Next we mention a very interesting symmetry which is obviously false, i.e., *reversibility in time*.⁵

The world view underlying Laplace's quotation was based on the belief that all physical systems are—if analysed in their inmost structure—ultimately mechanical systems. Since a clock was understood as a paradigm example of a mechanical system, the main thesis of the mechanistic world view could be expressed by saying that all complex systems (things) of the world—even most complicated ones like gases, swarms of mosquitoes, or clouds—are ultimately (i.e., if we would have enough knowledge of the detailed interaction of the particles) clocks. Or, put in the words Popper used in his A.H. Compton Memorial Lecture: "All clouds are clocks"⁶

After the discovery of statistical laws in thermodynamics and later in other areas, there was a general doubt with respect to the mechanistic and

3 Laplace (1814), ch.2

4 Cf. Prigogine (1993).

5 Feynman (1997), p.28.

6 Popper (1965).

deterministic interpretation of the world. One of the first philosophers who noticed that a certain imperfection in all “clocks” allows to enter chance and randomness was Charles Sanders Peirce.⁷ The question was now: Could it not be the case that all laws are statistical and the deterministic outlook is only on the surface of macroscopic phenomena? That is, all complex systems (things) of the world are in fact—in their inmost structure, i.e. on the atomic level—like gases or swarms of mosquitoes or clouds. This led to another extreme picture discussed by Popper: “All clocks are clouds”.⁸

But neither of these extreme pictures—reduction to dynamical laws “all clouds are clocks” or reduction to statistical laws “all clocks are clouds”—proved satisfactory as an explanation of everything. The heroic ideal to explain everything by one (or one kind of) principle had to be replaced by the aim to find relatively few (kinds of) principles (laws) for relatively many facts.

2. Experimental facts

Keeping the laws (time-reversal) symmetric and putting the responsibility for the time asymmetric phenomena into the initial or boundary conditions leads to explanations like the following ones: the thermodynamic asymmetry presupposes progenitor states far from the equilibrium; the CP asymmetry presupposes a spontaneous symmetry breaking of the Hamiltonian; the expansion of the universe presupposes a special singularity (big bang), etc.

However, many authors have also discussed time-symmetric models of the universe.⁹ In this case the universe undergoes expansion and contraction in a symmetric way such that we have periodicity. However there are at least two difficulties: The improbable recurrence and the T -violation in weak interactions. The first is expressed by the following quotation:

The difficulty with the time-symmetric models is their implausibility. They require a very finely tuned set of boundary conditions, for which no explanation is offered.¹⁰

The second is that since CPT (charge-parity-time) invariance (of laws) is

7 Peirce (1960) ch.6.47.

8 Popper (1965).

9 Gell-Mann, Hartle, (1994), ch.22.5.

10 Haliwell (1994) p.374.

generally satisfied—all fundamental field equations are *CPT* invariant—but *CP* invariance is slightly violated in weak interactions, *T* has to outbalance the difference. Therefore *T* invariance (time-reversal invariance) is not completely satisfied for the fundamental laws of nature.¹¹ *CPT* invariance—one of the most important symmetries of Quantum Field Theory—says that physical laws seem to be symmetric with respect to the complex exchange of particle-antiparticle, right-left and past-future. This *CPT* symmetry has remarkable consequences: It implies that the mass of (any) particle must be the same as that of its respective antiparticle. The same holds for their lifetimes. Their electric charges must have the same magnitude but opposite signs, their magnetic moments must agree.

Moreover as it appears from recent experiments *T*-reversal symmetry seems to be violated directly, too, and not only via *CPT* symmetry and *CP* violation. The violation concerns weak interactions. But since weak interaction concerns all elementary particles except photons the experimental result appears to be very important. There have been two different series of experiments independently made at CERN¹² and FERMILAB¹³ which seem to prove the violation. The experiment made at CERN concerns time dependent rates for the strangeness-oscillation process with neutral kaons which are different for $K^0 \rightarrow -K^0$ and its inverse $-K^0 \rightarrow K^0$. The experiment made at FERMILAB is of a more complicated structure.

3. Non recurrence versus irreversibility

Skiing in fresh powder snow is a great pleasure. But if the slope is small and one is skiing down frequently, the slope will be filled with traces and after some time no new space (powder snow) is left and thus one has to use one's own traces again (recurrence). This illustration tells us already some important conditions: The motion has to be area-preserving (the skier is not supposed to leave the slope) and in a finite region. Observe now that just by raising the complexity of the system recurrence becomes very improbable: Imagine that there are thousands of skiers on the slopes (the cable

11 This holds, provided there are no other ways out; for instance that the *T*-violation is not due to an asymmetry in the cosmological boundary conditions or to an asymmetry of our particular epoch and spatial location. Cf. Gell-Mann, Hartle (1994), p.329.

12 Angelopoulos et al. (1998).

13 Schwarzschild (1999).

cars and lifts of a big ski region in Austria can take up about 60,000 people per hour). The probability that at some later time all skiers would be in the same position again such that the whole state of this system would recur has much lower probability.

Boltzmann's example

Instead of living organisms take the molecules in a litre of gas (air) at temperature $T = 0^\circ\text{C}$ (273 K) and atmospheric pressure ($1.033 \cdot 10^3 \text{ g/cm}^2$). A litre of air (at temperature and pressure mentioned) consists of $2.688 \cdot 10^{22}$ molecules. It will be understandable that this system of $2.688 \cdot 10^{22}$ molecules can be in a huge number of different (micro-)states. The number is about $10^{5 \cdot 10^{22}}$ so as to realise the macrostate "litre of air under the conditions mentioned". Thus the same (for our lungs the same) macrostate can be realised by a huge number of different microstates. Boltzmann's discovery was that the probability of such a macrostate can be defined as the number of microstates which can realise the macrostate and that this number (more accurately the logarithm of it) is the entropy.

We might ask the following questions: Will all the $10^{5 \cdot 10^{22}}$ microstates of the litre of air be realised at all? And in what time? This leads to an interesting cosmological question: Assume that we are asking how many possible microstates are in the whole universe in order to calculate the entropy of the whole universe. Then the question arises whether every microstate can be realised within the life time of the universe, if the life time is finite. Since the number of microstates is extremely huge they probably will not all be realisable within the life time calculated by the Standard (Big Bang) Theory. If this is so, then there are more possible universes than the actual universe, which obey the same laws of nature and differ from each other only w.r.t. some microstates. In other words the laws of nature have more (possible) models than the one actually realised.

Irreversibility locally violated

Schrödinger raised the question: How can we understand a living system in terms of Boltzmann's theory?¹⁴ Or, how can these systems manage to keep, or even to increase, a low entropy level despite of the validity of the law of entropy? The answer, which was partially already given by Schrödinger, includes the following points:

14 Schrödinger (1944).

- (i) Living systems (organisms) are not thermodynamically closed; they are open systems.
- (ii) Living systems receive high-grade energy (energy with low entropy) from their environment via metabolism, but they pass on low-grade energy (energy with high entropy).
- (iii) By process (ii) the living systems are capable of achieving orthogenesis (maturation), i.e. increasing order, quality, and differentiation.

Cosmological investigations show that also planets, especially the earth, behave in a similar way as living systems. It receives high grade energy as electromagnetic radiation (with Planck-temperature of 5600 Kelvin) from the sun and passes on low-grade energy as heat radiation (with Planck-temperature of only about 300 Kelvin) into its environment.

From the above considerations it will be understandable that we want to avoid the term ‘irreversibility’ for three reasons:

- (i) What thermodynamic processes—and many others like radiation, cosmological expansion, processes of measurement, biological and psychological processes—really show is that recurrence of the state of the whole system is very improbable but not that recurrence or time reversal is impossible.
- (ii) From the last example described, it is plain that thermodynamic processes can be reversed locally without violating the second law.
- (iii) Moreover one can show independently that non-recurrence and time-irreversibility are not equivalent notions: since we have cases of non-recurrent phase density and time reversibility in chaotic motion of dynamical chaos.

Summing up

The difference between dynamical and statistical laws which is usually viewed as the most striking one—time-reversal invariance of the laws versus irreversibility of the laws—has to be taken with care. Strict time-reversal symmetry is no longer valid on the microlevel and irreversibility on the macrolevel should be better replaced by very improbable recurrence. However, the differences in this respect are sufficiently large to forbid reducibility of one type of law to the other. But the more careful interpretation paves the way for the compatibility of both types of laws.

4. Is the “arrow of time” compatible with dynamical laws?

We shall discuss this question w.r.t. three different descriptions of time (a)–(c).

(a) *Time flows*

That time *flows* we grasp from change, mutation and movement (i) w.r.t. to an ordered sequence (ii).

(i) Without any change time would “stand still” such that change is a necessary condition of time—at least for our understanding: “It is utterly beyond our power to measure the changes of things by time. Quite the contrary, time is an abstraction, at which we arrive by means of the changes of things.”¹⁵

That time presupposes change was already pointed out by Aristotle in his definition of time.¹⁶ Cf. also Kant in the *Critique of Pure Reason*.¹⁷

That there is no past to future direction of time in regions that are at equilibrium was pointed out by Boltzmann.¹⁸ He compared this with gravitation: As there is no downward direction in regions of space where there is no (net) gravitational force, there is also no past to future direction of time in regions that are at equilibrium.

(ii) Thus it seems better to speak of the asymmetry of a flowing process of a sequence of successive states which are ordered by a partial ordering instead of a “flowing time”. In such a sequence we distinguish past and future states and we measure the distance between them with the help of time units (produced by another physical periodic process in a clock).¹⁹

The presupposed ordered sequence is best described by the chronology of time or by the basic axioms of tense logic which assume partial ordering, transitivity, antisymmetry, irreflexibility and density.

Newton’s interpretation:

Absolute, true and mathematical time, of itself, and from its own nature, flows equably without relation to anything external ...²⁰

15 Mach, E. (1933), p.273.

16 Phys., 219b1.

17 KRV, B233.

18 Boltzmann (1897), p.583.

19 Cf. also the critical remarks by Paul Davies in his (1994).

20 Newton, *Principia*, Scholium.

Einstein's interpretation in GR:

The relative time of an observer plus reference frame is measured by standard clocks. By stipulation it flows equably but it depends on the distribution of matter, fields, and boundary conditions. Every observer plus reference frame has its own time scale and there is no universal time scale that is relevant for all observers.

In analogy to Newton we may paraphrase Einstein's interpretation thus: "Relative, true and physical time ... flows equably only locally, but in general it flows unequally *with* relation to something external i.e. to the distribution of matter, fields and boundary conditions."

(b) *Time flows only in one direction*

The subsequent considerations in (b) and (c) will show that the frequently used expression of the "arrow of time" is misleading; the arrow is intrinsic *in process* (not "in" time).

Concerning the question of how to distinguish a (particle's) movement on a spatial coordinate (in GR: space geodesic) from a (particle's) movement on a time coordinate (in GR: either time-like geodesic or null geodesic²¹), we may formulate two subquestions: (α) Can the coordinates (geodesics) be distinguished by their directions (vectors)? (β) Can the coordinates (geodesics) be distinguished by their closure conditions? Both questions can be answered with: Yes. The answer to the first subquestion is very well expressed by the following quotation from Wigner: "The difference between the two cases arises from the fact that a particle's world line can cross the $t = \text{constant}$ line only in one direction (in the direction of increasing t); it can cross the $x = \text{constant}$ line in both directions."²²

The answer to the second subquestion is the following: According to GR, the space of the universe is closed (even if the universe is expanding); that is, there are closed spatial coordinates or closed space-like geodesics. On the other hand, we usually assume that the time coordinate is not closed; i.e., we assume that the non-space-like geodesics (time-like geodesics and null geodesics) are not closed. This assumption has been called the *chronology condition* of spacetime²³. This condition plays an important role for the concept of causality. Causality would break down and one could travel into

21 See Hawking, Ellis (1973).

22 Wigner (1972), p.239.

23 Cf. Hawking, Ellis (1973), p.189.

one's own past, if the chronology condition is not satisfied.²⁴

The fact, that the time coordinate is distinguished from the spatial coordinates such that a particle's world line can cross the $t = \text{constant}$ line only in the positive direction of increasing t , is not determined by dynamical laws; because dynamical laws permit both directions, positive and negative. From this, it follows that (b), i.e. *time flows in one direction*, is compatible with dynamical laws, because dynamical laws allow both directions.

(c) *Time is connected with directional processes*

In addition to the directional feature of time described in (b) which is not at all in conflict with dynamical laws, there are two further features of time which are connected with directional processes.

(i) Assume a very large (long) but finite sequence of decimal places after 0, say the (finite) sequence of natural numbers, i.e. 0, 1 2 3 ... 10 11 12 ... 99 100 101 ... etc. It can be proved that this sequence has a normal distribution. It will therefore be easily understandable that the probability of recurrence for the three numbers 1, 4, 2 (in this order) on decimal places will be not very low. It occurs in 142, in 1420, 1421 etc. In contradistinction to that the probability of the recurrence of an ordered sequence of 10^{10} numbers on decimal places as part of the above sequence will be very much lower. This has nothing to do with entropy or with the increasing of a certain physical magnitude. But it has to do with "direction" and asymmetry.

(ii) Directional processes: First it should be clear that chronological time scales as they are used for time measurement do not define a direction of time even if they indicate that "time flows" in the sense of representing a sequence of partial ordering. Assuming events (states) $S_1, S_2, \dots (\in S)$, reference frames plus observer, RF, and chronometrical scales CS (mappings of durations onto real numbers, standard clock) the following postulates are basic for time T and time interval t :

- α) T is a function $\{S_1, S_2, \text{RF}, \text{CS}\} \rightarrow \mathfrak{R}^+$ with $T\{S_1, S_1, \text{RF}, \text{CS}\} = 0$
- β) For every state S_1 relative to RF and CS, and for any value $t \in \mathfrak{R}^+$ there exists a second state S_2 such that $T(S_1, S_2, \text{RF}, \text{CS}) = t$
- γ) Transitivity. For any triple of states (S_1, S_2, S_3) relative to RF and CS it holds $T(S_1, S_2, \text{RF}, \text{CS}) + T(S_2, S_3, \text{RF}, \text{CS}) = T(S_1, S_3, \text{RF}, \text{CS})$

²⁴ See ch.9 of Mittelstaedt and Weingartner (2005) for causality expressed by laws of nature.

From these postulates it follows that for any two events S_1S_2 relative to RF and CS it holds that: $T(S_1, S_2, \text{RF}, \text{CS}) = -T(S_2, S_1, \text{RF}, \text{CS})$. This shows clearly that the above basic assumptions and postulates do not define a direction “in” time. They tell us only that the direction from S_1 to S_2 is the opposite of the direction from S_2 to S_1 ; but not which event is first in nature.

On the other hand so-called directional processes (of nature) tell us unambiguously which event is first and which is second. Penrose²⁵ lists seven such directional processes:

- (1) The decay of neutral K mesons in weak interactions.
- (2) The process of measurement in quantum mechanics, especially the so-called “collapse of the wave function”.
- (3) All processes in which entropy increases.
- (4) All processes of radiation.
- (5) All conscious mental processes.
- (6) The process of expansion of the universe.
- (7) The process of gravitational collapse ending in a *black hole*.

Of these processes (1), (3), (4) and (6) are experimentally very well confirmed. (5) is very well confirmed by introspection and by the descriptions of the psychology of mental processes. The claim that (2) is a directional process is—at least to a considerable extent—a matter of interpretation of the quantum mechanical process of measurement. The time reversal of (7), leading to a *white hole* (no experimental evidence so far) is an open question such that (7) cannot be viewed as an unambiguous case of a directional process. Furthermore it is an open question whether processes (1), (4), (6) and perhaps (7) can be reduced ultimately to process (3).²⁶

References

- Angelopoulos, A. et al. 1998 *Physics Letters*, B444, p.43f.
 Aristotle 1985 (Phys) *Physics*, in: Barnes, J. (ed.) *The Complete Works of Aristotle*.

25 Penrose (1979).

26 Cf. Wheeler (1994). Concerning (7) the important question is whether the horizon area of the black hole can be proved to be proportional to measures of entropy which has been supported by Christodoulou, Beckenstein and Hawking.

- Boltzmann, L. 1897 „Zu Herrn Zermelos Abhandlung ‘Über die mechanische Erklärung irreversibler Vorgänge’, in: *Wissenschaftliche Abhandlungen*, vol. III, § 120, New York 1968.
- Davies, P.C.W. 1994 “Stirring up Trouble”, in: Halliwell, J.J. et al. (1994), pp.119–130.
- Dirac, P.A.M. 1937 “The Reversal Operator in Quantum Mechanics”, *Bulletin de l’Académie de Sciences de l’URSS* 1937, pp.569–575.
- Feynman, R.P. 1997 *Six Not so Easy Pieces*, Cambridge, Mass.
- Gell-Mann, M., Hartle, J.B. 1994 “Time Symmetry and Asymmetry in Quantum Mechanics and Quantum Cosmology”, in: Halliwell, J.J. et al. (1994), pp.311–345.
- Halliwell, J.J. 1994 “Quantum Cosmology and Time Asymmetry”, in: Halliwell, J.J. et al. (1994), pp.369–389.
- Halliwell, J.J., Perez-Mercader, J., Zurek, W.H. 1994 *Physical Origins of Time Asymmetry*, Cambridge.
- Hawking, S.W., Ellis, G.F.R. 1973 *The Large Scale Structure of Space-Time*, Cambridge.
- Kant, I. 1787 *Kritik der reinen Vernunft*.
- Kuhn, Th. 1978 *Black-Body Theory and the Quantum Theory, 1894–1912*, Oxford.
- Laplace, P. 1814 *Essai philosophique sur les probabilités*, Courcier, Paris, Engl. Translation: *A Philosophical Essay on Probabilities*, Dover, New York 1951.
- Mach, E. 1933 *Die Mechanik in Ihrer Entwicklung*. Leipzig, 9th edition.
- Mittelstaedt, P., Weingartner, P. 2005 *Laws of Nature*, Springer, Heidelberg.
- Newton, I. 1934 *Mathematical Principles of Natural Philosophy*, Cajori, F. (ed.), Berkeley 1962.
- Peirce, Ch.S. 1960 *Collected Papers of Ch. S. Peirce*, ed. by Ch. Hartshorne and P. Weiss, Harvard U. P., Cambridge Mass.
- Penrose, R. 1979 “Singularities and Time-Asymmetry”, in: Hawking, S.W., Israel, W. (eds.) *General Relativity*, Cambridge.
- Popper, K.R. 1965 “Of Clouds and Clocks”, in: Popper: *Objective Knowledge*. Oxford 1972, pp.206–255.
- Prigogine, I. 1993 “Time, Dynamics and Chaos”, in: Holte, J. (ed.), *Nobel Conference XXVI*. Univ. Press of America.
- Schrödinger, E. 1944 *What is Life?*, Cambridge.
- Schwarzschild, B. 1999 *Physics Today*, Feb. 1999, pp.19–20.

- Wheeler, J.A. 1994 "Time Today", in: Halliwell, J.J. et al. 1994, pp.1–29.
- Wigner, E.P. 1972 "On the Time Energy Uncertainty Relation", in: Salam, A., Wigner, E.P. (ed.) *Aspects of Quantum Theory*, Cambridge.
- Wigner, E.P. 1932 "Über die Operation der Zeitumkehr in der Quantenmechanik", *Göttinger Nachrichten* 31, pp.546–459.