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FIL 217/317
Wittgenstein Studies

2nd lecture 28.08.2018

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Todays programme

- Background of the TLP
  - Frege
  - Russell

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Background

- Engineering, problems of mathematics and its foundations, logic -> Frege and Russell
- Wittgenstein inherits from Frege and Russell the view that logical analysis is fundamental: the analysis of inference (in mathematics, but by extension in any language) presupposes an analysis of the structure of propositions.

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Background

- *syntactic* analysis has priority
- Frege and Russel tried to develop a perspicuous logical notation, a means of representing the logical structure (syntax) of natural language
  - Modern logic as *the* tool for analyzing and understanding language.

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TLP, preface

How far my efforts agree with those of other philosophers I will not decide. Indeed what I have here written makes no claim to novelty in points of detail; and therefore I give no sources, because it is indifferent to me whether what I have thought has already been thought before me by another.

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TLP, preface

I will only mention that to the great works of Frege and the writings of my friend Bertrand Russell I owe in large measure the stimulation of my thoughts.

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- Gottlob Frege (1848-1925)
  - *Begriffsschrift, eine der arithmetischen nachgebildete Formelsprache des reinen Denkens (1879)*
  - *Die Grundlagen der Arithmetik: eine logisch mathematische Untersuchung über den Begriff der Zahl (1884)*
  - "Funktion und Begriff" (1891)
  - „Über Begriff und Gegenstand" (1892)
  - "Über Sinn und Bedeutung"(1892)

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Stimulation from Frege:

- Frege's logicism and his three principles
- The distinction between Sense and Reference (*Sinn und Bedeutung*)
- The idea of expressions a functions
- The idea of a *Begriffsschrift* as a tool for logical analysis

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Frege's logicism

- Mathematics, as well as other forms of "pure thought", must be grounded in logic.
- Logic is strictly universal. It contains its own substantive truths, and also is a canon for all correct inference.
- There is no thought outside of logic (no "illogical" thought), strictly speaking. Cf. *TLP* 5.473

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Frege's three principles

"In the enquiry that follows, I have kept to three fundamental principles:

always to separate sharply the psychological from the logical, the subjective from the objective;

never to ask for the meaning of a word in isolation, but only in the context of a proposition;

never to lose sight of the distinction between concept and object."

(In Introduction to *Grundlagen der Arithmetik*)

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1. anti-psychologism

- F works in a neo-Kantian tradition, but is strictly opposed to the attempts to assimilate philosophical and psychological questions
  - 'the extrusion of thoughts from the mind' (Dummett)
- we can understand thought, i.e. the process of concept formation without appeal to extraneous psychological considerations

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2. The context principle

„Nur im Zusammenhang eine Satzes bedeuten Wörter etwas." (cf. TLP 3.3 !)

- the basic unit of sense is the proposition/sentence, which is the smallest unit of language which can be used to say/think anything at all.
- The meaningfulness of names and predicates is a matter of the place they occupy in the sentence, and also whether the sentence is true.

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2. The context principle

- Asserts that the existence of objects comes down to whether sentences that contain terms referring to them are true.
- Ontological and epistemological questions about what exists, and how, are approached as questions about the truth of sentences.

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The context principle

- When it comes to the question of numbers, it is the linguistic behaviour of numerical expressions together with the truth of the propositions in which they are found that settles the fact that numbers are objects.
- Numbers are *logical objects*, not empirical generalizations, intuited forms, or entities existing in a Platonic world of ideas

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The context principle

- Sentences express propositions, thoughts or judgments. It is these that can be true or false, not just the bare signs.
- This is also a leading principle in the TLP, and lies behind, eg., the central distinction between sign and symbol.

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3. The distinction between object and concept

- Different parts of the proposition play different roles
- Names (such as ‘John’, ‘the cat’, etc) refer to objects, and predicates (such as ‘is good’, ‘was asleep’, etc) to concepts.

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- Frege uses “name” broadly, It is meant to to apply to any definite singular noun phrase — including both proper names (‘Cicero’, ‘Plato’, ‘Fluffy’) and definite descriptions (‘the most famous Roman orator’, ‘the teacher of Aristotle’, “my favourite cat”).

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Sinn und Bedeutung

- What about the following?
- “The current president of the United States is orange.”
- “The 45th president of the United States is orange.”
- In one respect, they seem to say the same thing, in another respect, not.

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- Frege would say that the two sentences have the same reference, but have two different senses.
- They express different thoughts, even though they both pick out the same individual (under current circumstances).

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Frege on ”Sinn” (sense) and ”Bedeutung” (reference)

- Frege distinguishes between ”Sinn” and ”Bedeutung” of an expression (1892):
  - ”Evening Star” and ”Morning Star” have the same *Bedeutung* (namely the planet Venus) - but they have different *Sinn*
- (Mostly) equivalent distinctions:
  - Sinn vs. Bedeutung (Frege)
  - Sense vs. Reference (most common translation)
  - Sense vs. Nominatum (in some translations)
  - Bedeutung vs. Referenzgegenstand (standard German philosophical terminology)
  - Sense vs. Meaning (Ogden)
  - Meaning vs. Denotation (Russell)
  - Intension vs. Extension (Carnap)
  - Meaning vs. Reference (Quine)

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Frege on ”Sinn” (sense) and ”Bedeutung” (reference)

- The *Bedeutung* of a name is the object that the expression refers to.
- The *Sinn* of a name is the "mode of presentation" of the referent, or the cognitive content associated with the expression in virtue of which the reference is picked out.

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"Ein Eigenname (Wort, Zeichen, Zeichenverbindung, Ausdruck) drückt aus seinen Sinn, bedeutet oder bezeichnet seine Bedeutung. Wir drücken mit einem Zeichen dessen Sinn aus und bezeichnen mit ihm dessen Bedeutung."

(Frege, SB 203)

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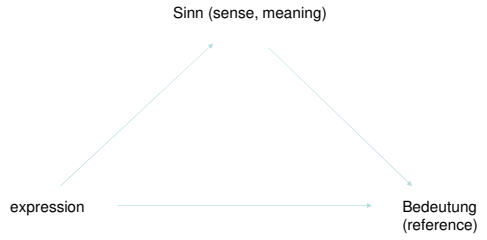
The «semantic triangle»



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- “The Morning Star” and “the Evening Star” are two different modes of presenting the same object (the second planet from the sun).
- Thoughts, senses, or modes of presentation generally, are common intellectual property for Frege. Two people can grasp the same thought. The same thought can be expressed by different sentences.

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Concepts

- Frege treats concepts as functions
- In arithmetic, complex terms like '2^2' and '3 + 1' are formed with the help of 'incomplete expressions' such as the squaring function '( )^2' and the binary addition function '( )+( )'
- Similarly, a statement like "Donald Trump is president of the US" can be analysed into two parts:
  'DT' and 'is president of the US'
- The second part is incomplete or «unsaturated»(i.e.contains an «empty place»)
- Only when this empty place is properly filled up, complete sense appears

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- A concept is a function from objects to truth-values
- e.g. from Donald Trump to truth (or falsity)
- A proposition is bi-polar (takes on the values "true" or "false")

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Sense and reference (and truth) of sentences

- The sense of a sentence is a thought (proposition) : Nonsensical sentences do not express thoughts
- The reference of a sentence is its truth value (a sentence which is true refers to the True)
- The truth value of a sentence is a function of the references of its parts
- To understand a sentence is to have grasped its truth-conditions
  - i.e. you must understand how the world must be in order for the sentence to be true.

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- NB: this means we have to understand a sentence in order to judge its truth value: meaning (sense) comes before truth!

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The idea of logical analysis

- The linguistic form of a sentence often disguises the logical form of the proposition it can be used to express (cf. TLP 3.323-3.325; 4.002)
  - "The man saw the boy with the binoculars." (syntactical ambiguity)

  Every sentence expressing a thought (proposition) must have a determinate sense.

  The aim of the analysis is to reveal the logical grammar or syntax of a proposition.

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Frege's "Begriffsschrift"

- Hence we need a suitable symbolism that precisely reflects logical structure and eliminates vagueness and ambiguity. Cf. *TLP* 3.325
- Frege calls his devised symbolism a *Begriffsschrift*, or concept script. In devising it (1879), he founded modern logic.
- The aim is to construct a universal, logically ideal language for the analysis and advancement of science and human knowledge (at least in all areas of "pure thinking" – mathematics and the foundations of natural science): "*eine der arithmetischen nachgebildete Formelsprache des reinen Denkens"*

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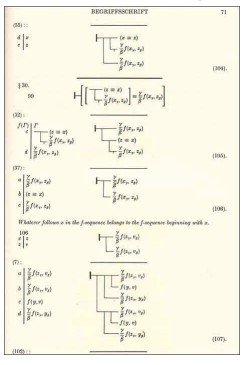
A page from the *Begriffsschrift*



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Frege's notation vs. modern notations

Basic concept	Frege's notation	Modern notations
Judging	$\vdash A, \Vdash A$	$p(A) = 1$ $p(A) = i$
Negation	¬A	$\neg A, \sim A$
Conditional (implication)	A B	$B \to A$ $B \supset A$
Universal quantification	F(x)	$\forall x\colon F(x)$
Existential quantification	F(x)	$\sim \forall x\colon \sim F(x)$ $\exists x\colon F(x)$
Content identity (equivalence/identity)	$A \equiv B$	A ↔ B $A \equiv B$ $A = B$

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Frege's *Begriffsschrift*

- A *Begriffsschrift* should
  - show where one does not say anything (though one thinks one does)
  - show what one actually says (what one actually says may be different from what one thinks that one says)
  - provide a symbolism which permits to say what one wants to say clearly and precisely, and thus helps avoid misunderstandings and renders in a precise way logical distinctions that are blurred in ordinary language

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Frege's *Begriffsschrift*

- A standard example of the functionality of a *Begriffsschrift* regards the use of "is" (TLP 3.323).
  - Cf. "Alois is diligent" vs. "2 times 2 is four" vs. "God is". Through the use of disambiguating logical language the three cases can be distinguished and the "is" can be rendered accordingly in different ways:
    - *P(a)* (predication)
    - 2x2=4 (sign of equivalence, identity)
    - ∃(x): *G(x)* (existential quantification)

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Problems with truth and reference of names

- Do proper names ("Alois Pichler") have both sense and reference?
- How to deal with "empty names" ("Pegasus", "Sherlock Holmes")?
- Can there be sense without reference?
  - "The greatest integer"
- What is the relation between names and "definite descriptions" (e.g. "the director of the Wittgenstein archives", "the teacher of Plato", "the current emperor of the USA")?
- Do functional symbols in logic (including connectives, negation, and quantifiers) have sense and reference?

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Problems with truth and reference of sentences

- Truth, reference and sense of sentences
  - Can a sentence be true and still senseless?
  - Can a sentence be senseless and false?
  - Can it be that while parts of a sentence have reference and sense, the whole sentence does not make sense?
  - Can a sentence have sense but no reference?
- TLP attempts to solve these problems

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Stimulation from Russell

- Russell's paradox and the theory of types
- Logical analysis and the problem of descriptions (philosophy as "critique of language“, cf. TLP 4.0031)

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- Bertrand Russell (1872-1970)
  - *The Principles of Mathematics* (1903)
  - "On Denoting" (1905)
  - *Principia Mathematica* (1910-13) (with A. N. Whitehead)

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Differences from Frege:

- a. Ostensive meaning (non-contextual)
- b. More psychologistic
- c. More empiricist
- d. Less universalist in logic in order to avoid Frege’s contradiction - Theory of Types(cf. *TLP* 3.331-3.334)

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Russell’s ”theory of types” began with …

- Frege’s definition of "number" in the *Grundlagen der Arithmetik* :
  - A number is a class of classes with as many members as a given class.
  - Can we find a class whose size is guaranteed by logic?
  - **0** is defined as the class of all classes with as many members as the class of objects which are not identical to themselves (the null-class)
  - **1** is defined as the class of classes with as many members as the class of null-classes
  - **2** is defined as the class of classes with as many members as the class whose members are zero and one
  - …etc

Russell's "theory of types" began with …

- Frege's logicist system presupposes that all classes must be capable of being members of other classes.
- The set-theoretical foundation leads to a paradox which Russell points out in 1902: Does "the class of all classes which don't contain themselves as their own elements" contain itself as an element or not?
- Frege was devastated by this, because it means there is a contradiction in his system, and thus the whole idea of building mathematics upon logic is jeopardized
- Russell proposed his "theory of types" as a solution to the paradox

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Other antinomies

- The Barber paradox:
  - The barber is a man in town who shaves those and only those men in town who do not shave themselves. **Who shaves the barber?**
- "This sentence is false"
  - Is it true or false?
- "I am always lying."
  - Always false?

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Russell's solution: A "theory of types" (1908)

To solve the paradox, Russell develops the theory of types / of hierarchy of classes.

- There are principal differences of *type* between
  (A) Classes of individuals and
  (B) Classes of classes of individuals
  - The elements of (A) are individuals; the elements of (B) are classes. One must not mix the two and make a class an element of itself.

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Russell's Theory of Descriptions (1905)

- A solution to Frege's problems with names and descriptions
- "The present king of Norway is bald." **True**
- "The present king of France is bald."
  - False or meaningless?
- "The present king of France" looks like it functions as a name in the sentence, but it is really a disguised existential claim.

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Result of analysis:

$\exists x(Fx \,\&\, \forall y(Fy \rightarrow x{=}y) \,\&\, Gx)$

- F: a is a king of France
- G: a is bald

- So now we see that the original sentence is false, with no resort to anything tricky.
- The analysed prop SHOWS us what must be the case if the prop is to be true.

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- We also see that the logical form of the proposition is more complex than we might have expected from the “surface” form of the sentence
- 4.0031 All philosophy is "Critique of language" […]. Russell's merit is to have shown that the apparent logical form of the proposition need not be its real form.

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Presuppositions about logical analysis shared by Frege and Russell (but *not* by W)

- logic is an essential framework of all thought, a system of maximally general truths (universalist conception of logic)
  - logic is conceived of as a science of objective laws of truth; laws of logic are distinguished from the laws of the special sciences only by their absolute generality

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Presuppositions about logical analysis shared by Frege and Russell (*and* by W)

- it must be possible to give a clear, completely explicit and unambiguous expression to the propositional contents judged true or false (otherwise we are dealing with nonsense)

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