A. Pichler & S. Säätelä: *Introduction to Wittgenstein*

5th lecture 18.9.2018: Truth functions; Logic; The colour exclusion problem

Today’s program

1st hour: Molecular propositions
- Truth functions and logic
- Some questions to the Tractatus

2nd hour: From the TLP to the later philosophy
- The colour-exclusion problem
- From the TLP to the later philosophy

Molecular propositions

Truth functions

How can I find out whether a specific sentence is true?

- 2.151 Die Form der Abbildung ist die Möglichkeit, daß sich die Dinge so zu einander verhalten, wie die Elemente des Bildes.
- 2.1511 Das Bild ist *so* mit der Wirklichkeit verknüpft; es reicht bis zu ihr.
- 2.1512 Es ist wie ein Maßstab an die Wirklichkeit angelegt.
- 2.221 Was das Bild darstellt, ist sein Sinn.
- 2.222 In der Übereinstimmung oder Nichtübereinstimmung seines Sinnes mit der Wirklichkeit besteht seine Wahrheit oder Falschheit.
- 2.223 Um zu erkennen, ob das Bild wahr oder falsch ist, müssen wir es mit der Wirklichkeit vergleichen.
- 4.024 Einen Satz verstehen, heißt, wissen was der Fall ist, wenn er wahr ist. …

How can I find out whether a specific *molecular* sentence is true?

N.B.: Frege and LW say: Logic is inherent in ***your*** thought and language!

- You already know the answer to the question above!

Logical connectives (”Junktoren”)

- ~p
  - N.B.: Negation does not really produce a *molecular* proposition.
- p & q*
- p v q
- p → q

* p and q are placeholders for propositions, e.g. ”It rains” and ”My cat is grey”

~	p
F	W
W	F
(2)	(1)

p	&	q
W	W	W
W	F	F
F	F	W
F	F	F
(1)	(3)	(2)

p	v	q
W	W	W
W	W	F
F	W	W
F	F	F
(1)	(3)	(2)

p	→	q
W	W	W
W	F	F
F	W	W
F	W	F
(1)	(3)	(2)

Logical connectives: Summary

p	q	~	&	v	→
W	W	F	W	W	W
W	F	W	F	W	F
F	W		F	W	W
F	F		F	F	W

The «laws» of logic

Are the ”laws of logic” laws?

No!

They are just the ways we think.

Non-contradiction

~	(p	&	~	p)
W	W	F	F	W
W	F	F	W	F
(4)	(1)	(3)	(2)	(1)

Excluded middle

p	v	~	p
W	W	F	W
F	W	W	F
(1)	(3)	(2)	(1)

The laws of logic do not tell us anything about the world

- TLP #4.0312: The logical connectives stand for nothing in the world. *Each logical connective can be expressed through a truth value series!*
- The laws of logic are tautologies (TLP #6.1). Tautologies do not tell us anything about the world. They do not express a thought and do not have content (TLP #6.111, #6.121 …) Cf. “It rains or it does not rain.”
- TLP #6.124: Tautologies and contradictions are the *limits* of meaningful (potentially true / false) language use: tautologies are always true, contradictions are always false. Both are sense-less (”sinnlos”).
- Tautologies describe the ”Gerüst der Welt” (TLP #6.124). We do not compare them with reality to find out whether they are true or false. In order to find that out it is sufficient to to run them through the truth tables.

➢ Logic is restricted to, and is happy with, tautologies and truth-functionality - no metalanguage and no ”school master” language police are needed.

The ”laws of logic”

- The laws of logic are no laws, but tautologies:
  - The law of excluded middle: p v ~p
  - The law of non-contradiction: ~ (p & ~p)
  - The law of identity: A = A
- The laws of logic cannot be verified or falsified; there is nothing in the world which corresponds with them.
- One can do without the laws / sentences of logic.
- None of the tautologies / truth value series of logic has primacy over the others.

➢ Therefore, logic is not collecting the most general laws of thought, but simply showing the structure of thought and at most collecting *tautologies*.

”~ (p v q)” and ”~p & ~q” have precisely the same truth value series and are therefore equivalent –

”~ (p v q) ↔ ~p & ~q” is a *tautology*!

~	(p	v	q)	↔	~	p	&	~	q
F	W	W	W	W	F	W	F	F	W
F	W	W	F	W	F	W	F	W	F
F	F	W	W	W	W	F	F	F	W
W	F	F	F	W	W	F	W	W	F
(7)	(1)	(6)	(2)	(8)	(3)	(1)	(5)	(4)	(2)

Molecular sentences and equivalence:

No real need for the logical connectives

Molecular sentences result from logical operations on atomic sentences

- Through logical connectives, a molecular sentence is built out of atomic sentences. Molecular sentences result from logical operations on atomic sentences.
  - ”Function” is not enough!
- Examples of molecular sentences:
  - ”It rains and my cat is grey”*:* p & q
  - ”It is *not* raining”*:* ~p
  - ”It rains or my cat is grey”*:* p v q
  - ”If it rains, my cat is grey”: p → q

How can I find out whether a specific molecular sentence is true?

- 5. Der Satz ist eine Wahrheitsfunktion der Elementarsätze.
  (Der Elementarsatz ist eine Wahrheitsfunktion seiner selbst.)
- 5.01 Die Elementarsätze sind die Wahrheitsargumente des Satzes.
  - Cf. „y = 2x“ (argument x, function value y)
- 5.13 Daß die Wahrheit eines Satzes aus der Wahrheit anderer Sätze folgt, ersehen wir aus der **Struktur** der Sätze.
- 5.23 Die Operation ist das, was mit dem einen Satz geschehen muß, um aus ihm den anderen zu machen.
- 5.3 Alle Sätze sind Resultate von Wahrheitsoperationen mit den Elementarsätzen. … Jeder Satz ist das Resultat von Wahrheitsoperationen mit Elementarsätzen.
- 5.4 Hier zeigt es sich, daß es „logische Gegenstände”, „logische Konstante” (im Sinne Freges und Russells) nicht gibt.

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Truth tables: How can I know whether a molecular proposition is true?

- The truth value of a molecular sentence is (exclusively!) a function of the truth values of the atomic sentences of which the molecular sentence is built through logical operation.
- By running a molecular sentence through the truth table calculus we show how its truth value series is a function of the truth values of the atomic sentences it is composed of.
- ”&” is *shown* by the T, F, F, F truth value series; ”~” is *shown* by the F, T truth value series ...
- 4.31 The truth-possibilities can be presented by schemata of the following kind ("T" means "true", "F" "false". The rows of T's and F's under the row of the elementary propositions mean their truth-possibilities in an easily intelligible symbolism).
- 5.01: The elementary propositions are the truth-arguments of propositions.
- 5.101 The truth-functions of every number of elementary propositions can be written in a schema of the following kind:
  - (W W W W)
  - (F W W W)
  - (W F W W)
  - (W W F W)
  - (W W W F)
  - (F F W W)
  - (F W F W)
  - (F W W F)
  - (W F F W)
  - (W F W F)
  - (W W F F)
  - (F F F W)
  - (F F W F)
  - (F W F F)
  - (W F F F)
  - (F F F F)

  ...

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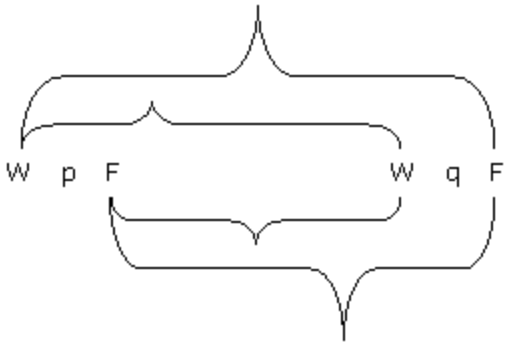
Reducability of logical connectives to truth value series

- 6.1203 Die Wahrheitskombinationen drücke ich durch Klammern aus, …

p		q
W	W	W
W	W	F
F	W	W
F	F	F



Equivalence

- ”Logic is not more fundamental than knowledge and cannot therefore be the basis for it. Logical analysis can however show us the true structure of our knowledge, thought and language.”
- Remember the truth value series for →? It’s W F W W.
- And what would it be for ~ (p & ~q)?
- And what for ~p v q?
- And what for ~q → ~p?
- It’s always W F W W!

➢ Logic shows us that these apparently different operations all have the same structure, that they are euivalent! (Which might not be easy to see otherwise.)

➢ *The really important things are the truth value series, not the logical connectives!*

Interdefinability («Möglichkeit des kreuzweisen Definierens», TLP #5.42) and reducability of logical connectives to truth value series

- The logical connectives can thus be reduced to truth value series. Sentences with **different** logical connectives can have completely **identical** truth value series and thus the **same** logical structure! Cf. TLP #5.101.
  - ”p v q” and ”~(~p & ~q)” have the same truth value series T T T F – they are equivalent; check it!
  - ”~ (p v q)” and ”~p & ~q” have the same truth value series F F F T – they are equivalent; check it!
  - ”~(p & ~q)” and ”p → q” and ”~p v q” and ”~q → ~p” have the same truth value series T F T T – they are equivalent; check it!
- The Sheffer stroke can be used to define all other logical connectives. See TLP #5.1311.
  - Sheffer stroke: «not both», «N(ot)and»: «p | q» = «~(p & q)»
- The logical connectives are as such not important and, in the end, there is only *one* ”primitive sign” in logic – die ”allgemeine Satzform”, the general form of proposition (TLP #4.5, #5.47-#5.472, #6): :
  - 5.5 Jede Wahrheitsfunktion ist ein Resultat der sukzessiven Anwendung der Operation (-----W) (ξ, ...) auf Elementarsätze.
    Diese Operation verneint sämtliche Sätze in der rechten Klammer und ich nenne sie die Negation dieser Sätze.
  - 6.001 Dies sagt nichts anderes, als daß jeder Satz ein Resultat der sukzessiven Anwendung der Operation N(ξ)* auf die Elementarsätze ist.

* ξ with overline.

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Cf. *Notes dictated to Moore (*1914)

- Cf. what Wittgenstein already in 1914 had said to G. E. Moore (Notes dictated to G. E. Moore in Norway 1914):

  Logical so-called propositions *shew* [the] logical properties of language and therefore of [the] Universe, but *say* nothing.

  This means that by merely looking at them you can *see* these properties; whereas, in a proposition proper, you cannot see what is true by looking at it.

  It is impossible to *say* what these properties are, because in order to do so, you would need a language, which hadn't got the properties in question, and it is impossible that this should be a *proper* language. Impossible to construct [an] illogical language.

Some questions to the Tractatus

Some questions to the Tractatus

- Was nun Ihre eigene Schrift anbetrifft, so nehme ich gleich an dem ersten Satze Anstoss. Nicht, dass ich ihn für falsch hielte, sondern weil mir der Sinn unklar ist. "Die Welt **ist** alles, was der Fall ist". Das "ist" wird entweder als blosse Copula gebraucht, oder wie das Gleichheitszeichen in dem volleren Sinne von "ist dasselbe wie". Während das "ist" des Nebensatzes offenbar blosse Copula ist, kann ich das "ist" des Hauptsatzes nur in dem Sinne eines Gleichheitszeichens verstehen. Bis hier ist, glaube ich, kein Zweifel möglich. Aber ist die Gleichung als Definition zu verstehen? Das ist nicht so deutlich. Wollen sie sagen: "Ich will unter 'Welt' verstehen alles, was der Fall ist? Dann ist "die Welt" der erklärte Ausdruck, "alles was der Fall ist" der erklärende. In diesem Falle wird nichts damit behauptet von der Welt oder von dem, was der Fall ist, sondern, wenn etwas behauptet werden soll, so ist es etwas über den Sprachgebrauch des Schriftstellers. Ob und wieweit dieser etwa mit dem Sprachgebrauch des Lebens übereinstimme, ist eine Sache für sich, auf die aber für den Philosophen wenig ankommt, nachdem er seinen Sprachgebrauch einmal festgestellt hat. … (Frege in a letter to Wittgenstein. In: Ludwig Wittgenstein: Gesamtbriefwechsel/ Complete Correspondence. Electronic Edition, 3.4.1920, InteLex http://pm.nlx.com)

N. Malcolm, *Ludwig Wittgenstein A Memoir,* p.70

- ”I asked Wittgenstein whether, when he wrote the *Tractatus*, he had ever decided upon anything as an *example* of a 'simple object'. His reply was that at that time his thought had been that he was a *logician*; and that it was not his business, as a logician, to try to decide whether this thing or that was a simple thing or a complex thing, that being a purely *empirical* matter! It was clear that he regarded his former opinion as absurd.”

World vs. Reality

- TLP #2.04: The totality of existent atomic facts is the world.
- TLP #2.06: The existence and non-existence of atomic facts is the reality.
- TLP #2.063: The total reality is the world.
- From #2.04 and #2.06 one is tempted to conclude that ”world” denotes a subset of ”reality” – but this seems contradicted by #2.063.

Also see «Sachverhalt»

- Wittgenstein's letter to Russell, 19.8.1919: "What is the difference between Tatsache and Sachverhalt?" Sachverhalt is, what corresponds to an Elementarsatz if it is true. Tatsache is what corresponds to the logical product of elementary props when this product is true. The reason why I introduce Tatsache before introducing Sachverhalt would want a long explanation.
- «Sachverhalt is, what corresponds to an Elementarsatz if it is true.» - Yes, this is correct; but Sachverhalt is also, what corresponds to an Elementarsatz if it is *false*, isn't it? See TLP #2: Was der Fall ist, die Tatsache, ist das **Bestehen** von Sachverhalten. Thus, Sachverhalte can also „***nicht*** bestehen", ***not*** obtain! What corresponds to a Sachverhalt which does not obtain? An Elementarsatz which is false.

”Nonsense”

- TLP #4.124, #5.5351, #6.54: Attempts at describing the logic of our language – though important they may be – are condemned to fail to make sense since they attempt at *saying* what only can be shown: what *can* be shown of the ”Gerüst der Welt” *is* shown by every use of language.
  - The sentences of logic, mathematics, the foundations of the natural sciences, ethics and philosophy are all ”pseudo-propositions” (either senseless or nonsensical).
- Tautologies (and contradictions) are senseless (”sinnlos”).
- Sentences which contain formal concepts are nonsensical (”unsinnig”).
  - Examples for formal concepts include ”object”, ”complex”, ”number” (TLP #4.126ff)
  - TLP #4.1272: So one cannot say, *e.g.* “There are objects …”
- Sentences which contain value concepts are nonsensical (”unsinnig”).
- A proposition is nonsensical if it contains a sign without meaning. (TLP #5.4733)

”Nonsense”

- Russell, in his preface to TLP: “… Mr Wittgenstein manages to say a good deal about what cannot be said …”
- Is the context-principle part of the ladder to be thrown away? (TLP 3.3 a.o.)
- Is the sign-symbol distinction part of the ladder to be thrown away? (TLP 3.32 a.o.)
- …

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Simple objects

- ”Real” (material particles of physics) or phenomenal (points in the visual field, objects of acquaintance)?
- ”Things” only or also properties and relations? If elementary propositions of the form ”a ε P” are to be possible, then simple objects have to include also properties?!
  - See Ms-102,147r[3] (date: 19150616): Auch Relation und Eigenschaften etc. sind Gegenstände.
- Realist or idealist (or quietist) interpretation?

The colour exclusion problem

”Can you give me an example …?”

- The *Tractatus* doesn’t give a single example of a simple object.
- The *Tractatus* doesn’t give a single example of an elementary proposition.
- Our everyday language sentences are not elementary propositions:
  - They contain hidden or explicit logical operators.
  - They contain complex expressions referring to complex objects.
  - They are mostly *not* logically independent of each other.
  - A great many of them are not truth value capable (not bipolar).
  - With some of them it is not at all clear what their structure is or whether they at all have a structure („Hi!“).

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Colour statements: candidates for elementary propositions?

- Sense-data statements:
  - «Here red»
  - «There green»
  - «This heavy»
  - «Here pain»
  - …
- Could it be that sense-data statements are elementary propositions?
  - «This is red», «This is green» …

Colour exclusion as an example of logical impossibility

TLP 6.375

- Just as the only necessity that exists is *logical* necessity, so too the only impossibility that exists is *logical* impossibility.

TLP 6.3751

- For example, the simultaneous presence of two colours at the same place in the visual field is impossible, in fact logically impossible, since it is ruled out by the logical structure of colour.
- Let us think how this contradiction appears in physics: more or less as follows – a particle cannot have two velocities at the same time; that is to say, it cannot be in two places at the same time; that is to say, particles that are in different places at the same time cannot be identical.
- (It is clear that the logical product of two elementary propositions can neither be a tautology nor a contradiction. **The statement that a point in the visual field has two different colours at the same time is a contradiction.**)

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Colour exclusion as an example of logical impossibility

TLP 6.375

- Wie es nur eine *logische* Notwendigkeit gibt, so gibt es auch nur eine *logische* Unmöglichkeit.

TLP 6.3751

- Daß z.B. zwei Farben zugleich an einem Ort des Gesichtsfeldes sind, ist unmöglich, und zwar logisch unmöglich, denn es ist durch die logische Struktur der Farbe ausgeschlossen.
- Denken wir daran, wie sich dieser Widerspruch in der Physik darstellt: Ungefähr so, daß ein Teilchen nicht zu gleicher Zeit zwei Geschwindigkeiten haben kann; das heißt, daß es nicht zu gleicher Zeit an zwei Orten sein kann; das heißt, daß Teilchen an verschiedenen Orten zu Einer Zeit nicht identisch sein können.
- (Es ist klar, daß das logische Produkt zweier Elementarsätze weder eine Tautologie noch eine Kontradiktion sein kann. **Die Aussage, daß ein Punkt des Gesichtsfeldes zu gleicher Zeit zwei verschiedene Farben hat, ist eine Kontradiktion.)**

- **„This point is green“**
- **„This point is red “**
- **~(G & R): „*logisch* unmöglich“?**

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The colour exclusion problem and its consequences

1) The truth values of elementary propositions are independent of each other.
2) Since colour statements can stand in a relation of mutual exclusion to each other, they cannot be elementary propositions.
3) Since colour statements cannot be elementary propositions, they must be analysable into simpler propositions, and their analysis must eventually yield elementary propositions (that do not exclude each other).
4) If the analysis of colour statements into such elementary propositions (that do not exclude each other) cannot be successfully achieved, we may want to recognize the colour statements themselves as elementary propositions which would imply that we accept elementary propositions which *do* exclude each other.
5) Now, it seems indeed to be the case that colour statements cannot be analysed further into elementary propositions that do *not* exclude each other. Should we therefore just go for (4) and
   a. conceive of the colour statements themselves as elementary propositions,
   b. accept that there are elementary propositions that do exclude each other and thus are *not* independent of each other!?
6) If there are at least some elementary propositions that are not independent from each other, we may just as well through the whole concept of elementary proposition over board!?

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If we no longer have elementary propositions …

→

A whole lot is being thrown over board!

The colour exclusion problem (premiss 1)

- Elementary propositions are logically independent of each other; they cannot exclude each other.
- Therefore mutually exclusive color statements cannot be elementary propositions.
- If they cannot be elementary propositions, they must be further analyzable (TLP #4.211, #6.3751).
- **(1) In the end, the analysis of "*a* is red" and "*a* is green" must each yield elementary propositions that do *not* exclude each other.**

The colour exclusion problem (premiss 2)

- Some color statements are mutually exclusive: "*a* is red" excludes "*a* is green".
- There is only *logical* necessity or impossibility; therefore the analysis must show that the mutual exclusion of '"*a* is red" and "*a* is green"' is of a logical kind.
- The logical product of color statements such as "*a* is red" and "*a* is green" must amount to a *logical contradiction.*
  - *"a* is red & *a* is green" must be a logical contradiction.
- **(2) Logical analysis must show that the logical product of "*a* is red" and "*a* is green" is something like "*a is* red and *a is* not red" – thus a logical contradiction.**

Some Remarks on Logical Form and other writings from 1929-30

- Logical analysis shall show that
  - (1) both "*a* is red" and "*a* is green" can be analyzed into elementary propositions which no longer exclude each other
  - (2) "*a* is red & *a* is green" is a logical contradiction
- *Some Remarks on Logical Form* (SRLF) undertakes the logical analysis of color statements.
- But SRLF does not succeed in showing that
  - (1) color statements are composed of simpler statements (e.g. statements of degree) which would no longer exclude each other.
  - (2) in showing that *"a* is red" and "*a* is green" are mutually exclusive on the basis of logical syntax alone, and thus, in showing that their logical product amounts to a logical contradiction.

Some Remarks on Logical Form and other writings from 1929-30

- If color statements cannot be analyzed further into statements that lead to elementary propositions which are logically independent of each other - are they maybe themselves elementary propositions? We may try to answer Yes. But …
- … if colour statements are elementary propositions, what about the *independency* view of elementary propositions?
  - Then at least some elementary propositions are mutually exclusive and not independent of each other!
- Three Tractatus views are at stake:
  - The *independency* view of elementary propositions
  - The view that elementary propositions are *simple*
  - The view that logic "must take care of itself" (TLP #5.473)
    - We seem to need more than logical necessity / possibility only! On the basis of logical syntax / logical analysis alone we cannot show how color statements can exclude each other!

Conclusions from the colour exclusion problem

- Wittgenstein concludes that color statements such as "*a* is red" and "*a* is green" should be regarded themselves elementary propositions, but then
  - elementary propositions can be mutually exclusive ("*a* is red" and "*a* is green" exclude each other for "phenomen(ologic)al" impossibility)
    - There is no longer only logical impossibility.
  - elementary propositions, states of affairs and facts are no longer independent of each other
  - elementary propositions are no longer *simple*
    - Colour statements can be analyzed further into statements of colour degree, and propositions ascribing degree are not simple.

➢ **The *Tractatus* conception of elementary propositions can just as well be given up!?**

My whole task consists in explaining the nature of the proposition. (NB p. 39, 22.1.1915)

Language (Sprache)	Reality / World (Wirklichkeit / Welt)
complex proposition (zusammengesetzter Satz)	a group of states of affairs (Sachlage?)
elementary proposition (Elementarsatz) [sense]	state of affairs (Sachverhalt)
true elementary proposition (wahrer Elementarsatz) [truth]	fact (Tatsache)
name (einfaches Zeichen, Name) [have reference only in the context of an elementary proposition]	simple object (einfacher Gegenstand)