Michael A.R. Biggs

Editing Wittgenstein's "Notes on Logic"

Vol. 2

Skriftserie fra
Wittgensteinarkivet ved
Universitetet i Bergen
Nr 11, 1996

Working Papers from the
Wittgenstein Archives at
the University of Bergen
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Wittgensteinarkivet ved Universitetet i Bergen

Wittgensteinarkivet er et forskningsprosjekt ved Filosofisk institutt ved Universitetet i Bergen. Prosjektet ble startet 1. Juni 1990. Dets hovedmålsetting er å gjøre Ludwig Wittgensteins etterlatte skrifter (Nachlaß) tilgjengelige for forskning. Wittgensteinarkivet produserer derfor en komplett, maskinleselig versjon av Wittgensteins Nachlaß og utvikler programvare for presentasjon og analyse av tekstene.

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The Wittgenstein Archives at the University of Bergen

The Wittgenstein Archives is a research project at the Department of Philosophy at the University of Bergen, Norway. Established in June 1990, its main purpose is to make Ludwig Wittgenstein's collected writings (Nachlaß) available for research. To serve this purpose, the Wittgenstein Archives is producing a machine-readable transcription of Wittgenstein's Nachlaß, and will supply tools for computer-assisted display and analysis of the textual corpus.

In cooperation with the Wittgenstein Trustees and Oxford University Press, the Nachlaß will be published in electronic form. The entire Nachlaß will first be published as an electronic facsimile on CD-ROM. This facsimile will then be supplemented by transcriptions prepared at the Wittgenstein Archives. In the meantime, visiting scholars to the Wittgenstein Archives have access to the texts and tools prepared by the project as well as to a complete copy of Wittgenstein's Nachlaß.

Working papers from the Wittgenstein Archives at the University of Bergen

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Preface

This monograph was originally conceived as one volume but is here presented as two. The first contains a discussion of, and detailed comparison between, the two published editions of Wittgenstein’s "Notes on Logic". It also contains tables and concordances by which the published editions may be compared with one another, and to four scripts of the work which are still extant. The second volume provides the reader with a typographical representation of each of these four scripts in its entirety and made available in published form for the first time. Examination of these scripts supports the arguments concerning provenance and chronology in the first volume.

The reason for the separation of these two intimately related parts is that the second volume is being simultaneously published as an electronic text, in support of the objectives of The Wittgenstein Archives at the University of Bergen to provide primary texts in this format. Readers may therefore either avail themselves of the numerous possibilities associated with the electronic medium, or use the typographic presentation of the second volume in traditional book form.

Editorial Preface to Volume 2

Parallel to this paper edition, Wittgenstein’s "Notes on Logic" are also being issued in electronic format. This double-release marks the beginning of an ambitious publication plan. In the course of the coming years Wittgenstein’s entire *Nachlaß* will be made available as machine-readable text. Correctly employed, this medium is sufficiently flexible to satisfy the needs of everyone from the casual browser to the most demanding scholar.

The advantages of a well prepared electronic edition are numerous. Text can be presented, either on a computer screen or as paper printout, in a range of different styles, with or without cross-references and footnotes, as a simplified reading version or with all the variant readings and false starts of an author’s original manuscript. The same material can be searched in seconds for occurrences of a particular term or internally compared section for section so as to reveal recurrent phrases, and so on.

The foundation of an electronic edition is invariably a transcription of the source material interspersed with codes which enable a computer to distinguish the numerous textual features. The Wittgenstein Archives at the University of Bergen uses a "mark-up" language (an encoding syntax plus a vocabulary of code names) called MECS-WIT for the transcription of Wittgenstein’s *Nachlaß*. This language was specially developed to handle the features typical of an opus which is largely handwritten and unfinished, such as frequent deletions, later additions, rearrangement of material etc.

The transcription of "Notes on Logic" from which the current publication is derived was originally prepared by Michael Biggs. It was then proof-read by Michael Biggs and Peter Cripps. The transcription of item 201a-1 (McMaster's catalogue no.s RA1.710.057822 and RA1.710.057823) is presented here in two formats: "normalised" and "diplomatic". Items 201a-2 (McMaster's catalogue no. RA1.710.057824) and 201a-3 (referred to in this publication as TS_x), being essentially variants of 201a-1, are presented in "diplomatic" format only.

The electronic version of this publication will be made available on the Internet under the URL **http://www.hd.uib.no/wab/**

"Normalised" Transcription of Item 201a-1

Legend

Bold	Editorial comments and marks
Underlined	Underlined with straight solid line
Doubly underlined	Underlined with two straight solid lines
Doubly underlined and italicised	Underlined with three or more straight solid lines
Italics	Letterspaced
@	Unreadable text
<!>	Editorial instruction or intertextual reference - cf diplomatic version

Bold numbers (**A1**, **A2**,... **B1**, **B2**,...) in the left margin at the beginning of sections (paragraphs) refer to page numbers in the original, - cf diplomatic version. If the page shift occurs inside a section, its position is marked ‖.

Deleted, overwritten and substituted text has been left out, orthographic errors tacitly corrected.

Item 201a-1 (RA1.710.057822 and (RA1.710.057823)

A1 ## Summary

One reason for thinking the old notation wrong is that it is very unlikely that from every proposition `p` an infinite number of other propositions `not-not-p`, `not-not-not-not-p`, etc., should follow.

If only those signs which contain proper names were complex then propositions containing nothing but apparent variables would be simple. Then what about their denials?

The verb of a proposition cannot be „is true" or „is false", but whatever is true or false must already contain the verb.

Deductions only proceed according to the laws of deduction, but these laws cannot justify the deduction.

One reason for supposing that not all propositions which have more than one argument are relational propositions is that if they were, the relations of judgement and inference would have to hold between an arbitrary number of things.

Every proposition which seems to be about a complex can be analysed into a proposition about its constituents and about the proposition which describes the complex perfectly; i.e., that proposition which is equivalent to saying the complex exists.

The idea that propositions are names of complexes suggests that whatever is not a proper name is a sign for a relation.

Because spatial complexes[1] consist of Things and Relations only and the idea of a complex is taken from sp**ace.**

In a proposition convert all its indefinables into variables; there then remains a class of propositions which is not all propositions but a type.

A2 There are thus two ways in which signs are similar. The names Socrates and Plato are similar: they are both names. But whatever they have in common must not be introduced before Socrates and Plato are introduced. The same applies to subject-predicate form etc. Therefore, thing, proposition, subject-predicate form, etc., are not indefinables, i.e., types are not indefinables.

When we say A judges that etc., then we have to mention a whole proposition which A judges. It will not do either to mention only its constituents, or its constituents and form, but not in the proper order. This shows that a proposition itself must occur in the statement that it is judged; however, for instance, „not-p" may be explained. The question „What is negated" must have a meaning**.**

To understand a proposition p it is not enough to know that p implies ‚„p" is true', but we must also know that ~p implies „p is false". This shows the bipolarity of the proposition. W-F = Wahr-Falsch

To every molecular function a WF scheme corresponds. Therefore we may use the WF scheme itself instead of the function. Now what the WF scheme does is, it correlates the letters W and F with each proposition. These two letters are the poles of atomic propositions. Then the scheme correlates another W and F to these poles. In this notation all that matters is the

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1 you — for instance imagine every fact as a spatial complex.

correlation of the outside poles to the pole of the atomic propositions. Therefore not-not-p is the same symbol as p. And therefore we shall never get two symbols for the same molecular function.

A3 The meaning of a proposition is the fact which actually corresponds to it.

As the ab functions of atomic propositions are bi-polar propositions again we can perform ab operations on them. We shall, by doing so, correlate two new outside poles via the old outside poles to the poles of the atomic propositions.

The symbolising fact in a–p–b is that, say[1] a is on the left of p and b on the right of p; then the correlation of new poles is to be transitive, so that for instance if a new pole a in whatever way i.e. via whatever poles is correlated to the inside a, the symbol is not changed thereby. It is therefore possible to construct all possible ab functions by performing one ab operation repeatedly, and we can therefore talk of all ab functions as of all those functions which can be obtained by performing this ab operation repeatedly.

[Note by Bertrand Russell]

[NB. ab means the same as WF, which means true-false.]

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1 This is quite arbitrary but if we such have fixed on which sides the poles have to stand we must of course stick to our convention. If for instance „apb" says p then bpa says nothing. (It does not say ~p.) But a–apb–b is the same symbol as apb the ab function vanishes automatically for here the new poles are related to the same side of p as the old ones. The question is always: how are the new poles correlated to p compared with the way the old poles are correlated to p.

Naming is like pointing. A function is like a line dividing points of a plane into right and left ones; then „p or not-p" has no meaning because it does not divide the plane.

But though a particular proposition „p or not-p" has no meaning, a general proposition „for all p's, p or not-p" has a meaning because this does not contain the nonsensical function „p or not-p" but the function „p or not-q" just as „for all x's xRx" contains the function „xRy".

A4 A proposition is a standard to which all facts behave, with names it is otherwise; it is thus bi-polarity and sense comes in; just as one arrow behaves to another arrow by being in the same sense or the opposite, so a fact behaves to a proposition.

The form of a proposition has meaning in the following way. Consider a symbol „xRy". To symbols of this form correspond couples of things whose names are respectively „x" and „y". The things **x y** stand to one another in all sorts of relations, amongst others some stand in the relation R, and some not; just as I single out a particular thing by a particular name I single out all behaviours of the points x and y with respect to the relation R. I say that if an x stands in the relation R to a y the sign „x R y" is to be called true to the fact and otherwise false. This is a definition of sense.

In my theory p has the same meaning as not-p but opposite s**en**se. The meaning is the fact. The proper theory of judgment must make it impossible to judge nonsense.

It is not strictly true to say that we understand a proposition p if we know that p is equivalent to „p is true" for this would be the case if accidentally both were true or false. What is wanted is the formal equivalence with respect to the forms of the proposition, i.e., all the general indefinables involved. The sense of an ab function of a proposition is a function of its sense. There are only unasserted propositions. ‖ Assertion is merely psychological. In not-p, p is exactly the same as if it stands alone; this point is absolutely fundamental. Among the facts which make „p or q" true there are also facts which make „p and q" true; if propositions have only meaning, we ought, in such a case, to say that these two propositions are identical, but in fact, their sense is different for we have introduced sense by talking of all p's and all q's. Consequently the molecular propositions will only be used in cases where there ab function stands under a generality sign or enters into another function such as „I believe that, etc.," because then the sense enters.

A5 In „a judges p" p cannot be replaced by a proper name. This appears if we substitute „a judges that p is true and not p is false". The proposition „a judges p" consists of the proper name a, the proposition p with its 2 poles, and a being related to both of these poles in a certain way. This is obviously not a relation in the ordinary sense.

The ab notation makes it clear that not and or are dependent on one another and we can therefore not use them as simultaneous indefinables. <!> Same objections in the case of apparent variables to old indefinables, as in the case of molecular functions: The application of the ab notation to apparent-variable propositions becomes clear if we consider that, for instance, the proposition „for all x, φx" is to be true when φx is true for all x's and false when φx is false for some x's. We see that some and all occur simultaneously in the proper apparent variable notation.

A6 The Notation is:

for $(x)\ \varphi x$; $a - (x) - a\ \varphi\ x\ b - (\exists\ x) - b$

and

for $(\exists x)\ \varphi\ x$: $a - (\exists x) - a\ \varphi\ x\ b - (x) - b$

Old definitions now become tautologous.

In aRb it is not the complex that symbolises but the fact that the symbol a stands in a certain relation to the symbol b. Thus facts are symbolised by facts, or more correctly: that a certain thing is the case in the symbol says that a certain thing is the case in the world.

Judgment, question and command are all on the same level. What interests logic in them is only the unasserted proposition. Facts cannot be named.

A proposition cannot occur in itself. This is the fundamental truth of the theory of types.

Every proposition that says something about one thing is a subject-predicate proposition, and so on.

Therefore we can recognize a subject-predicate proposition if we know it contains only one name and one form, etc. This gives the construction of types. Hence the type of a proposition can be recognized by its symbol alone.

A7 What is essential in a correct apparent-variable notation is this:— (1) it must mention a type of propositions; (2) it must show which components of a proposition of this type are constants.

[Components are forms and constituents.]

Take $(\varphi).\varphi!x$. Then if we describe the kind of symbols, for which $\varphi!$ stands and which, by the above, is enough to determine the type, then automatically „$(\varphi).\varphi!x$” cannot be fitted by this description, because it *contains* „$\varphi!x$” and the description is to describe all that symbolizes in symbols of the $\varphi!$ — kind. If the description is thus complete vicious circles can just as little occur as for instance in $(\varphi).(x)_\varphi$ (where $(x)_\varphi$ is a subject-predicate proposition).

B1

First MS.

Indefinables are of two sorts: names, and forms. Propositions cannot consist of names alone; they cannot be classes of names. A name can not only occur in two different propositions, but can occur in the same way in both.

Propositions [which are symbols having reference to facts] are themselves facts: that this inkpot is on this table may express that I sit in this chair.

It can never express the common characteristic of two objects that we designate them by the same name but by two different ways of designation, for, since names are arbitrary, we might also choose different names, and where then would be the common element in the designations? Nevertheless one is always tempted, in a difficulty, to take refuge in different ways of designation.

Frege said „propositions are names”; Russell said „propositions correspond to complexes”. Both are false; and especially false is the statement „propositions are names of complexes”.

It is easy to suppose that only such symbols are complex as contain names of objects, and that accordingly „$(\exists x,\varphi).\varphi x$” or „$(\exists x,R,y).xRy$” must be simple. It is then natural to call the first of these the name of a form, the second the name of a relation. But in that case what is the meaning of (e.g.) „~(∃x,y).xRy”? Can we put „not” before a name?

B2 The reason why „~Socrates” means nothing is that „~x” does not express a property of *x*.

There are positive and negative facts: if the proposition „this rose is not red” is true, then what it signifies is negative. But the occurrence of the word „not” does not indicate this unless we know that the signification of the proposition „this rose is red” (when it is true) is positive. It is only from both, the negation and the negated proposition, that we can conclude to a characteristic of the significance of the whole proposition. (We are not here speaking of negations of *general* propositions, i.e. of such as contain apparent variables. Negative facts only justify the negations of atomic propositions.)

Positive and *negative* facts there are, but not *true* and *false* facts.

If we overlook the fact that propositions have a *sense* which is independent of their truth or falsehood, it easily seems as if true and false were two equally justified relations between the sign and what is signified. (We might then say e.g. that „*q*” *signifies* in the true way what „not-q” *signifies* in the false way). But are not true and false in fact equally justified? Could we not express ourselves by means of false propositions just as well as hitherto with true ones, so long as we know that they are meant falsely? ‖ No! For a proposition is then true when it is as we assert in this proposition; and accordingly if by „*q*” we mean „not-*q*”, and it is as we mean to assert, then in the new interpretation „*q*” is actually true and *not* false. But it is important that we *can* mean the same by „*q*” as by „not-*q*”, for it shows that neither to the symbol „not” nor to the manner of its combination with „q" does a characteristic of the denotation of „q" correspond.

B4

2nd MS.

We must be able to understand propositions which we have never heard before. But every proposition is a new symbol. Hence we must have general indefinable symbols; these are unavoidable if propositions are not all indefinable.

Whatever corresponds in reality to compound propositions must not be more than what corresponds to their several atomic propositions.

Not only must logic not deal with [particular] things, but just as little with relations and predicates.

There are no propositions containing real variables.

What corresponds in reality to a proposition depends upon whether it is true or false. But we must be able to understand a proposition without knowing if it is true or false.

What we know when we understand a proposition is this: We know what is the case if the proposition is true, and what is the case if it is false. But we do not know [necessarily] whether it is true or false.

Propositions are not names.

We can never distinguish one logical type from another by attributing a property to members of the one which we deny to members of the other.

Symbols are not what they seem to be. In „aRb", „R" looks like a substantive, but is not one. What symbolizes in „aRb" is that R occurs between a and b. Hence „R" is not the indefinable in „aRb". Similarly in „φx", „φ" looks like a substantive but is not one; in „~p", „~" looks like „φ" but is not like it. This is the first thing that indicates that there may not be logical constants.

A reason against them is the generality of logic: logic cannot treat a special set of things.

B5 Molecular propositions contain nothing beyond what is contained in their atoms; they add no material information above that contained in their atoms.

All that is essential about molecular functions is their T-F schema [i.e. the statement of the cases when they are true and the cases when they are false].

Alternative indefinability shows that the indefinables have not been reached.

Every proposition is essentially true-false: to understand it, we must know both what must be the case if it is true, and what must be the case if it is false. Thus a proposition has two <u>poles</u>, corresponding to the case of its truth and the case of its falsehood. We call this the <u>sense</u> of a proposition.

In regard to notation, it is important to note that not every feature of a symbol symbolizes. In two molecular functions which have the same T-F schema, what symbolizes must be the same. In „not-not-<u>p</u>”, „not-<u>p</u>” does not occur; for „not-not-<u>p</u>” is the same as ”<u>p</u>”, and therefore, if „not-p” occurred in „not-not-<u>p</u>”, it would occur in „<u>p</u>”.

Logical indefinables cannot be predicates or relations, because propositions, owing to sense, cannot have predicates or relations. Nor are „not” and „or”, like judgment, <u>analogous</u> to predicates or relations, because they do not introduce anything new.

Propositions are always complex even if they contain no names.

B6 A proposition must be understood when <u>all</u> its indefinables are understood. The indefinables in „<u>a</u>R<u>b</u>” are introduced as follows:

„a” is indefinable;

„b” is indefinable;

Whatever „x” and „y” may mean, „xRy” says something indefinable about their meanings.

A complex symbol must never be introduced as a single indefinable. (Thus e.g. no proposition is indefinable.) For if one of its parts occurs also in another connection, it must there be re-introduced. And would it then mean the same?

The ways by which we introduce our indefinables must permit us to construct all propositions that have sense from these indefinables alone. It is easy to introduce „all” and „some” in a way that will make the construction of (say) „(x,y).xRy” possible from „all” and „xRy” as introduced before.

B7

3rd MS.

An analogy for the theory of truth: Consider a black patch on white paper; then we can describe the form of the patch by mentioning, for each point of the surface, whether it is white or black. To the fact that a point is black corresponds a positive fact, to the fact that a point is white (not black) corresponds a negative fact. If I designate a point of the surface (one of Frege’s „truth-values”), this is as if I set up an assumption to be decided upon. But in order to be able to say of a point that it is black or that it is white, I must first know when a point is to be called black and when it is to be called white. In order to be able to say that „p” is true (or false), I must first have determined under what circumstances I call a proposition true, and thereby I determine the sense of a proposition. The point in which the analogy fails is this: I can indicate a point of the paper that is white and black, but to a proposition without sense nothing corresponds, for it does not designate a thing (truth-value), whose properties might be called „false" or „true"; the verb of a proposition is not „is true" or „is false", as Frege believes, but what is true must already contain the verb.

The comparison of language and reality is like that of retinal image and visual image: to the blind spot nothing in the visual image seems to correspond, and thereby the boundaries of the blind spot determine the visual image — as true negations of atomic propositions determine reality.

B8 Logical inferences can, it is true, be made in accordance with Frege's or Russell's laws of deduction, but this cannot justify the inference; and therefore they are not primitive propositions of logic. If p follows from q, it can also be inferred from q, and the „manner of deduction" is indifferent.

Those symbols which are called propositions in which „variables occur" are in reality not propositions at all, but only schemes of propositions, which only become propositions when we replace the variables by constants. There is no proposition which is expressed by „$x = x$", for „x" has no signification; but there is a proposition „$(x).x = x$" and propositions such as „Socrates = Socrates" etc.

In books on logic, no variables ought to occur, but only the general propositions which justify the use of variables. It follows that the so-called definitions of logic are not definitions, but only schemes of definitions, and instead of these we ought to put general propositions; and similarly the so-called primitive ideas (Urzeichen) of logic are not primitive ideas, but the schemes of them. The mistaken idea that there are things called facts or complexes and relations easily leads to the opinion that there must be a relation of questioning to the facts, and then the question arises whether a relation can hold between an arbitrary number of things, since a fact can follow from arbitrary cases.

It is a fact that the proposition which e.g. expresses that q follows from p and $p \supset q$ is this: $p . p \supset q . \supset_{p.q} . q$.

B9 At a pinch, one is tempted to interpret „not-p" as „everything else, only not p". That from a single fact p an infinity of others, not-not-p etc., follow, is hardly credible. Man possesses an innate capacity for constructing symbols with which some sense can be expressed, without having the slightest idea what each word signifies. The best example of this is mathematics, for man has until lately used the symbols for numbers without knowing what they signify or that they signify nothing.

Russell's „complexes" were to have the useful property of being compounded, and were to combine with this the agreeable property that they could be treated like „simples". But this alone made them unserviceable as logical types, since there would have been significance in asserting, of a simple, that it was complex. But a property cannot be a logical type.

Every statement about apparent complexes can be resolved into the logical sum of a statement about the constituents and a statement about the proposition which describes the complex completely. How, in each case, the resolution is to be made, is an important question, but its answer is not unconditionally necessary for the construction of logic.

B10 That „or" and „not" etc. are not relations in the same sense as „right" and „left" etc., is obvious to the plain man. The possibility of cross-definitions in the old logical indefinables shows, of itself, that these are not the right indefinables, and, even more conclusively, that they do not denote relations.

If we change a constituent a of a proposition $\varphi(a)$ into a variable, then there is a class

$$\tilde{p} \{ (\exists x) . \varphi(x) = p \}.$$

This class in general still depends upon what, by an arbitrary convention, we mean by „φ(x)”. But if we change into variables all those symbols whose significance was arbitrarily determined, there is still such a class. But this is now not dependent upon any convention, but only upon the nature of the symbol „φ(x)”. It corresponds to a logical type.

Types can never be distinguished from each other by saying (as is often done) that one has these but the other has that properties, for this presupposes that there is a meaning in asserting all these properties of both types. But from this it follows that, at best, these properties may be types, but certainly not the objects of which they are asserted.

B11 At a pinch, we are always inclined to explanations of logical functions of propositions which aim at introducing into the function either only the constituents of these propositions, or only their forms, etc. etc.; and we overlook that ordinary language would not contain the whole propositions if it did not need them: However, e.g., „not-p” may be explained, there must always be a meaning given to the question „what is denied?”

The very possibility of Frege's explanations of „not-p” and „if p then q”, from which it follows that not-not-p denotes the same as p, makes it probable that there is some method of designation in which „not-not-p” corresponds to the same symbol as „p”. But if this method of designation suffices for logic, it must be the right one.

Names are points, propositions arrows — they have sense. The sense of a proposition is determined by the two poles true and false. The form of a proposition is like a straight line, which divides all points of a plane into right and left. The line does this automatically, the form of proposition only by convention.

B12 Just as little as we are concerned, in logic, with the relation of a name to its meaning, just so little are we concerned with the relation of a proposition to reality, but we want to know the meaning of names and the sense of propositions — as we introduce an indefinable concept „A" by saying: „‚A' denotes something indefinable", so we introduce e.g. the form of propositions $\underline{a}R\underline{b}$ by saying: „For all meanings of „x" and „y", „xRy" expresses something indefinable about $\underline{x}$ and $\underline{y}$".

In place of every proposition „p", let us write „$^{a}_{b}p$". Let every correlation of propositions to each other or of names to propositions be effected by a correlation of their poles „a" and „b". Let this correlation be transitive. Then accordingly „$^{a-a}_{b-b}p$" is the same symbol as „$^{a}_{b}p$". Let $\underline{n}$ propositions be given. I then call a „class of poles" of these propositions every class of $\underline{n}$ members, of which each is a pole of one of the $\underline{n}$ propositions, so that one member corresponds to each proposition. I then correlate with each class of poles one of two poles ($\underline{a}$ and $\underline{b}$). The sense of the symbolizing fact thus constructed I cannot define, but I know it.

If p = not-not-p etc., this shows that the traditional method of symbolism is wrong, since it allows a plurality of symbols with the same sense; and thence it follows that, in analyzing such propositions, we must not be guided by Russell's method of symbolizing.

B13 It is to be remembered that names are not things, but classes: „A" is the same letter as „A". This has the most important consequences for every symbolic language.

Neither the sense nor the meaning of a proposition is a thing. These words are incomplete symbols.

It is impossible to dispense with propositions in which the same argument occurs in different positions. It is obviously useless to replace $\varphi(a,a)$ by $\varphi(a,b).a = b$.

Since the ab-functions of p are again bi-polar propositions, we can form ab-functions of them, and so on. In this way a series of propositions will arise, in which in general the symbolizing facts will be the same in several members. If now we find an ab-function of such a kind that by repeated application of it every ab-function can be generated, then we can introduce the totality of ab-functions as the totality of those that are generated by application of this function. Such a function is $\sim p \vee \sim q$.

B14 It is easy to suppose a contradiction in the fact that on the one hand every possible complex proposition is a simple ab-function of simple propositions, and that on the other hand the repeated application of one ab-function suffices to generate all these propositions. If e.g. an affirmation can be generated by double negation, is negation in any sense contained in affirmation? Does „p” deny „not-p” or assert „p”, or both? And how do matters stand with the definition of „$\supset$” by „$\vee$” and „$\cdot$ ”, or of „$\vee$” by „$\cdot$ ” and „$\supset$”? And how e.g. shall we introduce $p|q$ (i.e. $\sim p \vee \sim q$), if not by saying that this expression says something indefinable about all arguments p and q? But the ab-functions must be introduced as follows: The function $p|q$ is merely a mechanical instrument for constructing all possible symbols of ab-functions. The symbols arising by repeated application of the symbol „|” do not contain the symbol „$p|q$”. We need a rule according to which we can form all symbols of ab-functions, in order to be able to speak of the class of them; and we now speak of them e.g. as those symbols of functions which can be generated by repeated application of the operation „|”. And we say now: For all p's and q's, „$p|q$” says something indefinable about the sense of those simple propositions which are contained in p and q.

B15 The assertion-sign is logically quite without significance. It only shows, in Frege and Whitehead and Russell, that these authors hold the propositions so indicated to be true. „⊢" therefore belongs as little to the proposition as (say) the number of the proposition. A proposition cannot possibly assert of itself that it is true.

Every right theory of judgment must make it impossible for me to judge that this table penholders the book. Russell's theory does not satisfy this requirement.

It is clear that we understand propositions without knowing whether they are true or false. But we can only know the meaning of a proposition when we know if it is true or false. What we understand is the sense of the proposition.

The assumption of the existence of logical objects makes it appear remarkable that in the sciences propositions of the form „$p \vee q$", „$p \supset q$", etc. are only then not provisional when „$\vee$" and "$\supset$" stand within the scope of a generality-sign [apparent variable].

B16 4th MS.

If we formed all possible atomic propositions, the world would be completely described if we declared the truth or falsehood of each. [I doubt this.]

The chief characteristic of my theory is that, in it, p has the same meaning as not-p.

A false theory of relations makes it easily seem as if the relation of fact and constituent were the same as that of fact and fact which follows from it. But the similarity of the two may be expressed thus: $\varphi a . \supset ._{\varphi, a} a = a$.

If a word creates a world so that in it the principles of logic are true, it thereby creates a world in which the whole of mathematics holds; and similarly it could not create a world in which a proposition was true, without creating its constituents.

Signs of the form „`p∨~p`" are senseless, but not the proposition „`(p).p ∨ ~p`". If I know that this rose is either red or not red, I know nothing. The same holds of all ab-functions.

To understand a proposition means to know what is the case if it is true. Hence we can understand it without knowing if it is true. We understand it when we understand its constituents and forms. If we know the meaning of „a" and „b", and if we know what „`xRy`" means for all x's and y's, then we also understand „`aRb`".

I understand the proposition „`aRb`" when I know that either the fact that `aRb` or the fact that `not aRb` corresponds to it; but this is not to be confused with the false opinion that I understand „`aRb`" when I know that „`aRb` or `not-aRb`" is the case.

B17 But the form of a proposition symbolizes in the following way: Let us consider symbols of the form „`xRy`"; to these correspond primarily pairs of objects, of which one has the name „x", the other the name „y". The x's and y's stand in various relations to each other, among others the relation R holds between some, but not between others. I now determine the sense of „`xRy`" by laying down: when the facts behave in regard to „`xRy`" so that the meaning of „x" stands in the relation R to the meaning of „y", then I say that they [the facts] are „of like sense" [„gleichsinnig"] with the proposition „`xRy`"; otherwise, „of opposite sense" [**„**entgegengesetzt"]; I correlate the facts to the symbol „`xRy`" by thus dividing them into those of like sense and those of opposite sense. To this correlation corresponds the correlation of name and meaning. Both are psychological. Thus I understand the form „`xRy`" when I know that it discriminates the behaviour of x and y according as these stand in the relation R or not. In this way I extract from all possible relations the relation R, as, by a name, I extract its meaning from among all possible things.

Strictly speaking, it is incorrect to say: We understand the proposition p when we know that ‚„p” is true’ ≡ p; for this would naturally always be the case if accidentally the propositions to right and left of the symbol „≡” were both true or both false. We require not only an equivalence, but a formal equivalence, which is bound up with the introduction of the form of p.

The sense of an ab-function of p is a function of the sense of p.

B18 The ab-functions use the discrimination of facts, which their arguments bring forth, in order to generate new discriminations.

Only facts can express sense, a class of names cannot. This is easily shown.

There is no thing which is the form of a proposition, and no name which is the name of a form. Accordingly we can also not say that a relation which in certain cases holds between things holds sometimes between forms and things. This goes against Russell's theory of judgment.

It is very easy to forget that, tho. the propositions of a form can be either true or false, each one of these propositions can only be either true or false, not both.

Among the facts which make „p or q” true, there are some which make „p and q” true; but the class which makes „p or q” true is different from the class which makes „p and q” true; and only this is what matters. For we introduce this class, as it were, when we introduce ab-functions.

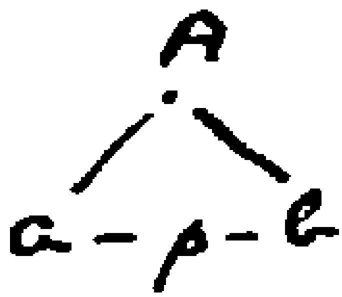
A very natural objection to the way in which I have introduced e.g. propositions of the form xRy is that by it propositions such as (∃x,y).xRy and similar ones are not explained, which yet obviously have in common with aRb what cRd has in common with aRb. But when we introduced propositions of the form xRy we mentioned no one particular proposition of this form; and we only need to introduce $(\exists x,y).\varphi(x,y)$ for all φ's in any way which makes the sense of these propositions dependent on the sense of all propositions of the form $\varphi(a,b)$, and thereby the justification of our procedure is proved.

B19 The indefinables of logic must be independent of each other. If an indefinable is introduced, it must be introduced in all combinations in which it can occur. We cannot therefore introduce it first for one combination, then for another; e.g., if the form xRy has been introduced, it must henceforth be understood in propositions of the form aRb just in the same way as in propositions such as $(\exists x,y).\ xRy$ and others. We must not introduce it first for one class of cases, then for the other; for it would remain doubtful if its meaning was the same in both cases, and there would be no ground for using the same manner of combining symbols in both cases. In short, for the introduction of indefinable symbols and combinations of symbols the same holds, mutatis mutandis, that Frege has said for the introduction of symbols by definitions.

It is a priori likely that the introduction of atomic propositions is fundamental for the understanding of all other kinds of propositions. In fact the understanding of general propositions obviously depends on that of atomic propositions.

Cross-definability in the realm of general propositions leads to the quite similar questions to those in the realm of ab-functions.

B20 When we say „A believes p", this sounds, it is true, as if here we could substitute a proper name for „p"; but we can see that here a sense, not a meaning, is concerned, if we say „A believes that ‚p' is true"; and in order to make the direction of p even more explicit, we might say „A believes that ‚p' is true and ‚not-p' is false". Here the bi-polarity of p is expressed, and it seems that we shall only be able to express the proposition „A believes p" correctly by the ab-notation; say by making „A" have a relation to the poles „a" and „b" of a-p-b.



The epistemological questions concerning the nature of judgment and belief cannot be solved without a correct apprehension of the form of the proposition.

The ab-notation shows the dependence of or and not, and thereby that they are not to be employed as simultaneous indefinables.

Not: „The complex sign ‚aRb'" says that a stands in the relation R to b; but that ‚a' stands in a certain relation to ‚b' says that aRb.

In philosophy there are no deductions: it is purely descriptive.

Philosophy gives no pictures of reality.

Philosophy can neither confirm nor confute scientific investigation.

B21 Philosophy consists of logic and metaphysics: logic is its basis.

Epistemology is the philosophy of psychology.

Distrust of grammar is the first requisite for philosophizing.

Propositions can never be indefinables, for they are always complex. That also words like „ambulo” are complex appears in the fact that their root with a different termination gives a different sense.

Only the doctrine of general indefinables permits us to understand the nature of functions. Neglect of this doctrine leads to an impenetrable thicket.

Philosophy is the doctrine of the logical form of scientific propositions (not only of primitive propositions).

The word „philosophy” ought always to designate something over or under, but not beside, the natural sciences.

Judgment, command and question all stand on the same level; but all have in common the propositional form, which does interests us.

The structure of the proposition must be recognized, the rest comes of itself. But ordinary language conceals the structure of the proposition: in it, relations look like predicates, predicates like names, etc.

Facts cannot be named.

B22 It is easy to suppose that „individual”, „particular”, „complex” etc. are primitive ideas of logic. Russell e.g. says „individual” and „matrix” are „primitive ideas”. This error presumably is to be explained by the fact that, by employment of variables instead of the generality-sign, it comes to seem as if logic dealt with things which have been deprived of all properties except thing-hood, and with propositions deprived of all properties except complexity. We forget that the indefinables of symbols [Urbilder von Zeichen] only occur under the generality-sign, never outside it.

Just as people used to struggle to bring all propositions into the subject-predicate form, so now it is natural to conceive every proposition as expressing a relation, which is just as incorrect. What is justified in this desire is fully satisfied by Russell's theory of manufactured relations.

One of the most natural attempts at solution consists in regarding „not-p" as „the opposite of p", where then „opposite" would be the indefinable relation. But it is easy to see that every such attempt to replace the ab-functions by descriptions must fail.

B23 The false assumption that propositions are names leads us to believe that there must be logical objects: for the meanings of logical propositions will have to be such things.

A correct explanation of logical propositions must give them a unique position as against all other propositions.

No proposition can say anything about itself, because the symbol of the proposition cannot be contained in itself; this must be the basis of the theory of logical types.

Every proposition which says something indefinable about a thing is a subject-predicate proposition; every proposition which says something indefinable about two things expresses a dual relation between these things, and so on. Thus every proposition which contains only one name and one indefinable form is a subject-predicate proposition, and so on. An indefinable simple symbol can only be a name, and therefore we can know, by the symbol of an atomic proposition, whether it is a subject-predicate proposition.

B24 I. Bi-polarity of propositions: sense and meaning, truth and falsehood.

II. Analysis of atomic propositions: general indefinables, predicates, etc.

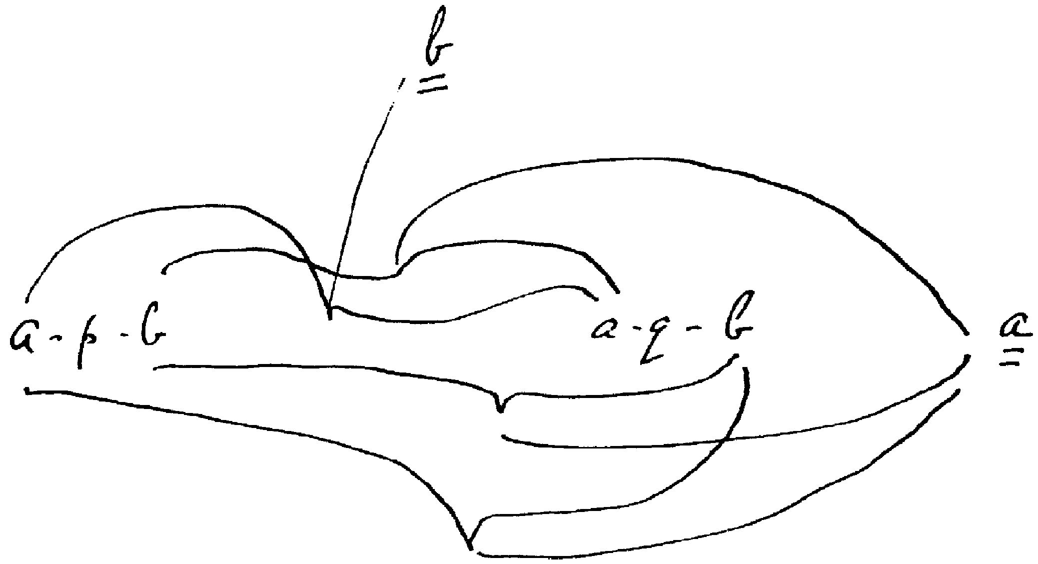
III. Analysis of molecular propositions: ab-functions.

IV. Analysis of general propositions.

V. Principles of symbolism: what symbolizes in a symbol. Facts for facts.

VI. Types.

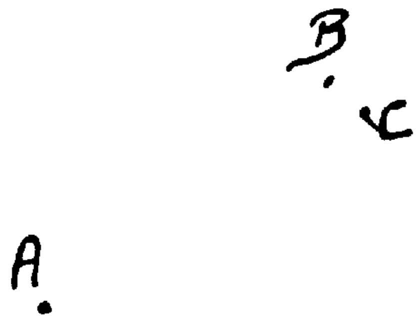
B25

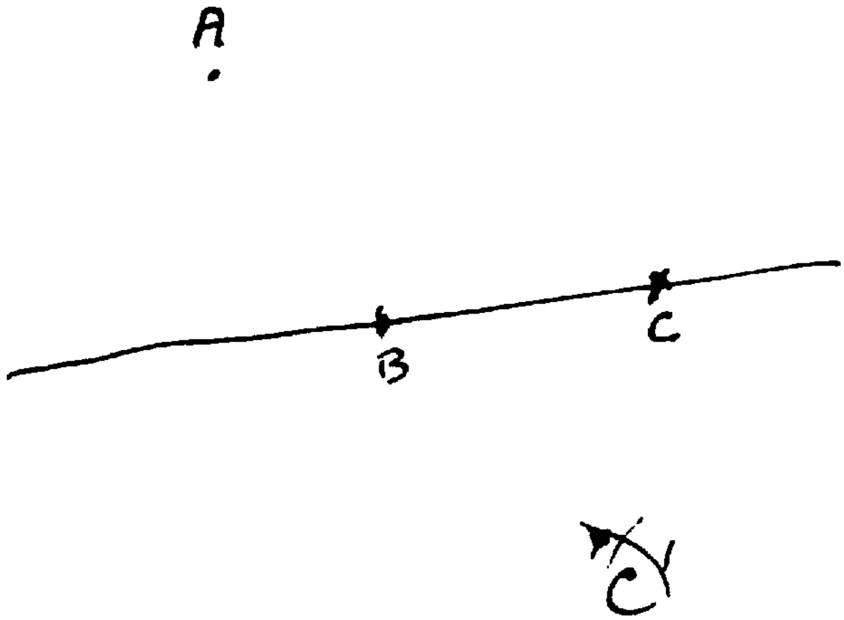


This is the symbol for

~p ∨ ~q

B26





"Diplomatic" Transcriptions

Legend

Bold	Editorial comments and marks
Underlined	Underlined with straight solid line
Doubly underlined	Underlined with two straight solid lines
Doubly underlined and italicised	Underlined with three or more straight solid lines
Outlined	Underlined with wavy, broken (dashed) or dotted line(s)
Redline	Deleted
[Redline|Shadow**]**	**[**Overwritten|overwriting**]**
⌈ ⌉	Marked insertion above line
⌊ ⌋	Marked insertion below line
< >	Insertion in line
〈 〉	Text in margin
⊏ ⊐	Relocated text or change of order
@	Unreadable text

Item 201a-1 (RA1.710.057822 and RA1.710.057823)

Transcriber(s): Michael Biggs, Peter Cripps
Proofreader(s): Peter Cripps
Comments:
Transcriptions of material from The Bertrand Russell Archives, McMaster University, Canada; their catalogue no.s:
TS section (pp. numbered 1-7) - RA1.710.057822,
MS section (pp. numbered 1-23 plus 3 unnumbered folios) - RA1.710.057823.
This item consists of two parts: a typescript of 7ff. with corrections in Russell's hand and additions in Wittgenstein's hand, and a manuscript of 26ff. in Russell's hand. In the TS part: non-typewriter characters are inserted by hand; fully typed pages have 24 or 25 lines of type; each new sentence is separated from the last by three spaces. True double spaced blank lines used occasionally but not consistently.
Hands:
s = handwritten insertions that belong to first pass, ie generally the insertion of symbols which are not found on the typewriter such as φ and ∃,
H5 = Ludwig Wittgenstein, or cases where the transcriber has felt unable to decide whether Ludwig Wittgenstein or Bertrand Russell
S1 = Bertrand Russell

<Summary>[1]

The[2] One[1] reason for thinking the old notation wrong is that it is very unlikely that from every proposition p an infinite number of other propositions not-not-p, not-not-not-not-p, etc., should follow.

If only th[e|o[3]]se signs which contain proper names were complex then propositions containing nothing but apparent variables would be simple. Then what about their denials?

The verb of a proposition cannot be ”is true” or ”is false”, but whatever is true or false must already contain the verb.

The[2] [d|D[3]]eductions only proceed according to the laws of deduction<,>[4] but these laws cannot justify the deduction.

The[2] One[1] reason for supposing that not all propositions which have more than one argument are relational propositions is that ⌜if⌝[1] they were<,>[1] the relations of judgement and inference that[2] would[1] have to hold between an arbitrary number of things.

Every proposition which seems to be about a complex can be analysed into a proposition about those[2] ⌜its⌝[1] cons[i|t]ituents and about the proposition which describes a[2] the[1] complex perfectly; i.e., that proposition which is equivalent to saying a[2] the[1] complex exists.

The idea that propositions are names of complexes between[5] L.W.[4] suggestions[5]<s>[4] 〈?〉[6] L.W.[4] that whatever is not a proper name is a sign for a relation. <Because spatial complexes* consist of Things & Relations only & the idea of a complex is taken from sp>[4]

In a proposition convert all its indefinables into variables; there then remains a class of propositions which has[2] is[1] not all propositions but a type.

⟨* <u>you</u>[7] - for instance imagine every fact as a spatial complex⟩[8]

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2

There are thus two ways in which signs are similar. The names Socrates and Plato are similar: they are both names. But whatever they have in common must not be introduced before Socrates and Plato are introduced. The same applies to subject-predicate form etc. Therefore, thing, proposition, subject-predicate form, etc., are not indefinables, <u>i.e</u>., types are not indefinables.

When we say a[2] A[1] judge<s>[1] is[2] that etc., then we have to mention a whole proposition which a[2] A[1] judge<s>[1] is[2]. It will not do either to mention only its constituent<s,>[1] or its constituent<s>[1] and form, but not in the proper order. This shows that a proposition itself must occur in the statement that it is judged; however, for instance, ”not-p” may be explained[.|,[9]] <<u>p</u>[10] must occur in it.>[1 5] ⟨[t|𝕋[9]]he question, „What is negated” must have a meaning⟩[11]

Always a que[x|𝕤]tion that is negated must have a meaning.[5] ⟨Rott!⟩[12]

To understand a proposition p it is not enough to know that ”[5]p implies <’”>[4]p” is true<’>[4], but we must also know that p also implies <’>[4]”not-p” is false<’>[4 5] ~p implies ”p is false”[4]. This shows the ┌bi┐[13]polarity of the proposition.

⟨W-F = Wahr-Falsch⟩[6]

To every molecular function ┌a┐[1] [wf|𝕎𝔽[3]] scheme corresponds. Therefore we may use the [wf|𝕎𝔽[3]] scheme itself instead of the function. Now what the [wf|𝕎𝔽[3]] scheme does is, it correlates the letters [w|𝕎[3]] and [f|𝔽[3]] with each proposition. These two letters are the poles of atomic propositions. Then ┌the scheme┐[1] corresponds[2]lates[1] another [f|𝕎[3]] and [w|𝔽[3]] to these poles. In this notation all that it[2] matters is the correlation of the outside poles to the pole of its[2] ┌the┐[1] atomic propositions. Therefore not-not-p is the same symbol as p. <And>[1] Therefore we shall never get two symbols for the same molecular functions[5].

The meaning of a proposition is the fact which actually corresponds to it.

As the ab functions of atomic propositions are by[2]i[1]<->[1]polar propositions again we can perform ab[7] operations on them. We [wi|sha[9]]ll<,>[4] b[e|y[9]] doing so, correlate two new outside poles via the old outside poles to the poles of the atomic propositions.

The symbolising fact in a-p-b is that, say[7]* a[7] is on the left of p[7] and b[7] on the right of p[7][,|;[9]] then the correlation of new poles is to be transitive, such[2] so[1] that ┌for instance┐[4] if a new pole a[7] in whatever way ┌i.e. via whatever poles┐[4] is correlated to the inside a[7], the symbol is not changed thereby. It is therefore possible to construct all possible ab[7] functions by performing one ab[7] operation repeatedly, and we can therefore talk of all ab[7] functions as of all th[e|o[9]]se functions which can be obtained by performing this ab[7] operation repeatedly.

⟨[Note by B.R.]⟩[6]

[NB. ab[7] means the same as [wf|WF[3]], which means true-false.]

Naming is like pointing. A function is like a line dividing points ┌of a plane┐[4] into right and left ones; then <”>[1]p or not-p<”>[1] has no meaning because it does not divide [a|the[9]] plane.

But though a particular proposition ”p”[5] or a ”[5]not-p” has no meaning<,>[4] a general proposition <”>[4]for all p’s, ”[5]p”[5] or ”[5]not-p” has a meaning because this does not contain [a|the[9]] nonsensical function <”>[4]p [n|o]r not-p<”>[4] but [a|the[9]] function <”>[4]p or ”[5]not-q” just as <”>[4]for all ”[5]x’s xRx<”>[1] contains the function <”>[4]xRy”.

⟨* This is quite arbitrary but if we such have fixed on which sides the poles have to stand we

must of course stick to our convention. If for instance „apb" says p then bpa says <u>nothing</u>[7]. (It does not say ~p) But a-apb-b is the same symbol as apb⟩[8]
⟨the ab function vanishes automatically) for here the new poles are⟩[14] ⟨related to the same side of p as the old ones. The question is allways: how are the new poles correlated to p compared with the way the old poles are correlated to p.⟩[15]

A proposition is a standard to which all facts behave, that[2] with[1] names it ┌is┐[1] otherwise; it is then[2]us[1] by[2]i[1]<->[1]polarity and sense comes in<;>[1] just as one error[2] arrow[1] behaves to another error[2] arrow[1] by being in the same sense or the opposite, so a fact behaves to a proposition.

The form of a proposition has meaning in the following way. Consider a symbol <”>[4]xRy<”>[4]. To symbols of this form correspond couples of things whose names are respectively <”>[4]x<”>[4] and <”>[4]y<”>[4]. The things x[7]</>[4]y[7] stand to one another in all sorts of relations<,>[4] amongst others some stand in the relatio[j|n] of[2] R[1], and some not; just as I single out a particular thing by a particular name I single out all behaviours of the point<s>[1] x and y the one between[2] with respect to[1] the relation <R.>[1] of the other[2]. I say that if an x stands in the relation of[2] R[1] to a y the sign <”>[4]x of[2] R[1] y<”>[4] is to be called true to the fact and otherwise false. This is a definition of sense.

⟨!⟩[16]

In my theory p has the same meaning as not-p but opposite snese. The meaning is the fact. The proper theory of judgment must make it impossible to judge nonsense.

It is not strictly true to say that we understand a proposition p if we know that p is equivalent to ”p is true” for this would be the case if accidentally both were true or false. What is wanted is the formal equivalence with respect to the forms of the proposition[.|,[9]] i.e., [A|a[9]]ll the general indefinables involved. The sense of an ab[7] function of a proposition is a function of its sense[:|.[9]] [t|T[9]]here are only unasserted propositio<ns.>[4]

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5

Assertion is merely psychological. If[2] n[1] not-p<,>[1] ┌p[10]┐[1] is exactly the same as if it stands alone<;>[1] this point is absolutely fundamental. Among the facts which make ”p or q” true[1] there are also facts which make ”p and q” true<;>[1] if propositions do only mean[2] ┌have only meaning┐[1]<,>[1] we ought<,>[1] to know[2] ┌in┐[1] such a case, to[1] say that these two propositions are identical, but in fact, their sense is different for we have introduced sense by talking of all p’s and all q’s. Consequently the molecular propositions will only be used in cases where there ab[7] function stands under a generality sign or enters into another function such as <”>[4]I believe that, etc.,<”>[4] because then[4] the sense enters.

In ”a judges p” p cannot be replaced by a proper name. This appears if we substitute ”a judges that p is true and not p is false”. The proposition ”a judges p” consists of the proper name a[.|,[9]] [T|t[9]]he proposition p with its 2 poles<,>[4] and a[7] being related to both of these poles in a certain way. This is obviously not the[5] <a>[4] relation in the ordinary sense.

The ab notation and[5] for[4] apparent variables[5] make<s>[4] it clear that not and or are dependent on one another and we can therefore not use them as simultaneous indefinables. <|[17]>[4] Some[5] <Same>[4] objections ┌in the case of app. var.┐[4] to old indefinables, [a|A[9]]s[5] as[4] [18] in the case of molecular functions[19][,|:[9]] [t|T[9]]he application of the ab notation to apparently[5]<->[4] variable propositions become<s>[4] clear if we consider that, for instance, the proposition <”>[4]for all ”[5]x<,>[4] φx” is to be true when φx is true for all x’s and false when φx is false for some x’s. We see that some and all occur simultaneously in the proper apparent variable notation.

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6

The Notation is:

for (x) φx ; a - (x) - a φ x b - (∃ x) - b

and

for (φ[5]∃x) φ x : a - (∃x) - a φ x b - (x) - [v|b]

Old definitions now become tautologous.

In aRb it is not the complex that symbolises but the fact that the symbol a stands in a certain relation to the symbol b. Thus facts are symbolised by facts, or the[5] more correctly: that a certain thing is the case in the symbol says that a certain thing is the case in the world.

Judgment, question and command are all on the same level. What interests logic in them is only the unasserted proposition. Facts cannot be named.

A proposition cannot occur in itself. This is the fundamental truth of the theory of types.

Every proposition that says something important[5] about one thing is a subject-predicate proposition, and so on.

Therefore we can recognize a subject-predicate proposition if we know it contains only one name and one form, etc. This gives the construction of types. Hence the type of a proposition can be recognized by its symbol alone.

What is essential in a correct apparent┌-variable┐[1] notation is this:- (1) it must mention a type of propositions; (2) it must show which components of a proposition of this type are constants.@

[Components are forms and constituents.]

Take (φ).φ!x. Then if we describe the kind[7] of symbols[20]┌, for which φ! stands┐[4] <&>[4] which, by the above, is enough to determine the type, then automatically "([x|φ]).φ!x" cannot be fi[ll|tt]ed by this descri[l|p]tion[.|,[9]] <because it *contains*[7] „φ!x" & the description is to describe all[7] that symbolizes in symbols of the φ! - kind. If the description is thus[7] complete vicious circles can just as little occur as if[5] for instance in (φ).φ(x)[5] (φ).(x)$_φ$[4] (where (~~x~~)$_φ$ is a subject-predicat prop) >[4]

Item 201a-1 Recto Page B1

Wittgenstein 1

First MS.

Indefinables are of two sorts: names, & forms. Propositions cannot consist of names alone; they cannot be classes of names. A name can not only occur in two different propositions, but can occur in the same way in both.

Propositions [which are symbols having reference to facts] are themselves facts: that this inkpot is on this table may express that I sit in this chair.

It can never express the common characteristic @ of two objects that we denote designate them by the same name but by two different ways of designation, for, since names are arbitrary, we might <also> choose different names, & where then would be the common element in the designations? Nevertheless one is always tempted, in a difficulty, to take refuge in different ways of designation. @

Frege said "propositions are names"; Russell said "propositions correspond to complexes". Both are false; & especially false is the statement "propositions are names of complexes".

It is easy to suppose that only such symbols are complex as contain names of objects objects, & ┌that accordingly┐ "$(\exists x,\varphi).\varphi x$" or "$(\exists x,$ ┌R,┐ $y).xRy$" must be simple. It is then natural to call the first of these the name of a form, the second the name of a relation. But in that case what is the meaning of (e.g.) "$\sim(\exists x,y).xRy$"? Can we put "not" before a name?

Item 201a-1 Recto Page B2

The reason why "~Socrates" means nothing is that "~x" does not express a property of x.

There are positive & negative facts: if the proposition "this rose is not red" is true, then its ┌what it┐ signifies is negative. But the occurrence of the word "not" does not indicate this unless we know that the signification of ┌the proposition┐ "this rose is red" (when it is true) is positive. It is only from both, the negation & the negated proposition, that we can conclude to a characteristic of the significance of the whole proposition. (We are not here speaking of negations of general propositions, i.e. of such as contain apparent variables.) Negative facts only justify the negations of simpl atomic propositions.)

Positive & negative facts there are, but not true & false facts.

If we overlook the fact that propositions have a sense which is independent of their truth or falsehood, it easily seems as if true & false were two equally justified relations between the sign & what is signified. (We might then say e.g. that "q" signifies in the true way what "not-q" signifies in the false way). But are not true & false in fact equally justified? Could we not express ourselves by means of false propositions just as well as hitherto with true ones, so long as we know that they are meant falsely?

3

No! For a proposition is then true when it is as we assert in this proposition; & accordingly if by "_q_" we mean "not-_q_", & it is as we mean to assert, then in the new interpretation "_q_" is actually true & _not_ false. But it is important that we _can_ mean the same by "_q_" as by "not-_q_", for it shows that neither to the symbol "not" nor to the manner of its combination with "_q_" does a characteristic of the denotation of "_q_" correspond.

4

2nd MS.

We must be able to understand propositions which we have never heard before. But every proposition is a new symbol. Hence we must have general indefinable symbols; these are unavoidable if propositions are not all indefinable.

Whatever corresponds in reality to compound propositions must not be more than what corresponds to their several atomic propositions.

Not only must logic not deal with [particular] things, but just as little with relations & predicates.

There are no propositions containing real variables.

What corresponds in reality to a proposition depends upon whether it is true or false. But we must be able to understand a proposition without knowing if it is true or false.

What we know when we understand a proposition is this: We know what is the case if the proposition is true, & what is the case if it is false. But we do not know [necessarily] whether it is true or false.

Propositions are not names.

We can never distinguish one logical type from another by attributing a property to members of the one which we deny to members of the other.

Symbols are not what they seem to be. In "aRb", "R" looks like a substantive, but is not one. What symbolizes in "aRb" is that R occurs between a & b. Hence "R" is not the indefinable in "aRb". Similarly in "φx", "φ" looks like a substantive but is not one; in "~p", "~" looks like "φ" but is not like it. This is the first thing that indicates that there <u>may</u> not be logical constants. A reason against them is the generality of logic: logic cannot treat a special set of things.

Molecular propositions contain nothing beyond what is contained in their atoms; they add no material information above that contained in their atoms.

All that is essential about molecular functions is their T-F schema [i.e. the statement of the cases when they are true & the cases when they are false].

Alternative indefinability shows that the indefinables have not been reached.

Every proposition is essentially true-false: to understand it, we must know both what must be the case if it is true, & what must be the case if it is false. Thus a proposition has two poles, corresponding to the case of its truth & the case of its falsehood. We call this the sense of a proposition.

In regard to notation, it is important to note that not every feature of a symbol symbolizes. In two molecular functions which have the same T-F schema, what symbolizes must be the same. In ″not-not-p″, ″not-p″ does not occur; for ″not-not-p″ is the same as ″p″, & therefore, if ″not-p″ occurred in ″not-not-p″, it would occur in ″p″.

Logical indefinables cannot be predicates or relations, because propositions, owing to sense, cannot have predicates or relations. Nor are ″not″ & ″or″, like judgment, analogous to predicates or relations, because they do not introduce anything new.

Propositions are always complex even if they contain no names.

A proposition must be understood when <u>all</u> its indefinables are understood. The indefinables in ”<u>a</u>R<u>b</u>” are introduced as follows:

”<u>a</u>” is indefinable;

”<u>b</u>” is indefinable;

Whatever ”x” & ”y” may mean, ”<u>x</u>R<u>y</u>” says something indefinable[21] about their meanings.

A complex symbol must never be introduced as a single indefinable. (Thus e.g. no proposition is indefinable.) For if one of its parts occurs also in another connection, it must there be re-introduced. And would it then mean the same?

The ways by which we introduce our indefinables must permit us to construct all propositions that have sense [? meaning] from these indefinables <u>alone</u>. It is easy to introduce ”all” & ”some” in a way that will make the construction of (say) ”(x,y).xRy” possible from ”all” & ”xRy” <u>as introduced before</u>.

3rd MS.

A comparis An analogy for the theory of truth: Consider a black patch on white paper; then we can describe the form of the patch by mentioning, for each point of the surface, whether it is white or black. To the fact that a point is black corresponds a positive fact, to the fact that a point is white (not black) corresponds a negative fact. If I designate a point of the surface (one of Frege's "truth-values"), this is as if I set up an assumption to be decided upon. But in order to be able to say of a point that it is black or that it is white, I must first know when a point is to be called black & when it is to be called white. In order to be able to say that "p" is true (or false), I must first have determined under what circumstances I call a proposition true, & thereby I determine the sense of a proposition. The point [on|in] which the analogy depends fails is this: I can indicate a point of the paper what is white & black, but to a proposition without sense nothing corresponds, for it does not designate a thing (truth-value), whose properties might be called "false" or "true"; the verb of a proposition is not "is true" or "is false", as Frege believes, but what is true must already contain the verb.

The comparison of language & reality is like that of retinal image & visual image: to the blind spot nothing in the visual image seems to correspond, & thereby the boundaries of the blind spot determine the visual image - as true negations of atomic propositions determine reality.

Logical inferences can, it is true, be made in accordance with Frege's or Russell's laws of deduction, but this cannot justify the inference; & therefore they are not primitive propositions of logic. If p follows from q, it can also be inferred from q, & the "manner of deduction" is indifferent.

Those symbols which are called propositions in which "variables occur" are in reality not propositions at all, but only schemes of propositions, which only become propositions when we replace the variables by constants. There is no proposition which is expressed by "x = x", for "x" has no signification; but there is a proposition "(x).x = x" & propositions such as "Socrates = Socrates" etc.

In books on logic, no variables ought to occur, but only the general propositions which justify the use of variables. It follows that the so-called definitions of logic are not definitions, but only schemes of definitions, & instead of these we ought to put general propositions; & similarly the so-called primitive ideas ⌜(Urzeichen)⌝ of logic are not primitive ideas, but the schemes of them. The mistaken idea that there are things called facts or complexes & relations easily leads to the opinion that there must be a relation of questioning to the facts, & then the question arises whether a relation can hold between an arbitrary number of things, since a fact can follow from arbitrary cases. It is a fact that the proposition which e.g. expresses that q follows from p & p⊃q is this: $p.p\supset q.\supset_{p.q}.q$.

At a pinch, one is tempted to interpret "not-p" as "everything else, only not p". That from a single fact p an infinity of others, not-not-p etc., follow, is hardly credible. Man possesses an innate capacity for constructing symbols with which some sense can be expressed, without having the slightest idea what each word signifies. The best example of this is mathematics, for man has until lately used the symbols for numbers without knowing what they signify or that they signify nothing.

Russell's "complexes" were to have the useful property of being compounded, & were to combine with this the agreeable property that they could be treated ~~as~~ like "simples". But this alone made them unserviceable as logical types, since there would have been significance in asserting, of a simple, that it was complex. But a property cannot be a logical type.

Every statement about apparent complexes can be resolved into the logical sum of a statement about the constituents & a statement about the proposition which describes the complex completely. How, in each case, the resolution is to be made, is an important question, but its answer is not unconditionally necessary for the construction of logic.

That "or" & "not" etc. are not relations in the same sense as "right" & "left" etc., is obvious to the plain man. The possibility of cross-definitions in the old logical indefinables shows, of itself, that these are not the right indefinables, &, even more conclusively, that they do not denote relations.

If we change a constituent a of a proposition $\varphi(a)$ into a variable, then there is a class

$$\tilde{p}\,\{(\exists x).\varphi(x) = p\}.$$

This class in general still depends upon what, by an arbitrary convention, we have mean by "$\varphi(x)$". But if we change into variables all those symbols whose significance was arbitrarily determined, there is still such a class. But this is now not dependent upon any convention, but only upon the nature of the symbol "$\varphi(x)$". It corresponds to a logical type.

Types can never be distinguished from each other by saying (as is often done) that one has th[i|e]s<e> @ but the other has th[at|ose] propert[y|i]<es>, for this presupposes that there is a meaning in asserting all these properties of both types. But from this it follows that, at best, these properties may be types, but certainly not the objects of which they are asserted.

At a pinch, we are always inclined to explanations of ⌜logical⌝ functions of propositions which ⌜aim at introducing into the function⌝ either only contain the constituents of these propositions, or only their forms, etc. etc; & we overlook that ordinary language would not contain the whole propositions if it did not need them: However, e.g., "not-p" may be explained, there must always be a meaning given to the question "what is denied?"

The very possibility of Frege's explanations of "not-p" & "if p then q", from which it follows that "not-not-p" denotes the same as p, makes it probable that there is some method of designation in which "not-not-p" corresponds to the same symbol as "p". But if this method of designation suffices for logic, it must be the right one.

Names are points, sentences propositions arrows - they have sense. The sense of a proposition is determined by the two poles true & false. The form of a proposition is like a straight line, which divides all points of a plane into right & left. The line does this automatically, the form of proposition only by convention.

Just as little as we are concerned, in logic, with the relation of a name to its meaning, just so little are we concerned with the relation of a proposition to reality, but we want to know the meaning of names & the sense of propositions - as we introduce an indefinable concept ″A″ by saying: ″′A′ denotes something indefinable″, so we introduce e.g. the form of propositions aRb by saying: ″For all meanings of ″x″ & ″y″, ″xRy″ expresses something indefinable about x & y″.

In place of every proposition ″p″, let us write @ ″${}^{a}_{b}p$″. Let every correlation of propositions to each other or of names to propositions be effected by a correlation of their poles ″a″ & ″b″. Let this correlation be transitive. Then accordingly ″${}^{a-a}_{b-b}p$″ is the same symbol as ″${}^{a}_{b}p$″. Let n propositions be given. I then call a ″class of poles″ of these propositions every class of n members, of which each is a pole of one of the n propositions, so that one member corresponds to each proposition. I then correlate with each class of poles one of two poles (a & b). The sense of the symbolizing fact thus constructed I cannot define, but I know it.

If p = not-not-p etc., this shows that the traditional method of symbolism is wrong, since it allows a plurality of symbols with the same sense; & thence it follows that, in analyzing such propositions, we must not be guided by Russell′s method of symbolizing.

It is to be remembered that names are not things, but classes: "A" is the same letter as "A". This has the most important consequences for every symbolic language.

Neither the sense nor the meaning of a proposition is a thing. These words are incomplete symbols.

It is impossible to dispense with propositions in which the same argument occurs in different positions. It is obviously useless to replace φ(a,a) by φ(a,b).a = b.

Since the *ab*-functions of *p* are again bi-polar propositions, we can form *ab*-functions of them, & so on. In this way a series of propositions will arise, in which in general the *symbolizing* facts will be the same in several members. If now we find an *ab*-function of such a kind that by repeated application of it every ab-function can be generated, then we can define introduce the totality of ab-functions as the totality of those that are generated by application of this function. Such a function is ~p∨~q.

It is easy to suppose a contradiction in the fact that on the one hand all every possible complex proposition is a simple ab-function of simple propositions, & that on the other hand the repeated application of one ab-function suffices to generate all these propositions. If e.g. an affirmation can be generated by double negation, is negation in any sense contained in affirmation? Does "p" deny "not-p" or assert "p", or both? And how do matters stand with the definition of "⊃" by "∨" & "~" ".", or of "∨" by "." & "⊃"? And how e.g. shall we introduce p|q (i.e. ~p∨~q), if not by saying that this expression says something indefinable about all arguments p & q? But the ab-functions must be introduced as follows: The function p|q is merely a mechanical instrument for constructing all possible symbols of ab-functions. The symbols arising by repeated application of the symbol "|" do not contain the symbol "p|q". We need a rule according to which we can form all symbols of ab-functions, in order to be able to speak of the class of them; & we now speak of them e.g. as those symbols of functions which can be generated by repeated application of the operation "|". And we say now: For all p's & q's, "p|q" says something indefinable about the sense of those simple propositions which are contained in p & q.

The assertion-sign is logically quite without significance. It ┌only┐ shows, in Frege & Whitehead & Russell, that these authors hold the propositions so indicated to be true. "⊢" therefore belongs as little to the proposition as (say) the number of the proposition. A proposition cannot possibly assert of itself that it is true.

Every right theory of judgment must make it impossible for me to judge that this table penholders the book. Russell's theory does not satisfy this requirement.

It is clear that we understand propositions without knowing whether they are true or false. But we can only know the meaning of a proposition when we know if it is true or false. What we understand is the sense of the proposition.

The assumption of the existence of logical objects makes it appear remarkable that ┌in the sciences┐ propositions of the form "p[or|∨]q", "p⊃q", etc. ┌are┐ only then not provisional when "∨" & "⊃" stand within the scope of a generality-sign [apparent variable].

4th MS.

If we formed all possible atomic propositions, the world would be completely described if we declared the truth or falsehood of each. [I doubt this.]

The chief characteristic of my theory is that, in it, p has the same meaning as not-p.

A false theory of relations makes it easily seem as if the relation of fact & constituent were the same as that of fact & fact which follows from it. But the similarity of the two may be expressed thus: $\varphi a . \supset ._{\varphi , a}\ a = a$.

If a word creates a world so that in it the principles of logic are true, it thereby creates a world in which the whole of mathematics holds; & similarly it could not create a world in which a proposition was true, without creating its constituents.

Signs of the form "p∨~p" are senseless, but not the proposition "(p).p ∨ ~p". If I know that this rose is either red or not red, I know nothing. The same holds of all ab-functions.

To understand a proposition means to know what is the case if it is true. Hence we can understand it without knowing if it is true. We understand it when we understand its constituents & forms. If we know the meaning of "a" & "b", & if we know what "xRy" means for all x's & y's, then we also understand "aRb".

I understand the proposition "aRb" when I know that either the fact that aRb or the fact that not aRb corresponds to it; but this is not to be confused with the false opinion that I understand "aRb" when I know that "aRb or not-aRb" is the case.

But the form of a proposition symbolizes in the following way: Let us consider symbols of the form ″xRy″; to these correspond primarily pairs of objects, of which one has the name ″x″, the other the name ″y″. The x's & y's stand in various relations to each other, among others the relation R holds between some, but not between others. I know now determine the sense of ″xRy″ by laying down: when the facts behave in regard to ″xRy″ so that the meaning of ″x″ stands in the relation R to the meaning of ″y″, then I say that they [the facts] are ″of like sense″ [″gleichsinnig″] with the proposition ″xRy″; otherwise, ″of opposite sense″ [entgegengesetzt″]; I correlate the facts to the symbol ″xRy″ by thus dividing them into those of like sense & those of opposite sense. To this correlation corresponds the correlation of name & meaning. Both are psychological. Thus I understand the form ″xRy″ when I know that it discriminates the behaviour of x & y according as these stand in the relation R or not. In this way I extract from all possible relations the relation R, as, by a name, I extract its meaning from among all possible things.

Strictly speaking, it is incorrect to say: We understand the proposition p when we know that '″p″ is true' ≡ p; for this would naturally always be the case if accidentally the propositions to right & left of the symbol ″≡″ were both true or both false. We require not only an equivalence, but a formal equivalence, which is bound up with the introduction of the form of p.

The sense of an ab-function of p is a function of the sense of p.

The ab-functions use the discrimination of facts, which their arguments bring forth, in order to generate new discriminations.

Only facts can express sense, a class of names cannot. This is easily shown.

There is no thing which is the form of a proposition, & no name which is the name of a form. Accordingly we can also not say that a relation which in certain cases holds between things holds sometimes between forms & things. This goes against Russell's theory of judgment.

It is ⌜very⌝ easy to forget that, tho' the propositions of a form can be either true or false, each one of these propositions can only be either true or false, not both.

Among the facts which make "p or q" true, there are some which make "p & q" true; but the class which makes "p or q" true is different from the class which makes "p & q" true; & ⌜only⌝ this is what matters. For we introduce this class, as it were, when we introduce ab-functions.

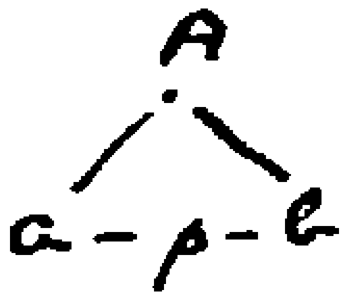
A very natural objection to the way in which I have introduced e.g. propositions of the form xRy is that by it propositions such as (∃x,y).xRy & similar ones are not explained, which yet obviously have in common with aRb what cRd has in common with aRb. But when we introduced propositions of the form xRy we mentioned no one particular proposition of this form; & we only need to introduce (∃x,y).φ(x,y) for all φ's in any way which makes the sense of these propositions dependent on the sense of all propositions of the form φ(a,b), & thereby the justification of our procedure is proved.

The indefinables of logic must be independent of each other. If an indefinable is introduced, it must be introduced in all combinations in which it can occur. We cannot therefore introduce it first for one combination, then for another; e.g., if the form xRy has been introduced, it must henceforth be understood in propositions of the form aRb just in the same way as in propositions such as (∃x,y). xRy & others. We must not introduce it first for one class of cases, then for the other; for it would remain doubtful if its meaning was the same in both cases, & there would be no ground for using the same manner of combining symbols in both cases. In short, for the introduction of indefinable symbols & classes combinations of symbols the same holds, mutatis mutandis, that Frege has said for the introduction of symbols by definitions.

It is a priori likely that the introduction of atomic propositions is fundamental for the understanding of all other kinds of propositions. In fact the understanding of general propositions obviously depends on that of atomic propositions.

Cross-definability in the realm of general propositions leads to the quite similar questions to those in the realm of ab-functions.

When we say "A believes p", this sounds, it is true, as if here we could substitute a proper name for "p"; but we can see that here a sense, not a meaning, is concerned, if we say "A believes that 'p' is true"; & in order to make the direction of p even more explicit, we might say "A believes that 'p' is true & 'not-p' is false". Here the bi-polarity of p is expressed, & it seems that we shall only be able to express the proposition "A believes p" correctly by the ab-notation; say by making "A" have a relation to the poles "a" & "b" of a-p-b.



The epistemological questions concerning the nature of judgment & belief cannot be solved without a correct apprehension of the form of the proposition.

The ab-notation shows the dependence of or & not, & thereby that they are not to be employed as simultaneous indefinables.

Not: "The complex sign 'aRb'" says that a stands in the relation R to b; but that 'a' stands in a certain relation to 'b' says that aRb.

In philosophy there are no deductions: it is purely descriptive.

Philosophy gives no pictures of reality.

Philosophy can neither confirm nor confute scientific investigation.

Philosophy consists of logic & metaphysics: logic is its basis.

Epistemology is the philosophy of psychology.

Distrust of grammar is the first requisite for philosophizing.

Propositions can never be indefinables, for they are always complex. That also words like ”ambulo” are complex appears in the fact that their root with a different termination gives a different sense.

Only the doctrine of general indefinables permits us to understand the nature of functions. Neglect of this doctrine leads to an impenetrable thicket.

Philosophy is the doctrine of the logical form of scientific propositions (not only of primitive propositions).

The word ”philosophy” ought always to designate something over or under, but not beside, the natural sciences.

Judgment, ⌜command &⌝ question [&|a]ll stand on the same level; but all have in common the propositional form, which does interests us.

The ~~construction~~ structure of the ~~sentence~~ proposition must be recognized, the rest comes of itself. But ordinary language conceals the structure of the proposition: in it, relations look like predicates, predicates like names, etc.

Facts cannot be named.

It is easy to suppose that ”individual”, ”particular”, ”complex” etc. are primitive ideas of logic. Russell e.g. says ”individual” & ”matrix” are ”primitive ideas”. This error presumably is to be explained by the fact that, by employment of variables instead of ⌜the⌝ generality-signs, it comes to seem as if logic dealt with things which have been deprived of all properties except thing-hood, & with propositions deprived of all properties except complexity. We forget that the indefinables of symbols [Urbilder von Zeichen] only occur under the generality-sign, never outside it.

Just as people used to struggle to bring all propositions into the subject-predicate form, so now it is natural to conceive every proposition as expressing a relation, which is just as incorrect. What is justified in this desire is fully satisfied by Russell’s theory of manufactured relations.

One of the most natural attempts at solution consists in regarding ”not-p” as ”the opposite of p”, where then ”opposite” would be the indefinable relation. But it is easy to see that every such attempt to replace the ab-functions by descriptions must fail.

The false assumption that propositions are names leads us to believe that there must be logical objects: for the meanings of logical propositions will have to be such things.

A correct explanation of logical propositions must give them a unique position as against all other propositions.

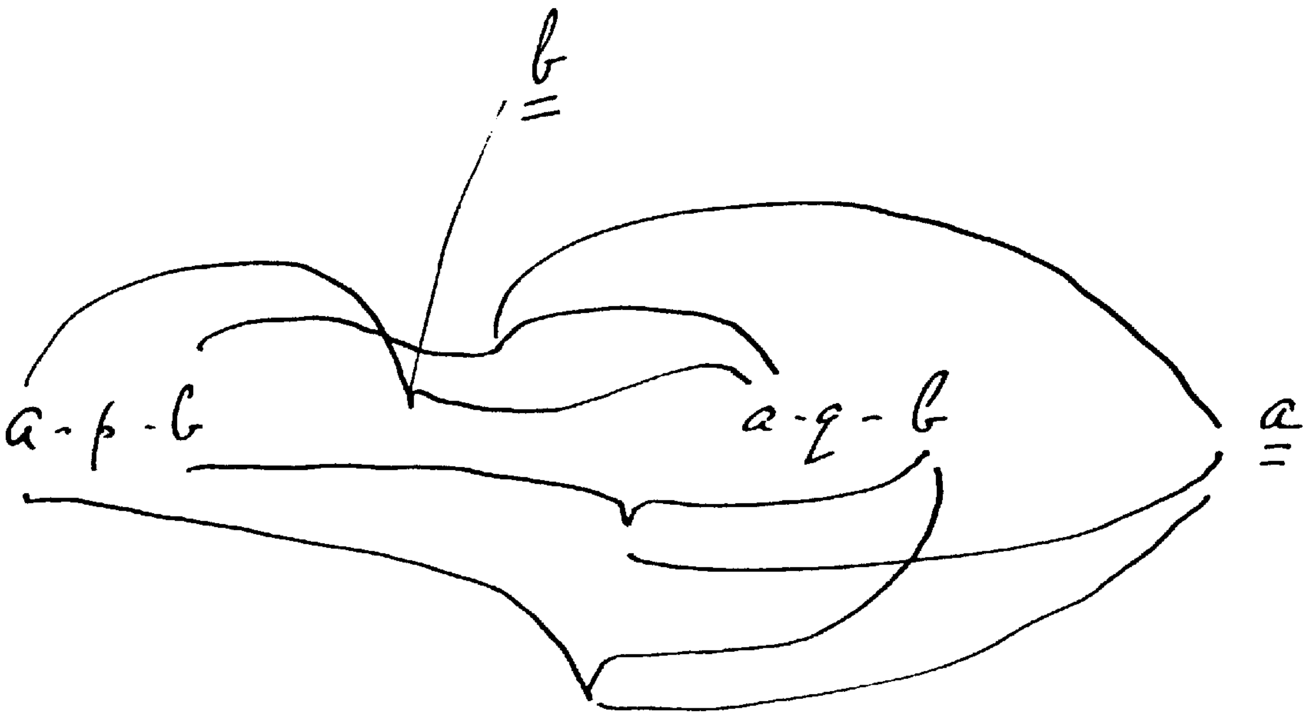
No proposition can say anything about itself, because the symbol of the proposition cannot be contained in itself; this must be the basis of the theory of logical types.

Every proposition which says something indefinable about a thing is a subject-predicate proposition; every proposition which says something indefinable about two things expresses a dual relation between these things, & so on. Thus every proposition which contains only one name & one indefinable form is a subject-predicate proposition, & so on. An indefinable simple sign symbol can only be a name, & therefore we can know, by the symbol of an atomic proposition, whether it is a subject-predicate proposition.

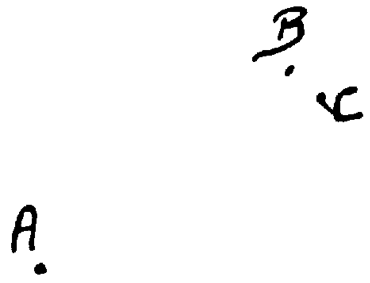
Item 201a-1 Recto Page B24

Wittg.

I. Bi-polarity of propositions: sense & meaning, truth & falsehood.
II. Analysis of atomic propositions: general indefinables, predicates, etc.
III. Analysis of molecular fu propositions: ab-functions.
IV. Analysis of general propositions[22]
[IV|V]. Principles of symbolism: what symbolizes in a symbol. Facts for facts.
V<I>. Types



This is the symbol for
$\sim p \vee \sim q$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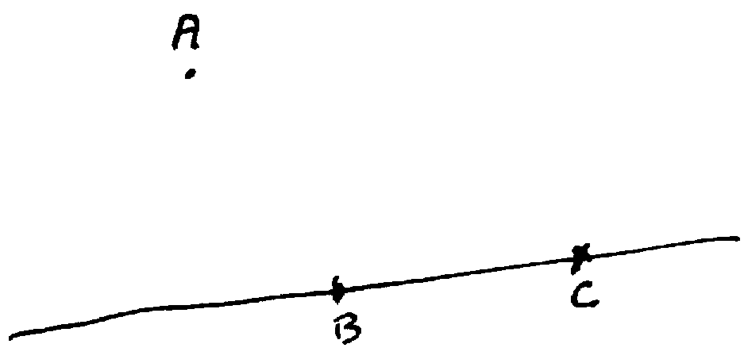
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C

Item 201a-2 (RA1.710.057824)

Transcriber(s): Michael Biggs, Peter Cripps
Proofreader(s): Peter Cripps
Comments:
Transcriptions of material from The Bertrand Russell Archives, McMaster University, Canada; their catalogue no.: RA1.710.057824.
This is a typescript of two parts (8ff.+25ff.). The first part is a copy of The Bertrand Russell Archives item no. RA1.710.057822 incorporating the handwritten corrections. The second part is a copy of The Bertrand Russell Archives item no. RA1.710.057823. The whole has handwritten Roman numerals in the left margin against each paragraph indicating the sections of Russell's subsequent rearrangement of Wittgenstein's text (the so-called Costello Version, von Wright's catalogue no. 201b). Non-typewriter characters are inserted by hand; fully typed pages have 24 or 25 lines of type; each new sentence is separated from the last by three spaces. True double spaced blank lines used occasionally but not consistently.
Hands:
s = handwritten insertions that belong to first pass, ie generally the insertion of symbols which are not found on the typewriter such as φ and ∃
S1 = Bertrand Russell

Item 201a-2 Recto Page TP

SUMMARY[2]

<Notes on Logic
by
Ludwig Wittgenstein
September 1913.>[1]

SUMMARY.

<III> One reason for thinking the old notation wrong is that it is very unlikely that from every proposition p an infinite number of other propositions not-not-p, not-not-not-not-p, etc., should follow.

<IV> If only those signs which contain proper names were complex then propositions containing nothing but apparent variables would be simple. Then what about their denials?

<I> The verb of a proposition cannot be ”is true” or ”is false”, but whatever is true or false must already contain the verb.

Deductions only proceed according to the laws of deduction, but these laws cannot justify the deduction.

<I> One reason for supposing that not all propositions which have more than one argument are relational propositions is that if they were, the relations of judgment and inference would have to hold between an arbitrary number of things.

<II> Every proposition which seems to be about a complex can be analysed into a proposition about its constituents and about the proposition which describes the complex perfectly; i.e., that proposition which is equivalent to saying the complex exists.

Item 201a-2 Recto Page A2

<I> The idea that propositions are names of complexes suggests that whatever is not a proper name is a sign for a relation. Because spatial complexes* * consists of Things and Relations only and the idea of a complex is taken from space.

<VI> In a proposition convert all its indefinables into variables; there then remains a class of propositions which is not all propositions but a type.

<VI> There are thus two ways in which signs are similar. The names Socrates and Plato are similar: they are both names. But whatever they have in common must not be introduced before Socrates and Plato are introduced. The same applies to a subject-predicate form etc. Therefore, thing, proposition, subject-predicate form, etc., are not indefinables, i.e., types are not indefinables.

<I> When we say A judges that etc., then we have to mention a whole proposition which A judges. It will not do either to mention only its constituents, or its constituents and form, but not in the proper order. This shows that a proposition itself must occur in the statement that it is judged; however, for instance, ”not-p” may be explained, the question, ”What is negated” must have a meaning.

⟨* Russell - for instance imagines every fact as a spatial complex.⟩[23]

<I> To understand a proposition p it is not enough to know that p implies ’”p” is true’, but we must also know that ~p implies ”p is false”. This shows the bi-polarity of the proposition.

<III> To every molecular function a WF* scheme corresponds. Therefore we may use the WF scheme itself instead of the function. Now what the WF scheme does is, it correlates the letters W and F with each proposition. These two letters are the poles of atomic propositions. Then the scheme correlates another W and F to these poles. In this notation all that matters is the correlation of the outside poles to the poles of the atomic propositions. Therefore not-not-p is the same symbol as p. And therefore we shall never get two symbols for the same molecular function.

<I> The meaning of a proposition is the fact which actually corresponds to it.

<III> As the ab functions of atomic propositions are bi-polar propositions again we can perform <u>ab</u> operations on them. We shall, by doing so, correlate two new outside poles via the old outside poles to the poles of the atomic propositions.

⟨* W-F = Wahr-Falsch.⟩[23]

<III> The symbolising fact in a-p-b is that, say* a is on the left of p and b on the right of p; then the correlation of new poles is to be transitive, so that for instance if a new pole a in whatever way i.e. via whatever poles is correlated to the inside a, the symbol is not changed thereby. It is therefore possible to construct all possible ab functions by performing one ab operation repeatedly, and we can therefore talk of all ab functions as of all those functions which can be obtained by performing this ab operation repeatedly.

[Note by B.R. ab means the same as WF, which means true-false.]

<III> Naming is like pointing. A function is like a line dividing points of a plane into right and left ones; then ”p or not-p” has no meaning because it does not divide the plane.

<III> But though a particular proposition ”p or not-p” has no meaning, a general proposition ”for all p’s, p or not-p” has a meaning because this does not contain the nonsensical function ”p or not-p” but the function ”p or not-q” just as ”for all x’s xRx” contains the function ”xRy”.

⟨* This is quite arbitrary but, if we once have fixed on which order the poles have to stand we must of course stick to our convention. If for instance ”a p b” says p then b p a says nothing. (It does not say ~p) But a - a p b - b is the same symbol as apb (here the ab function vanishes automatically) for here the new poles are related to the same side of p as the old ones. The question is always: how are the new poles correlated to p compared with the way the old poles are correlated to ~p.⟩[23]

<I> A proposition is a standard to which all facts behave, with names it is otherwise; it is thus bi-polarity and sense comes in; just as one arrow behaves to another arrow by being in the same sense or the opposite, so a fact behaves to a proposition.

<II> The form of a proposition has meaning in the following way. Consider a symbol ”xRy”. To symbols of this form correspond couples of things whose names are respectively ”x” and ”y”. The things <u>x</u> <u>y</u> stand to one another in all sorts of relations, amongst others some stand in the relation R, and some not; just as I single out a particular thing by a particular name I single out all behaviours of the points x and y with respect to the relation R. I say that if an x stands in the relation R to a y the sign ”xRy” is to be called true to the fact and otherwise false. This is a definition of sense.

In my theory p has the same meaning as not-p but opposite sense. The meaning is the fact. The proper theory of judgment must make it impossible to judge nonsense.

<I> It is not strictly true to say that we understand a proposition p if we know that p is equivalent to ”p is true” for this would be the case if accidentally both were true or false. What is wanted is the formal equivalence with respect to the forms of the proposition, <u>i.e.</u>, all the general indefinables involved. <u>The sense of</u> an <u>ab</u> function of a proposition is a function of its sense. There are only unasserted propositions.

<I> Assertion is merely psychological. In not-p, p is exactly the same as if it stands alone; this point is absolutely fundamental. Among the facts that make ″p or q″ true there are also facts which make ″p and q″ true; if propositions have only meaning, we ought, in such a case, to say that these two propositions are identical, but in fact, their sense is different for we have introduced sense by talking of all p's and all q's. Consequently the molecular propositions will only be used in cases where their ab function stands under a generality sign or enters into another function such as ″I believe that, etc″., because then the sense enters.

<I> In ″a judges p″ p cannot be replaced by a proper name. This appears if we substitute ″a judges that p is true and not p is false″. The proposition ″a judges p″ consists of the proper name a, the proposition p with its 2 poles, and a being related to both of these poles in a certain way. This is obviously not a relation in the ordinary sense.

<III><V> The ab notation makes it clear that not and or are dependent on one another and we can therefore not use them as simultaneous indefinables. Same objections in the case of apparent variables to the usual[1] old indefinables, as in the case of molecular functions. The application of the ab notation to apparent variable propositions becomes clear if we consider that, for instance, the proposition ″for all x, φx″ is to be true when φx is true for all x's and false when φx is false for some x's. We see that some and all occur simultaneously in the proper apparent variable notation.

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<IV> The notation is:

for $(x)\ \varphi x$: a - (x) - a φx b - $(\exists\ x)$ - b
and

for $(\exists x)\ \varphi x$: a - $(\exists x)$ - a φx b - (x) - b

Old definitions now become tautologous.

<V> In aRb it is not the complex that symbolises but the fact that the symbol a stands in a certain relation to the symbol b. Thus facts are symbolised by facts, or more correctly: that a certain thing is the case in the symbol says that a certain thing is the case in the world.

<I> Judgment, question and command are all on the same level. What interests logic in them is only the unasserted proposition. Facts cannot be named.

<VI> A proposition cannot occur in itself. This is the fundamental truth of the theory of types.

Every proposition that says something indefinable about one thing is a subject-predicate proposition, and so on.

<VI> Therefore we can recognise a subject-predicate proposition if we know it contains only one name and one form, etc. This gives the construction of types. Hence the type of a proposition can be recognized by its symbol alone.

What is essential in a correct apparent-variable notation is this:- (1) it must mention a type of propositions; (2) it must show which components of a proposition of this type are constants.

<VI> [Components are forms and constituents.]

Take (φ). φ!x. Then if we describe the <u>kind</u> of symbols, for which φ! stands and which, by the above, is enough to determine the type, then automatically "(φ). φ! x" cannot be fitted by this description, because it <u>CONTAINS</u> "φ!x" and the description is to describe <u>ALL</u> that symbolises in symbols of the φ! kind. If the description is <u>thus</u> complete vicious circles can just as little occur as for instance (φ). (X)φ (where (X)φ is a subject-predicate proposition).

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First MS.

<II> Indefinables are of two sorts: names, and forms. Propositions cannot consist of names alone; they cannot be classes of names. A name can not only occur in two different propositions, but can occur in the same way in both.

Propositions [which are symbols having reference to facts] are themselves facts: that this inkpot is on this table may express that I sit in this chair.

<V> It can never express the common characteristic of two objects that we designate them by the same name but by two different ways of designation, for, since names are arbitrary, we might also choose different names, and where then would be the common element in the designations? Nevertheless one is always tempted, in a difficulty, to take refuge in different ways of designation.

<I> Frege said "propositions are names"; Russell said "propositions correspond to complexes". Both are false; and especially false is the statement "propositions are names of complexes."

<IV> It is easy to suppose that only such symbols are complex as contain names of objects, and that accordingly " $(\exists x,\varphi).\ \varphi x$" or "$(\exists x,y).\ x\ R\ y$" must be simple. It is then natural to call the first of these the name of a form, the second the name of a relation. But in that case what is the meaning of (e.g.) "$\sim(\exists x,y).\ x\ R\ y$"? Can we put "not" before a name?

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<III> The reason why "~Socrates" means nothing is that "~x" does not express a property of *x*.

<I> There are positive and negative facts: if the proposition "this rose is not red" is true, then what it signifies is negative. But the occurrence of the word "not" does not indicate this unless we know that the signification of the proposition "this rose is red" (when it is true) is positive. It is only from both, the negation and the negated proposition, that we can conclude to a characteristic of the significance of the whole proposition. (We are not here speaking of negations of *general* propositions, i.e. of such as contain apparent variables. Negative facts only justify the negations of atomic propositions.)

Positive and *negative* facts there are, but not *true* and *false* facts.

<I> If we overlook the fact that propositions have a *sense* which is independent of their truth or falsehood, it easily seems as if true and false were two equally justified relations between the sign and what is signified. (We might then say e.g. that "*q*"[24] *signifies* in the true way what "not-*q*" *signifies* in the false way). But are not true and false in fact equally justified? Could we not express ourselves by means of false propositions just as well as hitherto with true ones, so long as we know that they are meant falsely? No! For a proposition is then[25] true when

it is as we assert in this proposition; and accordingly if by "q" we mean "not-q", and it is as we mean to assert, then in the new interpretation "q" is actually true and not false. But it is important that we can mean the same by "q" as by "not-q", for it shows that neither to the symbol "not" nor to the manner of its combination with "q" does a characteristic of the denotation of "q" correspond.

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<u>Second MS</u>.

<II> We must be able to understand propositions which we have never heard before. But every proposition is a new symbol. Hence we must have <u>general</u> indefinable symbols; these are unavoidable if propositions are not all indefinable.

<III> Whatever corresponds in reality to compound propositions must not be more than what corresponds to their several atomic propositions.

Not only must logic not deal with [particular] things, but just as little with relations and predicates.

<IV> There are no propositions containing real variables.

<I> What corresponds in reality to a proposition depends upon whether it is true or false. But we must be able to understand a proposition without knowing if it is true or false.

What we know when we understand a proposition is this: We know what is the case if the proposition is true, and what is the case if it is false. But we do not know [necessarily] whether it is true or false.

Propositions are not names.

<VI> We can never distinguish one logical type from another by attributing a property to members of the one which we deny to members of the other.

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<II> Symbols are not what they seem to be. In "a R b", "R" looks like a substantive, but is not one. What symbolizes in "*a* R *b*" is that R occurs between *a* and *b*. Hence "R" is *not* the indefinable in "*a* R *b*". Similarly in "φx", "φ" looks like a substantive but is not one; in "~p", "~" looks like "φ" but is not like it. This is the first thing that indicates that there *may* not be logical constants. A reason against them is the generality of logic: logic cannot treat a special set of things.

<III> Molecular propositions contain nothing beyond what is contained in their atoms; they add no material information above that contained in their atoms.

All that is essential about molecular functions is their T-F schema [i.e. the statement of the cases when they are true and the cases when they are false].

<V> Alternative indefinability shows that the indefinables have not been reached.

<I> Every proposition is essentially true-false: to understand it, we must know both what must be the case if it is true, and what must be the case if it is false. Thus a proposition has two *poles*, corresponding to the case of its truth and the case of its falsehood. We call this the *sense* of a proposition.

<V> In regard to notation, it is important to note that not every feature of a symbol symbolizes. In two molecular functions which have the same T-F schema, what symbolizes must be the same. In ″not-not-p″, ″not-p″ does not occur; for ″not-not-p″ is the same as ″p″, and therefore, if ″not-p″ occurred in ″not-not-p″, it would occur in ″p″.

<III> Logical indefinables cannot be predicates or relations, because propositions, owing to sense, cannot have predicates or relations. Nor are ″not″ and ″or″, like judgment, analogous to predicates or relations, because they do not introduce anything new.

<IV> Propositions are always complex even if they contain no names.

<II> A proposition must be understood when all its indefinables are understood. The indefinables in ″a R b″ are introduced as follows:

″a″ is indefinable;
″b″ is indefinable;
Whatever ″x″ and ″y″ may mean, ″x R y″ says something indefinable about their meaning.

<V> A complex symbol must never be introduced as a single indefinable. [Thus e.g. no proposition is indefinable). For if one of its parts occurs also in another connection, it must there be re-introduced. And would it then mean the same?

The ways by which we introduce our indefinables must permit us to construct all propositions that have sense [? meaning] from these indefinables <u>alone</u>. It is easy to introduce ″all″ and ″some″ in a way that will make the construction of (say) ″(x, y). x R y″ possible from ″all″ and ″x R y″ <u>as introduced before</u>.

<u>3rd. MS</u>.

<I> An analogy for the theory of truth: Consider a black patch on white paper; then we can describe the form of the patch by mentioning, for each point of the surface, whether it is white or black. To the fact that a point is black corresponds a positive fact, to the fact that a point is white (not black) corresponds a negative fact. If I designate a point of the surface (one of Frege's "truth-values"), this is as if I set up an assumption to be decided upon. But in order to be able to say of a point that it is black or that it is white, I must first know when a point is to be called black and when it is to be called white. In order to be able to say that "p" is true (or false), I must first have determined under what circumstances I call a proposition true, and thereby I determine the <u>sense</u> of a proposition. The point in which the analogy fails is this: I can indicate a point of the paper what is white[26] and black, but to a proposition without sense nothing corresponds, for it does not designate a thing (truth-value), whose properties might be called "false" or "true"; the verb of a proposition is not "is true" or "is false", as Frege believes, but what is true must already contain the verb.

<I> The comparison of language and reality is like that of retinal image and visual image: to the blind spot nothing in the visual image seems to correspond, and thereby the boundaries of the blind spot determine the visual image - as true negations of atomic propositions determine reality.

<III> Logical inferences can, it is true, be made in accordance with Frege’s or Russell’s laws of deduction, but this cannot justify the inference; and therefore they are not primitive propositions of logic. If p follows from q, it can also be inferred from q, and the ”manner of deduction” is indifferent.

<IV> Those symbols which are called propositions in which ”variables occur” are in reality not propositions at all, but only schemes of propositions, which only become propositions when we replace the variables by constants. There is no proposition which is expressed by ”x = x”, for ”x” has no signification; but there is a proposition ”(x). x = x” and propositions such as ”Socrates = Socrates” etc.

<IV> In books on logic, no variables ought to occur, but only the general propositions which justify the use of variables. It follows that the so-called definitions of logic are not definitions, but only schemes of definitions, and instead of these we ought to put general propositions; and similarly the so-called primitive ideas (Urzeichen) of logic are not primitive ideas, but the schemes of them. The mistaken idea that there

are things called facts or complexes and relations easily leads to the opinion that there must be a relation of questioning[10]⟨(?)⟩[27] to the facts, and then the question arises whether a relation can hold between an arbitrary number of things, since a fact can follow from arbitrary cases. It is a fact that the proposition which e.g. expresses that q follows from p and $p \supset q$ is this: $p.\ p \supset q.\ \supset_{p.q}.q.$

<I> At a pinch, one is tempted to interpret "not-p" as "everything else, only not p". That from a single fact p an infinity of others, not-not-p etc., follow, is hardly credible. Man possesses an innate capacity for constructing symbols with which some sense can be expressed, without having the slightest idea what each word signifies. The best example of this is mathematics, for man has until lately used the symbols for numbers without knowing what they signify or that they signify nothing.

<II> Russell's "complexes" were to have the useful property of being compounded, and were to combine with this the agreeable property that they could be treated like "simples". But this alone made them unserviceable as logical types, since there would have been significance in asserting, of a simple, that it was complex. But a property cannot be a logical type.

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<II> Every statement about apparent complexes can be resolved into the logical sum of a statement about the constituents and a statement about the proposition which describes the complex completely. How, in each case, the resolution is to be made, is an important question, but its answer is not unconditionally necessary for the construction of logic.

<III> That ”or” and ”not” etc. are not relations in the same sense as ”right” and ”left” etc., is obvious to the plain man. The possibility of cross-definitions in the old logical indefinables shows, of itself, that these are not the right indefinables, and, even more conclusively, that they do not denote relations.

<VI> If we change a constituent <u>a</u> of a proposition $\varphi(a)$ into a variable, then there is a class

$$\tilde{p}\,\{(\exists x).\ \varphi(x) = p\}.$$

This class in general still depends upon what, by an <u>arbitrary</u> <u>convention</u>, we mean by ”$\varphi(x)$”. But if we change into variables all those symbols whose significance was arbitrarily determined, there is still such a class. But this is now not dependent upon any convention, but only upon the nature of the symbol ”$\varphi(x)$”. It corresponds to a logical type.

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<VI> Types can never be distinguished from each other by saying (as is often done) that one has these *but* the other has those properties, for this presupposes that there is a *meaning* in asserting all these properties of both types. But from this it follows that, at best, these properties may be types, but certainly not the objects of which they are asserted.

<I> At a pinch we are always inclined to explanations of logical functions of propositions which aim at introducing into the function either only the constituents of these propositions, or only their form, etc. etc.; and we overlook that ordinary language would not contain the whole propositions if it did not need them: However, e.g., "not-*p*" may be explained, there must always be a meaning given to the question "what is denied?"

<III> The very possibility of Frege's explanations of "not-p" and "if *p* then *q*", from which it follows that "not-not-p" denotes the same as *p*, makes it probable that there is some method of designation in which "not-not-*p*" corresponds to the same symbol as "*p*". But if this method of designation suffices for logic, it must be the right one.

<I> Names are points, propositions arrows - they have *sense*. The sense of a proposition is determined by the two poles *true* and *false*. The form of a proposition is like a straight line, which divides all points of a plane into right and left. The line does this autom┌at┐ically, the form of proposition only by convention.

<II> Just as little as we are concerned, in logic, with the relation of a name to its meaning, just so little are we concerned with the relation of a proposition to reality, but we want to know the meaning of names and the sense of propositions as we introduce an indefinable concept "A" by saying: "'A' denotes something indefinable", so we introduce e.g. the form of propositions *a* R *b* by saying: "For all meanings of "x" and "y", "x R y" expresses something indefinable about x and y".

<III> In place of every proposition "p", let us write "${}^{a}_{b}p$". Let every correlation of propositions to each other or of names to propositions be effected by a correlation of their poles "a" and "b". Let this correlation be transitive. Then accordingly "${}^{a-a}_{b-b}p$" is the same symbol as "${}^{a}_{b}p$". Let *n* propositions be given. I then call a "class of poles" of these propositions every class of *n* members, of which each is a pole of one of the *n* propositions, so that one member corresponds to each proposition. I then correlate with each class of poles one of two poles (*a* and *b*). The sense of the symbolizing fact thus constructed I cannot define, but I know it.

<III> If p = not-not-p etc., this shows that the traditional method of symbolism is wrong, since it allows a plurality of symbols with the same sense; and thence it follows that, in analyzing such propositions, we must not be guided by Russell's method of symbolizing.

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<V> It is to be remembered that names are not things, but classes: "A" is the same letter as "A". This has the most important consequences for every symbolic language.

<I> Neither the sense nor the meaning of a proposition is a thing. These words are incomplete symbols.

<V> It is impossible to dispense with propositions in which the same argument occurs in different positions. It is obviously useless to replace $\varphi(a, a)$ by $\varphi(a, b). a = b$.

<III> Since the <u>ab</u>-functions of <u>p</u> are again bi-polar propositions, we can form <u>ab</u>-functions of them, and so on. In this way a series of propositions will arise, in which in general the <u>symbolizing</u> facts will be the same in several members. If now we find an <u>ab</u>-function of such a kind that by repeated application of it every ab-function can be generated, then we can introduce the totality of ab-functions as the totality of those that are generated by application of this function. Such a function is $\sim p \vee \sim q$.

<III> It is easy to suppose a contradiction in the fact that on the one hand every possible complex proposition is a simple <u>ab</u>-function of simple propositions, and that on the other hand the repeated application of one <u>ab</u>-function suffices to generate all these propositions. If e.g. an affirmation can be generated by double negation, is negation in any sense contained in affirmation? Does "p" deny "not-p" or assert "p", or both? And how do

matters stand with the definition of ”⊃” by ”∨” and ”.”, or of ”∨” by ”.” and ”⊃”? And how e.g. shall we introduce p[”||]q (i.e. ~p ∨ ~q), if not by saying that this expression says something indefinable about all arguments <u>p</u> and <u>q</u>? But the <u>ab</u>-functions must be introduced as follows: The function p[”||]q is merely a mechanical instrument for constructing all possible <u>symbols</u> of <u>ab</u>-functions. The symbols arising by repeated application of the symbol ”|” do <u>not</u> contain the symbol ”p|q”. We need a rule according to which we can form all symbols of <u>ab</u>-functions, in order to be able to speak of the class of them; and we now speak of them e.g. as those symbols of functions which can be generated by repeated application of the operation ”|”. And we say now: For all p’s and q’s, ”p|q” says something indefinable about the sense of those simple propositions which are contained in p and q.

<I> The assertion-sign is logically quite without significance. It only shows, in Frege and Whitehead and Russell, that these authors hold the propositions so indicated to be true. ”⊢” therefore belongs as little to the proposition as (say) the number of the proposition. A proposition cannot possibly assert of itself that it is true.

Every right theory of judgment must make it impossible for me to judge that this table penholders the book. Russell’s theory does not satisfy this requirement.

<I> It is clear that we understand propositions without knowing whether they are true or false. But we can only know the <u>meaning</u> of a proposition when we know if it is true or false. What we understand is the <u>sense</u> of the proposition.

<III> The assumption of the existence of logical objects makes it appear remarkable that in the sciences propositions of the form "$p \vee q$", "$p \supset q$", etc. are only then not provisional when "$\vee$" and "$\supset$" stand within the scope of a generality-sign [apparent variable].

4th. MS.

<II> If we formed all possible atomic propositions, the world would be completely described if we declared the truth or falsehood of each. [I doubt this.]

<I> The chief characteristic of my theory is that, in it, p has the same meaning as not-p.

<II> A false theory of relations makes it easily seem as if the relation of fact and constituent were the same as that of fact and fact which follows from it. But the similarity of the two may be expressed thus: $\varphi a . \supset ._{\varphi, a} a = a$.

<II> If a word creates a world so that in it the principles of logic are true, it thereby creates a world in which the whole of mathematics holds; and similarly it could not create a world in which a proposition was true, without creating its constituents.

<III> Signs of the form "p ∨ ~p" are senseless, but not the proposition "(p). p ∨ ~p". If I know that this rose is either red or not red, I know nothing. The same holds of all ab-functions.

<I> To understand a proposition means to know what is the case if it is true. Hence we can understand it without knowing if it is true. We understand it when we understand its constituents and forms. If we know the meaning of "a" and "b", and if we know what "x R y" means for all x's and y's, then we also understand "a R b".

<I> I understand the proposition "a R b" when I know that either the fact that a R b or the fact that not a R b corresponds to it; but this is not to be confused with the false opinion that I understood "a R b" when I know that "a R b or not a R b" is the case.

<II> But the form of a proposition symbolizes in the following way: Let us consider symbols of the form "x R y"; to these correspond primarily pairs of objects, of which one has the name "x", the other the name "y". The x's and y's stand in various relations to each other, among others the relation R holds between some, but not between others. I now determine the sense of "x R y" by laying down: when the facts behave in regard to "x R y" so that the meaning of "x" stands in the relation R to the meaning of "y", then I say that they [the facts] are "of like sense" ["gleichsinnig"] with the proposition "x R y"; otherwise, "of opposite sense" [entgegengesetzt"]; I correlate the facts to the symbol "x R y" <u>by</u> thus dividing them into those of like sense and those of opposite sense. To this correlation corresponds the correlation of name and meaning. Both are psychological. Thus I understand the form "x R y" when I know that it discriminates the behaviour of x and y according as these stand in the relation R or not. In this way I extract from all possible relations the relation R, as, by a name, I extract its meaning from among all possible things.

<I> Strictly speaking, it is incorrect to say: we understand the proposition p when we know that '"p" is true' ≡ p; for this would naturally always be the case if accidentally the propositions to right and left of the symbol "≡" were both true or both false. We require not only an equivalence, but a formal equivalence, which is bound up with the introduction of the form of p.

<III> The sense of an ab-function of p is a function of the sense of p.

<III> The ab-functions use the discrimination of facts, which their arguments bring forth, in order to generate new discriminations.

<V> Only facts can express sense, a class of names cannot. This is easily shown.

<II> There is no thing which is the form of a proposition, and no name which is the name of a form. Accordingly we can also not say that a relation which in certain cases holds between things holds sometimes between forms and things. This goes against Russell's theory of judgment.

[28]**[**It is very easy to forget that, though the propositions of a form can be either true or false, each one of these propositions can only be either true or false, not both.**]**[28]

<III> Among the facts which make "p or q" true, there are some which make "p and q" true; but the class which makes "p or q" true is different from the class which makes "p and q" true; and only this is what matters. For we introduce this class, as it were, when we introduce ab-functions.

<IV> A very natural objection to the way in which I have introduced e.g. propositions of the form $x \, R \, y$ is that by it propositions such as $(\exists. x. y). x \, R \, y$ and similar ones are not explained, which yet obviously have in common with $a \, R \, b$ what $c \, R \, d$ has in common with $a \, R \, b$. But when we introduce propositions of the form $x \, R \, y$ we mentioned no one particular proposition of this form; and we only need to introduce $(\exists \, x, y). \varphi(x, y)$ for all φ's in any way which makes the sense of these propositions dependent on the sense of all propositions of the form $\varphi(a, b)$, and thereby the justification[2]ness[1] of our procedure is proved.

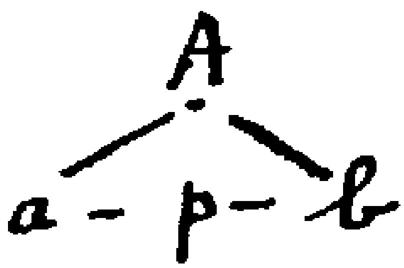
<V> The indefinables of logic must be independent of each other. If an indefinable is introduced, it must be introduced in all combinations in which it can occur. We cannot therefore introduce it first for one combination, then for another; e.g., if the form $x \, R \, y$ has been introduced, it must henceforth be understood in propositions of the form $a \, R \, b$ just in the same way as in propositions such as $(\exists x, y). x \, R \, y$ and others. We must not introduce it first for one class of cases, then for the other; for it would remain doubtful if its meaning was the same in

both cases, and there would be no ground for using the same manner of combining symbols in both cases. In short, for the introduction of indefinable symbols and combinations of symbols the same holds, mutatis mutandis, that Frege has said for the introduction of symbols by definitions.

<III> It is a priori likely that the introduction of atomic propositions is fundamental for the understanding of all other kinds of propositions. In fact the understanding of general propositions obviously depends on that of atomic propositions.

<IV> Cross-definability in the realm of general propositions leads to the[2] quite similar questions to those in the realm of *ab*-functions.

<I> When we say "A believes *p*", this sounds, it is true, as if here we could substitute a proper name for "*p*"; but we can see that here a *sense*, not a meaning, is concerned, if we say "A believes that '*p*' is true"; and in order to make the direction of *p* even more explicit, we might say "A believes that 'p' is true and 'not-p' is false". Here the bi-polarity of *p* is expressed and it seems that we shall only be able to express the proposition "A believes *p*" correctly by the *ab*-notation; say by making "A" have a relation to the poles "a" and "b" of a-p-b.



The epistemological questions concerning the nature of judgment and belief cannot be solved without a correct apprehension of the fo*ur*[10]rm[1] of the proposition.

<III> The ab-notation shows the dependence of or and not, and thereby that they are not to be employed as simultaneous indefinables.

<V> Not: "The complex sign 'a R b'" says that a stands in the relation R to b; but that 'a' stands in a certain relation to 'b' says that a R b.

([29]**[**⟨Preliminary⟩[6] In philosophy there are no
(deductions: it is purely descriptive.
(Philosophy gives no pictures of reality.
(Philosophy can neither confirm nor confute
(scientific investigation.
(Philosophy consists of logic and metaphysics:
(logic is its basis.
(Epistemology is the philosophy of psychology.
(Distrust of grammar is the first requisite
(for philosophizing.**]**[29]

[28]**[**Propositions can never be indefinables, for they are always complex. That also words like "ambulo" are complex appears in the fact that their root with a different termination gives a different sense.**]**[28]

<II> Only the doctrine of general indefinables permits us to understand the nature of functions. Neglect of this doctrine leads to an impenetrable thicket.

([29]**[**⟨Preliminary⟩[6]
(Philosophy is the doctrine of the logical
(form of scientific propositions (not only of
(primitive propositions).
(The word "philosophy" ought always to
(designate something over or under but not beside,
(the natural sciences.**]**[29]

<I>   Judgment, command and question all stand on the same level; but all have in common the propositional form, which does interest <u>us</u>.

<I>   The structure of the proposition must be recognized, the rest comes of itself. But ordinary language conceals the structure of the proposition: in it, relations look like predicates, predicates like names, etc.

Facts cannot be <u>named</u>.

<VI>   It is easy to suppose that "individual", "particular", "complex" etc. are primitive ideas of logic. Russell e.g. says "individual" and "matrix" are "primitive ideas". This error presumably is to be explained by the fact that, by employment of variables instead of the generality-sign it comes to seem as if logic dealt with things which have been deprived of all properties except thing-hood, and with propositions deprived of all properties except complexity. We forget that the indefinables of symbols [Urbilder von Zeichen] only occur under the generality-sign, never outside it.

<IV> Just as people used to struggle to bring all propositions into the subject-predicate form, so now it is natural to conceive every proposition as expressing a relation, which is just as incorrect. What is justified in this desire is fully satisfied by Russell's theory of manufactured relations.

<I> One of the most natural attempts at solution consists in regarding "not-p" as "the opposite of p", where then "opposite" would be the indefinable relation. But it is easy to see that every such attempt to replace the ab-functions by descriptions must fail.

<I> The false assumption that propositions are names leads us to believe that there must be logical objects: for the meanings of logical propositions will have to be such things.

⟨Preliminary⟩[6]

A correct explanation of logical propositions must give them a unique position as against all other propositions.

No proposition can say anything about itself, because the symbol of the proposition cannot be contained in itself; this must be the basis of the theory of logical types.

<VI> Every proposition which says something indefinable about a thing is a subject-predicate proposition; every proposition which says something indefinable about two things expresses a dual relation between these things, and so on. Thus every proposition which contains only one name and one indefinable form is a subject-predicate proposition, and so on. An

indefinable simple symbol can only be a name, and therefore we can know, by the symbol of an atomic proposition, whether it is a subject-predicate proposition.

Item 201a-3 (also referred to as TS_x)

Transcriber(s): Michael Biggs, Peter Cripps
Proofreader(s): Peter Cripps
Comments:
Transcriptions of material from The Wittgenstein Archives at the University of Bergen. Item referred to by Michael Biggs as TS_x (related to 201a). This is a typescript of 15ff. The first 2 folios are unnumbered, the foliation then runs from 2 to 14. Some logical notation symbols added by hand, others omitted. References to the published text of Tractatus Logico-Philosophicus are added by hand. Fully typed pages have between 61 and 65 lines of type; each new sentence is separated from the last by three spaces. Blank lines between paragraphs.
Hands:
s = handwritten insertions that belong to first pass, ie generally the insertion of symbols which are not found on the typewriter such as φ and ∃
S2 = handwritten insertions presumably made D. Schwayder.

Notes on Logic
by
Ludwig Wittgenstein
September 1913.

SUMMARY.

One reason for thinking the old notation wrong is that it is very unlikely that from every proposition p an infinite number of other propositions not-not-p, not-not-not-not-p, etc., should follow.

If only those signs which contain proper names were complex then propositions containing nothing but apparent variables would be simple. Then what about their denials?

The verb of a proposition cannot be ”is true” or ”is false”, but whatever is true or false must already contain the verb.

Deductions only proceed according to the laws of deduction, but these laws cannot justify the deduction.

One reason for supposing that not all propositions which have more than one argument are relational propositions is that if they were, the relations of judgment and inference would have to hold between an arbitrary number of things.

Every proposition which seems to be about a complex can be analysed into a proposition about its constituents and about the proposition which describes the complex perfectly; i.e., that proposition which is equivalent to saying the complex exists.

The idea that propositions are names of complexes suggests that whatever is not a proper name is a sign for a relation. Because spatial complexes* consist of Things and Relations only and the idea of a complex is taken from space.

In a proposition convert all its indefinables into variables; there then remains a class of propositions which is not all propositions but a type.

There are thus two ways in which signs are similar. The names Socrates and Plato are similar: they are both names. But whatever they have in common must not be introduced before Socrates and Plato are introduced. The same applies to a subject-predicate form etc. Therefore, thing, proposition, subject-predicate form, etc., are not indefinables, i.e., types are not indefinables.

When we say A judges that etc., then we have to mention a whole proposition which A judges. It will not do either to mention only its constituents, or its constituents and form, but not in the proper order. This shows that a proposition itself must occur in the statement that it is judged; however, for instance, "not-p" may be explained, the question, "What is negated" must have a meaning.

To understand a proposition p it is not enough to know that p implies '"p" is true', but we must also know that ~p implies "p is false". This shows the bi-polarity of the proposition.

To every molecular function a WF* scheme corresponds. Therefore we may use the WF scheme itself instead of the function. Now what the Wf scheme does is, it correlates the letters W and F with each proposition. These two letters are the poles of atomic propositions. Then the scheme correlates another W and F to these poles. In this notation all that matters is the correlation of the outside poles to the poles of the atomic propositions. Therefore not-not-p is the same symbol as p. And therefore we shall never get two symbols for the same molecular function.

⟨* Russell - for instance imagines every fact as a spatial complex.

* W-F = Wahr-Falsch.⟩[23]

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<⊗> The meaning of a proposition is the fact which actually corresponds to it.

As the ab functions of atomic propositions are bi-polar propositions again we can perform ab operations on them. We shall, by doing so, correlate two new outside poles via the old outside poles to the poles of the atomic propositions.

The symbolising fact in a-p-b is that, say* a is on the left of p and b on the right of p; then the correlation of new poles is to be transitive, so that for instance if a new pole a in whatever way i.e. via whatever poles is correlated to the inside a, the symbol is not changed thereby. It is therefore possible to construct all possible ab functions by performing one ab operation repeatedly, and we can therefore talk of all ab functions as of all those functions which can be obtained by performing this ab operation repeatedly.

(Note by B.R. ab means the same as WF, which means true-false.?)

Naming is like pointing. A function is like a line dividing points of a plane into right and left ones; then ”p or not-p” has no meaning because it does not divide the plane.

But though a particular proposition ”p or not-p” has no meaning, a general proposition ”for all p’s, p or not-p” has a meaning because this does not contain the nonsensical function ”p or not-p” but the function ”p or not-q” just as ”for all x’s xRx” contains the function ”xRy”.

<⊗> A proposition is a standard to which facts behave, with names it is otherwise; it is thus bi-polarity and sense comes in; just as one arrow behaves to another arrow by being in the same sense or the opposite, so a fact behaves to a proposition.

<⊗> The form of a proposition has meaning in the following way. Consider a symbol ”xRy”. To symbols of this form correspond couples of things whose names are respectively ”x” and ”y”. The things *x y* stand to one another in all sorts of relations, amongst others some stand in the relation R, and some not; just as I single out a particular thing by a particular name I single out all behaviours of the points x and y with respect to the relation R. I say that if an x stands in the relation R to a y the sign ”xRy” is to be called true to the fact and otherwise false. This is a definition of sense.

<✓⊗> In my theory p has the same meaning as not-p but opposite sense. The meaning is the fact. The proper theory of judgment must make it impossible to judge nonsense.

It is not strictly true to say that we understand a proposition p if we know that p is equivalent to ”p is true” for this would be the case if accidentally both were true or false. What is wanted is the formal equivalence with respect to the forms of the proposition, *i.e.*, all the general indefinables involved. *The sense of* an *ab* function of a proposition is a function of its sense. There are only unasserted propositions. Assertion is merely psychological. In *not-p*, *p* is exactly the same as if it stands alone; this point is absolutely fundamental. Among the facts that make "p or q" true there are also facts which make "p and q" true; if propositions have only meaning, we ought, in such a case, to say that these two

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⟨* This is quite arbitrary but, if we once have fixed on which order the poles have to stand we must of course stick to our convention. If for instance "a p b" says p then b p a says <u>nothing</u>. (It does not say p) But a - a p b - b is the same symbol as apb (here the ab function vanishes automatically) for here the new poles are related to the same side of p as the old ones. The question is always: how are the new poles correlated to p compared with the way the old poles are correlated to p.⟩[23]

propositions are identical, but in fact, their sense is different for we have introduced sense by talking of all p’s and all q’s. Consequently the molecular propositions will only be used in cases where their ab function stands under a generality sign or enters into another function such as ”I believe that, etc”., because then the sense enters.

In ”a judges p” p cannot be replaced by a proper name. This appears if we substitute ”a judges that p is true and not p is false”. The proposition ”a judges p” consists of the proper name a, the proposition p with its 2 poles, and a being related to both of these poles in a certain way. This is obviously not a relation in the ordinary sense.

The ab notation makes it clear that not and or are dependent on one another and we can therefore not use them as simultaneous indefinables. Same objections in the case of apparent variables to the usual indefinables, as in the case of molecular functions. The application of the ab notation to apparent variable propositions becomes clear if we consider that, for instance, the proposition ”for all x, ϕ x” is to be true when ϕ x is true for all x’s and false when ϕ x is false for some x’s. We see that some and all occur simultaneously in the proper apparent variable notation. The Notation is:

for (x) ϕ x : a - (x) - a ϕ x b - (x) - b

and

for (x) x : a - (x) - a ϕ x b - (x) - b

Old definitions now become tautologous.

[30][In aRb it is not the complex that symbolises but the fact that the symbol a stands in a certain relation to the symbol b. Thus facts are symbolised by facts, or more correctly: that a certain thing is the case in the symbol says that a certain thing is the case in the world.][30]

Judgment, question and command are all on the same level. What interests logic in them is only the unasserted proposition. Facts cannot be named.

A proposition cannot occur in itself. This is the fundamental truth of the theory of types.

Every proposition that says something indefinable about one thing is a subject-predicate proposition, and so on.

Therefore we can recognise a subject-predicate proposition if we know it contains only one name and one form, etc. This gives the construction of types. Hence the type of a proposition can be recognised by its symbol alone.

What is essential in a correct apparent-variable notation is this:- (1) it must mention a type of propositions; (2) it must show which components of a proposition of this type are constants.

(Components are forms and constituents.)

Tale (ϕ). [x|ϕ]!x. Then if we describe the <u>kind</u> of symbols, for which ϕ! stands and which, by the above, is enough to determine the type, then automatically " (ϕ). ϕ! x" cannot be fitted by this description, because it <u>CONTAINS</u> "ϕ! x" and the description is to describe <u>ALL</u> that symbolises in symbols of the ϕ! kind. If the description is <u>thus</u> complete vicious circles can just as little occur as for instance (ϕ). (X) ϕ (where (X) ϕ is a subject-predicate proposition).

First MS

Indefinables are of two sorts: names, and forms. Propositions cannot consist of names alone; they cannot be classes of names. A name can not only occur in two different propositions, but can occur in the same way in both.

[30] **[**Propositions (which are symbols having reference to facts) are themselves facts: that this inkpot is on this table may express that I sit in this chair.**]**[30]

It can never express the common characteristic of two objects that we designate them by the same name but by two different ways of designation, for, since names are arbitrary, we might also choose different names, and where then would be the common element in the designations? Nevertheless one is always tempted, in a difficulty, to take refuge in different ways of designation.

Frege said "propositions are names"; Russell said "propositions correspond to complexes". Both are false; and especially false is the statement "propositions are names of complexes."

It is easy to suppose that only such symbols are complex as contain names of objects, and that accordingly " $(\exists x, \phi) . \phi x$" or " $(\exists x,y) . x R y$" must be simple. It is then natural to call the first of these the name of a form, the second the name of a relation. But in that case what is the meaning of (e.g.) "$\sim(\exists x,y) . x R y$"? Can we put "not" before a name?

The reason why "~Socrates" means nothing is that "~x" does not express a property of *x*.

<⊗> There are positive and negative facts: if the proposition "this rose is not red" is true, then what it signifies is negative. But the occurrence of the word "not" does not indicate this unless we know that the signification of the proposition "this rose is red" (when it is true) is positive. It is only from both, the negation and the negated proposition, that we can conclude to a characteristic of the significance of the whole proposition. (We are not here speaking of negations of *general* propositions, i.e. of such as contain apparent variables. Negative facts only justify the negations of atomic propositions.)

<⊗> *Positive* and *negative* facts there are, but not *true* and false facts.

<⊗> If we overlook the fact that propositions have a *sense* which is independent of their truth or falsehood, it easily seems as if true and false were two equally justified relations between the sign and what is signified. (We might then say e.g. that "*q*" signifies in the true way what "not-q" *signifies* in the false way). But are not true and false in fact equally justified? Could we not express ourselves by means of false propositions just as well as hitherto with true ones, so long as we know that they are meant falsely? No! For a proposition is then true when it is as we assert in this proposition; and accordingly if by "q" we mean "not-*q*", and it is as we mean to assert, then in the new interpretation "*q*" is actually true and *not* false. But it is important that we *can* mean the same by "*q*" as by "not-q", for it shows that neither to the symbol "not" nor to the manner of its

combination with "<u>q</u>" does a characteristic of the denotation of "<u>q</u>" correspond. <Cf. 4.061, 4.062, 4.0621>[31]

Second MS

We must be able to understand propositions which we have never heard before. But every proposition is a new symbol. Hence we must have general indefinable symbols; these are unavoidable if propositions are not all indefinable. <Cf 4.02, 4.021, 4.027>[31]

Whatever corresponds in reality to compound propositions must not be more than what corresponds to their several atomic propositions.

Not only must logic not deal with (particular) things, but just as little with relations and predicates.

There are no propositions containing real variables.

<⊗> What corresponds in reality to a proposition depends upon whether it is true or false. But we must be able to understand a proposition without knowing if it is true or false. <cf. 4.024>[31]

<⊗> What we know when we understand a proposition is this: We know [t|w]hat is the case if the proposition is true, and what is the case if it is false. But we do not know (necessarily) whether it is true or false. <cf. 4.024>[31]

Propositions are not names. <cf. 3.144>[31]

We can never distinguish one logical type from another by attributing a property to members of the one which we deny to members of the other.

<⊗> Symbols are not what they seem to be. In "a R b", "R" looks like a substantive, but is not one. What symbolizes in "*a* R *b*" is that R occurs between *a* and *b*. Hence "R" is *not* the indefinable in "*a* R *b*". Similarly in " ϕ x" " ϕ " looks like a substantive but is not one; in "~p" "~" looks like " ϕ " but is not like it. This is the first thing that indicates that the re *may* not be logical constants. A reason against them is the generality of logic: logic cannot treat a special set of things. <Cf. 3.1432>[31]

Molecular propositions contain nothing beyond what is contained in their atoms; they add no material information above that contained in their atoms.

All that is essential about molecular functions is their T-F schema (i.e the statement of the cases when they are true and the cases when they are false).

Alternative indefinability shows that the indefinables have not been reached.

<⊗> [30][Every proposition is essentially true-false: to understand it, we must know both what must be the case if it is true, and what must be the case if it is false.][30] Thus a proposition has two *poles*, corresponding to the case of its truth and the case of its falsehood. We call this the *sense* of a proposition.

In regard to notation, it is important to note that not every feature of a symbol symbolizes. In two molecular functions which have the same T-F schema, what symbolizes must be the same. In "not-not-*p*", "not-*p*" does not occur; for "not-not-*p*" is the same as "*p*", and therefore, if "not-p" occurred in "not-not-p", it would occur in "p".

Logical indefinables cannot be predicates or relations, because propositions, owing to sense, cannot have predicates or relations. Nor are "not" and "or", like judgment, analogous to predicates or relations, because they do not introduce anything new.

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Propositions are always complex even if they contain no names.

[30]**[**A proposition must be understood when <u>all</u> its indefinables are understood. The indefinables in ″a R b″ are introduced as follows:

″<u>a</u>″ is indefinable;

″<u>b</u>″ is indefinable;

Whatever ″x″ and ″y″ may mean, ″x R y″ says something indefinable about their meaning.**]**[30]

A complex symbol must never be introduced as a single indefinable. (Thus e.g. no proposition is indefinable). For if one of its parts occurs also in another connection, it must there be re-introduced. And would it then mean the same?

The ways by which we introduce our indefinables must permit us to construct all propositions that have sence [? meaning][32] from these indefinables <u>alone</u>. It is easy to introduce ″all″ and ″some″ in a way that will make the construction of **[**″|(**]**say) ″(x, y) .x R y″ possible from ″all″ and ″x R y″ <u>as introduced before</u>.

Third MS

An analogy for the theory of truth: Consider a black patch on white paper; then we can describe the form of the patch by mentioning, for each point of the surface, whether it is white or black. To the fact that a point is black corresponds a positive fact, to the fact that a point is white (not black) corresponds a negative fact. If I designate a point of the surface (one of Frege's "truth-values"), this is as if I set up an assumption to be decided upon. But in order to be able to say of a point that it is black or that it is white, I must first know when a point is to be called black and when it is to be called white. In order to be able to say that "p" is true (or false), I must first have determined under what circumstances I call a proposition true, and thereby I determine the sense of a proposition. The point in which the analogy fails is this: I can indicate a point of the paper that is white and black, but to a proposition without sense nothing corresponds, for it does not designate a thing (truth-value), whose properties might be called "false" or "true"; the verb of a proposition is not "is true" or "is false", as Frege believes, but what is true must already contain the verb. <See 4.063>[31]

The comparison of language and reality is like that of retinal image and visual image: to the blind spot nothing in the visual image seems to correspond, and thereby the boundaries of the blind spot determine the visual image - as true negations of atomic propositions determine reality.

Logical inferences can, it is true, be made in accordance with Frege's or Russell's laws of deduction, but this cannot justify the inference; and therefore they are not primitive propositions of logic. If p follows from q, it can also be inferred from q, and the "manner of deduction" is indifferent. <Cf 5.132>[31]

Those symbols which are called propositions in which "variables occur" are in reality not propositions at all, but only schemes of propositions, which only become propositions when we replace the variables by constants. There is no proposition which is expressed by "x = x", for "x" has no signification; but there is a proposition "(x). x = x" and propositions such as "Socrates = Socrates" etc.

In books on logic, no variables ought to occur, but only the general propositions which justify the use of variables. It follows that the so-called definitions of logic are not definitions, but only schemes of definitions, and instead of these we ought to put general propositions; and similarly the so-called primitive ideas (Urzeichen) of logic are not primitive ideas, but the schemes of them. The mistaken idea that there are things called facts or complexes and relations easily leads to the opinion that there must be a relation of questioning[32] ⟨proposition⟩[33] to the facts, and then the question arises whether a relation can hold between an arbitrary number of things, since a fact can follow from arbitrary cases. It is a fact that the proposition which e.g. expresses that q follows from p and p ⊃ q is this: $p.p \supset q. \supset_{p.q} q$.

At a pinch, one is tempted to interpret "not-p" as "everything[34] else, only not p". That from a single fact *p* an infinity of others, not-not-p etc., follow, is hardly credible. Man possesses an innate capacity for constructing symbols with which *some* sense can be expressed, without having the slightest idea what each word signifies. The best example of this is mathematics, for man has until lately used the symbols for numbers without knowing what they signify or that they signify nothing. <Cf. 5.43>[31]

Russell's "complexes" were to have the useful property of being compounded, and were to combine with this the agreeable property that they could be treated like "simples". But this alone made them unserviceable as logical types, since there would have been significance in asserting, of a simple, that it was complex. But a property cannot be a logical type.

Every statement about apparent complexes can be resolved into the logical sum of a statement about the constituents and a statement about the proposition which describes the complex completely. How, in each case, the resolution is to be made, is an important question, but its answer is not unconditionally necessary for the construction of logic. <Cf 2.0201>[31]

That "or" and "not" etc. are not relations in the same sense as "right" and "left" etc., is obvious to the plain man. The possibility of cross-definitions in the old logical indefinables shows, of itself, that these are not the right indefinables, and, even more conclusively, that they do not denote relations. <See[31] 5.42>[31]

If we change a constituent a[35] of a proposition ϕ (a) into a variable, then there is a class

$$\hat{p}\ [(\exists x).\ \phi(x) = p]\ .$$

This class in general still depends upon what, by an arbitrary convention, we mean by "ϕ (x)". But if we change into variables all those symbols whose significance was arbitrarily determined, there is still such a class. But this is now not dependent upon any convention, but only upon the nature of the symbol "ϕ (x)". It corresponds to a logical type. <Cf 3.315>[31]

Types can never be distinguished from each other by saying (as is often done) that one has these *but* the other has those properties, for this presupposes that there is a *meaning* in asserting all these properties of both types. But from this it follows that, at best, these properties may be types, but certainly not the objects of which they are asserted. <[See|Cf[36]] 4.124>[31]

At a pinch we are always inclined to explanations of logical functions of propositions which aim at introducing into the functions either only the constituents of these propositions, or only their form, etc. etc.; and we overlook that ordinary language would not contain the whole propositions if it did not need them: However, e.g., "not-*p*" may be explained, there must always be a meaning given to the question "what is denied?"

The very possibility of Frege's explanations of "not-p" and "if *p* then *q*", from which it follows that "not-not-p" denotes the same as *p*, makes it probable that there is some method of designation in which "not-not-*p*" corresponds to the same symbol as "*p*". But if this method of designation suffices for logic, it must be the right one.

<⊗> Names are points, propositions arrows - they have *sense*. The sense of a proposition is determined by the two poles *true* and *false*. The form of a proposition is like a straight line, which divides all points of a plane into right and left. The line does this automatically, the form of proposition only by convention. See 3.144[31]

Just as little as we are concerned, in

logic, with the relation of a name to its meaning, just so little are we concerned with the relation of a proposition to reality, but we want to know the meaning of names and the sense of propositions as we introduce an ind[f|e]finable concept ”A” by saying: ”’A’ denotes something indefinable”, so we introduce e.g. the form of propositions a R b by saying: ”For all meanings of ”x” and ”y”, ”x R y” expresses something indefinable about x and y”.

In place of every proposition ”p[2|”], let us write ”$^{a}_{b}$ p”. Let every correlation of propositions to each other or of names to propositions be effected by a correlation of their poles "a" and "b". Let this correlation be transitive. Then accordingly "$^{a-a}_{b-b}$ p" is the same symbol as "$^{a}_{b}$ p". Let n propositions be given. I then call a "class of poles" of these propositions every class of n members, of which each is a pole of one of the n propositions, so that one member corresponds to each proposition. I then correlate with each class of poles one of two poles (a and b). The sense of the symbolizing fact thus constructed I cannot define, but I know it.

If p = not-not-p etc., this shows that the traditional method of symbolism is wrong, since it allows a plurality of symbols with the same sense; and thence it follows that, in analyzing such propositions, we must not be guided by Russell's method of symbolizing.

It is to be remembered that names are not things, but classes: "A" is the same letter as "A". This has the most important consequences for every symbolic language. <See 3.203>[31]

Neither the sense nor the meaning of a proposition is a thing. These words are incomplete symbols.

It is impossible to dispense with propositions in which the same argument occurs in different positions. It is obviously useless to replace ϕ (a, a) by ϕ (a, b). a = b.

Since the ab-functions of p are again bi-polar propositions, we can form ab-functions of them, and so on. In this way a series of propositions will arise, in which in general the *symbolizing* facts will be the same in several members. If now we find an *ab*-function of such a kind that by repeated application of it every ab-function can be generated, then we can introduce the totality of ab-functions as the totality of those that are generated by application of this function. Such a function is $\sim p \vee \sim q$.

It is easy to suppose a contradiction in the fact that on the one hand every possible complex proposition is a simple *ab*-function of simple propositions, and that on the other hand the repeated application of one *ab*-function suffices to generate all these propositions. If e.g. an affirmation can be generated by double negation, is negation in any sense contained in affirmation? Does "p" deny "not-p" or assert "p", or both? And how do matters stand with the definition of "$\supset$" by "$\vee$" and "**[**.|~**]**", or of "$\vee$" by "**[**.|~**]**" and "$\supset$"? And how e.g. shall we introduce p/q (i.e. $\sim p \vee \sim q$) if not by saying that this expression says something indefinable about all arguments *p* and *q*? But the *ab*-functions must be introduced as follows: The function p/q is merely a mechanical instrument for constructing all possible *symbols* of *ab*-functions. The symbols arising by repeated application of the symbol " | " do *not* contain the symbol "p|q". We need a rule according to which we can form all symbols of *ab*-functions, in order to be able to speak of the class of them; and we now speak of them e.g. as those symbols of functions which can be generated by repeated application of the operation " | ". And we say now: For all p's and q's, "p|q" says something indefinable about the sense of those simple propositions which are contained in p and q. <See 5.44>[31]

The assertion-sign is logically quite without significance. It only shows, in Frege and

Whitehead and Russell, that these authors hold the propositions so indicated to be true. ″⊢″ therefore belongs as little to the proposition as (say) the number of the proposition. A proposition cannot possibly assert of itself that it is true. <See 4.42>[31]

Every right theory of judgment must make it impossible for me to judge that this table penholders the book. Russell’s theory does not satisfy this requirement. <5.5422>[31]

<⊗> It is clear that we understand propositions without knowing

whether they are true or false. But we can only know the meaning of a proposition when we know if it is true or false. What we understand is the sense of the proposition. <Cf 4.024>[31]

The assumption of the existence of logical objects makes it appear remarkable that in the sciences propositions of the form ″$p \vee q$″, ″$p \supset q$″, etc. are only then not provisional when ″ $\vee$ ″ and ″ $\supset$ ″ stand within the scope of a generality-sign (apparent variable).

Fourth MS

If we formed all possible atomic propositions, the world would be completely described if we declared the truth or falsehood of each. I doubt this. R? DS. <See[31] 4.26.>[31]

<⊗> The chief characteristic of my theory is that, in it, p has the same meaning as not-p. <Cf 4.0621>[31]

A false theory of relations makes it easily seem as if the relation of fact and constituent were the same as that of fact and fact which follows from it. But the similarity of the two may be expressed thus: ϕ a. $\supset$ (φ, a) a = a.

If a word created[32]S[31] a world so that in it the principles of logic are true, it thereby creates a world in which the whole of mathematics holds; and similarly it could not create a world in which a proposition was true, without creating its constituents. Cf 5.123[31]

Signs of the form "p ∨ ~p" are senseless, but not the propositions "(p). p ∨ ~p". If I know that this rose is either red or not red, I know nothing. The same holds of all ab-functions. Cf 4.461[31]

To understand a proposition means to know what is the case if it is true. Hence we can understand it without knowing if it is true. We understand it when we understand its constituents and forms. If we know the meaning of "a" and "b", and if we know what "x R y" means for all x"s and y's, then we also understand "a R b". Cf 4.024[31]

I understand the proposition ″a R b″ when I know that either the fact that a R b or the fact that not a R b corresponds to it; but this is not to be confused with the false opinion that I understood ″a R b″ when I know that ″a R b or not a R b″ is the case.

But the form of a proposition symbolizes in the following way: Let us consider symbols of the form ″x R y″; to these correspond primarily pairs of objects, of which one has the name ″x″, the other the name ″y″. The x′s and y′s stand in various relations to each other, among others the relation R holds between some, but not between others. I now determine the sense of ″x R y″ by laying down: when the facts behave in regard to ″x R y″ so that the meaning of ″x″ stands in the relation R to the meaning of ″y″, then I say that they (the facts) are ″of like sense″ (″gleichsinnig″) with the proposition ″x R y″; otherwise, ″of opposite sense″ (entgegengesetzt″); I correlate the facts to the symbol ″x R y″ <u>by</u> thus dividing them into those of like sense and those of opposite sense. To this correlation corresponds the correlation of name and meaning. Both are psychological. Thus I understand the form ″x R y″ when I know that it discriminates the behaviour of x and y according as these stand in the relation R or not. In this way I extract from all possible relations the relation R, as, by a name, I extract its meaning from among all possible things.

Strictly speaking, it is incorrect to say: we understand the proposition <u>p</u> when we know that ′″p″ is true′ ≡ p; for this would naturally always be the case if accidentally the propositions to right and left of the symbol ″≡″ were both true or both false. We require not only an equivalence, but a formal equivalence, which is bound up with the introduction of the form of *p*.

The sense of an ab-function of *p* is a function of the sense of *p*. <See 5.2341>[31]

The *ab*-functions use the discrimination of facts, which their arguments bring forth, in order to generate new discriminations.

Only facts can express sense, a class of names cannot. This is easily shown. <See 3.142>[31]

There is no thing which is the form of a proposition, and no name which is the name of a form. Accordingly we can also not say that a relation which in certain cases holds between things holds sometimes between forms and things. This goes against Russell’s theory of judgment.

<X> It is very easy to forget that, though the propositions of a form can be either true or false, each one of these propositions can only be either true or false**[.|,]** not both

This was typed in but has e~~xce~~sses through it. (D.S.)

Among the facts which make ”p or q” true, there are some which make ”p and q” true; but the class which makes ”p or q” true is different from the class which makes ”p and q” true; and only this is what matters. For we introduce this class, as it were, when we introduce <u>ab</u>-functions. <Cf 5.1241>[31]

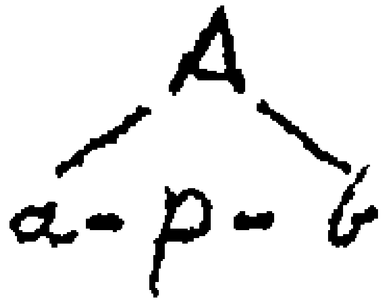
A very natural objection to the way in which I have introduced e.g. propositions of the form x R y is that by it propositions such as (∃ x. y). x R y and similar ones are not explained, which yet obviously have in common with a R b what c R d has in common~~x~~ with a R b. <u>But</u> when we introduced propositions of the form x R y we mentioned no one particular proposition of this form; and we only need to introduce (∃x, y). ϕ (x, y) for all ϕ’s in any way which makes the sense of these propositions dependent on the sense of all propositions of the form ϕ (a, b), and thereby the justness of our procedure is proved.

The indefinables of logic must be independent of each other. If an indefinable is introduced, it must be introduced in all combinations in which it can occur. We cannot therefore introduce it first for one combination, then for another; e.g., if the form x R y has been introduced, it must henceforth be understood in propositions of the form a R b just in the same way as in propositions such as (∃x, y). x R y and others. We must not introduce it first for one class of cases, then for the other; for it would remain doubtful if its meaning was the same in both cases, and there would be no ground for using the same manner of combining symbols in both cases. In short, for the introduction of indefinable symbols and combinations of symbols the same holds, mutatis mutandis, that Frege has said for the introduction of symbols by definitions. Cf 5.451[31]

It is a priori likely that the introduction of atomi[x|c] propositions is fundamental for the understanding of all other kinds of propositions. In fact the understanding of general propositions obviously depends on that of atomic propositions.

Cross-definability in the realm of general propositions leads to quite similar questions to those in the realm of *ab*-functions.

When we say "A believes *p*", this sounds, it is true, as if here we could substitute a proper name for "*p*"; but we can see that here a *sense*, not a meaning, is concerned, if we say "A believes that 'p' is true"; and in order to make the direction of *p* even more explicit, we might say "A believes that 'p' is true and 'not-p' is false". Here the bi-polarity of *p* is expressed and it seems that we shall only be able to express the proposition "A believes p" correctly by the *ab*-notation; say by making "A" have a relation to the poles "a" and "b" of a-p-b.



The epistemological questions concerning the nature of judgment and belief cannot be solved without a correct apprehension of the form of the proposition.

The *ab*-notation shows the dependence of *or* and *not*, and thereby that they are not to be employed as simultaneous indefinables.

Not: "The complex sign "a R b'" says that a stands in the relation R to b; but that 'a' stands in a certain relation to 'b' says that a R b. <3.143>[31]

In philosophy there are no deductions: it is purely descriptive.

Philosophy gives no pictures of reality.

Philosophy can neither confirm nor confute scientific investigation. <Cf 4.111>[31]

Philosophy consists of logic and metaphysics: logic is its basis.

Epistemology is the philosophy of psychology. <See 4.1126>[31]

Distrust of grammar is the first requisite for philosophizing.

Propositions can never be indefinables, for they are always complex. That also words like "ambulo" are complex appears in the fact that their root with a different termination gives a different sense. <4.032>[31] Crossed out but originally typed in. (D.S.)

Only the doctrine of general indefinables permits us to understand the nature of functions. Neglect of this doctrine leads to an impenetrable thicket.

Philosophy is the doctrine of the logical form of scientific propositions (not only of primitive propositions). <Cf 4.113>[31]

The word "philosophy" ought always to designate something over or under but not beside, the natural sciences. <See 4.111>[31]

<✓> Judgment, command and question all stand on the same level; but all have in common the propositional form, which does interest us.

<✓> The structure of the proposition must be recognized, the rest comes of itself. But ordinary language conceals the structure of the proposition: in it, relations look like predicates, predicates like names, etc. <Cf 4.002>[31]

Facts cannot be named. <See 3.144>[31]

It is easy to suppose that "individual", "particular", "complex" @.. etc. are primitive ideas of logic. Russell e.g. says "individual" and "matrix" are "primitive ideas". This error presumably is to be explained by the fact that, by employment of variables instead of the generality-sign, it come s to seem as if logic dealt with things which have been deprived of all properties except thing-hood, and with propositions deprived of all properties except complexity. We forget that the indefinables of symbols (Urbilder von Zeichen) only occur under the generality-sign, never outside it.

Just as people used to struggle to bring all propositions into the subject-predicate form, so now it is natural to conceive every proposition as expressing a relation, which is just as incorrect. What is justified in this desire is fully satisfied by Russell's theory of manufactured relations.

⟨ wrong termed? ⟩[33]

One of the most natural attempts at solution consists in regarding ”not-p” as ”the opposite of p”, where then ”opposite” would be the indefinable relation. But it is easy to see that every such attempt to replace the ab-functions by descriptions must fail.

The false assumption that propositions are names leads us to believe that there must be logical objects: for the meanings of logical propositions will have to be such things.

A correct explanation of logical propositions must give them a unique position as against all other propositions. <6.12>[31]

No proposition can say anything about itself, because the symbol of the proposition cannot be contained in itself; this must be the basis of the theory of logical types. <Cf 3.332>[31]

Every proposition which says something indefinable about a thing is a subject-predicate proposition; every proposition which says something indefinable about two things expresses a dual relation between these things, and so on. Thus every proposition which contains only one name and one indefinable form is a subject-predicate proposition, and so on. An indefinable simple symbol can only be a name, and therefore we can know, by the symbol of an atomic proposition, whether it is a subject-predicate proposition.

<

This is the symbol for $\sim p \vee \sim q$ >[31]

Notes

1 Insertion by S1
2 Deletion by S1
3 Overwriting by S1
4 Insertion by H5
5 Deletion by H5
6 Text in left margin by S1
7 Underlining by H5
8 Insertion by H5 in lower margin
9 Overwriting by H5
10 Underlining by S1
11 Insertion in right margin by H5
12 Text in right margin by H5
13 Point of insertion underlined by S1
14 Insertion by H5 in right margin
15 Insertion by H5 in upper margin
16 Text in left margin by H5
17 Cf. MS.; new paragraph?
18 Deletion by H5 - deletion cancelled by H5
19 Underlining with dotted line by H5
20 Underlining by H5 - underlining cancelled by H5
21 Cf. MS.; indentation?
22 Cf. MS.; inserted line
23 Insertion in lower margin
24 Cf. MS.; new line?
25 Possibly deleted.
26 Possibly underlined; doubt.
27 Text in right margin by S1
28 Text vertically or diagonally crossed through by S1
29 Curved line in left margin by S1
30 Straight line(s) in left margin by S2
31 Insertion by S2
32 Deletion by S2
33 Text in left margin by S2
34 Underlining by S2
35 Underlining cancelled by S2
36 Overwriting by S2

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