

Remarks on Bernays vs. Wittgenstein

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After his philosophical breakthrough, *Tractatus logico-philosophicus* (1921), and before his late masterpiece, *Philosophical Investigations* (1953), Ludwig Wittgenstein published next to nothing. This does not mean to say that he was short of philosophical inspiration during the thirty years from the early 1920s until the early 1950s. On the contrary, his unpublished work includes an immense amount of insightful and interesting material. Because of its fragmentary and partly unfinished nature, however, it is not always easy to relate this material to Wittgenstein's published contributions. Nevertheless, Wittgenstein also left behind material that was more or less ready for publication. For example, the first part of the posthumous *Remarks on the Foundations of Mathematics* (1956, hereafter: *RFM*) is an exact replica of Wittgenstein's original text. Also the sixth part of the second edition (1967) is almost the same as its original. The rest of the book consists of different kinds of materials from different periods of time.

In what follows, I shall first take a look at the origins of the materials of *RFM*. Thereafter I shall discuss Paul Bernays' article "Comments on Ludwig Wittgenstein's *Remarks on the Foundations of Mathematics*" (1959). I shall use the second edition of *RFM*, whereas Bernays naturally discussed the first edition. However, this does no injustice to Bernays, since when preparing the second edition, Wittgenstein's literary executors did not remove anything from the first edition. They just added to it some previously unpublished interesting and relevant materials.

The fragmentary and partly unfinished nature of *RFM* is not the only thing that is good to keep in mind when relating it to Wittgenstein's other contributions. It is also important to accept that it must not be taken as an independent treatise on the philosophy of mathematics. Rather, it provides auxiliary material for approaching Wittgenstein's other writings -- particularly *Philosophical Investigations*. The link between these two works is evident. The first part of *RFM* was originally the second part of an early version of the *Philosophical Investigations*. The latter part of this typescript, which concerns the concept of negation, is dated "Ende August 36" and entitled "Philosophische Untersuchungen. Versuch einer Umarbeitung". It is also worth knowing that only the first part of *RFM* is based on a typescript. The rest of it originates in pocket notebooks and other handwritten materials.

Wittgenstein wrote down almost everything that is included in *RFM* between September 1937 and April 1944. I have already said a few words about its first part. In

the second part Wittgenstein discusses in particular Russell's project of constructing a logicist philosophy of mathematics through the reduction of mathematics to logic. The original manuscript (MS 122) is titled simply "Band XVIII". Its first entry is dated October 16th, 1939, and the last one February 3rd, 1940. It continues in MS 117, entitled "Philosophische Bemerkungen". (The final part of this manuscript is entitled "Fortsetzung des Bandes XVIII".) Only parts of this manuscript are included in *RFM*. According to the editors, this material is stylistically and substantially immature. They regarded parts of it as worth publishing although they admitted that they were not totally satisfied with the result: "The task was very difficult, and the editors feel little satisfaction with the result" (von Wright *et al.* 1967, viie).

The third part of *RFM* is based on a pocket notebook (MS 125). It begins in December 1941 and ends in October 1942. The fourth part is both temporally and thematically closely related to the third. The respective notebooks (MSS 126, 127) are dated from October 1942 until April 1943. The latter one, which is entitled "Mathematik & Logik", focuses on the intuitionist philosophy of mathematics and related topics, such as, what to think about the law of the excluded middle and existence proofs in mathematics.

Finally, the fifth part written in June-July 1941 and in March-April 1944 is divided into two parts. Both of these parts belong to the same manuscript volume. This suggests that they were meant to be together. Their themes vary from consistency and rule-following to the formation of mathematical proofs and concepts. (Cf. Wittgenstein 1953, §§209-237.)

Let us now move on to Bernays' criticism of *RFM*. Bernays begins his paper by fairly admitting that Wittgenstein "[...] would doubtless have made extensive changes in the arrangement and selection of the material had he been able to complete the work" (Bernays 1959, 1). However, he seems repeatedly to forget this. He soon calls for more solid argumentation in support of some admittedly rather controversial and cryptic fragments, writes about "this book on the foundations of mathematics" (*ibid.*, 2) as if it could be regarded as a monograph, judges that much of what Wittgenstein holds simply does not go without saying, and remarks that "Wittgenstein often speaks with a certitude which strangely contrasts with his readiness to doubt so much of what is generally accepted" (*ibid.*, 5).

Bernays underlines two problematic tendencies which he finds characteristic of *RFM*. The first is behaviourism. Bernays understands that Wittgenstein does not deny the existence of the mental experiences of feeling, perceiving and imagining, and as such he is not an advocate of behaviourism. However, according to Bernays, "with regard to thinking [Wittgenstein's] attitude is distinctly behaviouristic" (*ibid.*, 2). It seems

to him that Wittgenstein assumes that images and perceptions are immediately followed by behaviour. He writes: "We do like this', that is usually the last word of understanding -- or else [Wittgenstein] relies upon a need as an anthropological fact, thought, as such, is left out" (*ibid.*, 2). In this connection Bernays points out that it is symptomatic that Wittgenstein conceives proofs as pictures or paradigms. Indeed, at one time Wittgenstein gives a mere figure as an example of a geometrical proof. What is more, Bernays claims that although Wittgenstein is critical of the method of formalizing proofs, he repeatedly takes the formal method of proof in the Russell's system as an example and is satisfied with only a characterization of inference in which one passes directly from a linguistic establishment of the premises to an action. Indeed, there are no instances of proper mathematical proofs to be found in *RFM*. (Cf. Wittgenstein 1967, I, §17.)

The another problematic tendency is nominalism. According to Bernays, Wittgenstein's nominalistic attitude springs from the strict separation of the linguistic and the factual. This leads to a situation where only one kind of factuality, namely concrete reality, is possible. In Bernays' view this kind of philosophical preconception implies nominalism which also corrupts the philosophy of mathematics elsewhere. In order to justify his nominalism, "Wittgenstein would have to go back further than he does in [*RFM*]. At all events he cannot here appeal to our actual mental attitude. For indeed he attacks our tendency to regard arithmetic, for example, 'as the natural history of the domain of numbers'" (Bernays 1959, 4). Bernays points out that Wittgenstein's conception of arithmetic is not the usual one of the mathematician. According to him, Wittgenstein concerns himself more with theories on the foundations of arithmetic than with arithmetic itself. (Cf. Wittgenstein 1967, III, §§ 11, 13).

What Bernays finds most disturbing about *RFM* (and about Wittgenstein's thought in general after *Tractatus*) is its lack of rigour and clarity. And most importantly, in Bernays' opinion, we can obtain from Wittgenstein no guidance on how basic logic could be replaced by something philosophically more efficient. Bernays cannot see how strict and exact methods of presentation could *prima facie* limit our freedom. In his view, what remains in *RFM* is the "negative attitude towards speculative thinking and the permanent tendency to disillusionize" (Bernays 1959, 1).

Bernays goes on to discuss Wittgenstein's philosophical method which he recognizes as eliminative. He characterizes Wittgenstein as a "free-thinker combatting superstition" (*ibid.*, 6). One might think that this would mean searching for freedom of the mind, but as far as Bernays is concerned, this is not the case. In his view Wittgenstein restricted his mind in many ways "through a mental ascetism for the benefit of an irrationality whose goal is undetermined" (*ibid.*). However, he does not forget to

acknowledge that Wittgenstein's mental ascetics was by no means as extreme during his late period as it was earlier. Moreover, Bernays voices his concern for Wittgenstein's influence on younger generations:

[I]n contrast to the assertive form of philosophical statement throughout the *Tractatus*, a mainly aporetic attitude prevails in [*RFM*]. There lies [...] a danger for philosophical pedagogics, especially as Wittgenstein's philosophy exerts a strong attraction on the young minds. The old Greek observation that philosophical contemplation frequently begins in philosophical astonishment today misleads many philosophers into holding the view that the cultivation of astonishment is in itself a philosophical achievement. One may certainly have one's doubt about the soundness of a method which requires young philosophers to be trained as it were in wondering. Wondering is heuristically fruitful only where it is the expression of an instinct of research. Naturally it cannot be demanded of any philosophy that it should make comprehensible all that is astonishing. (*Ibid.*)

This is, of course, substantially acceptable. Another thing is whether it is justified to accuse Wittgenstein of corrupting "the young minds" by cultivating astonishment when confronted with philosophical problems. Bernays keeps reviewing *RFM* more as a scholarly monograph than as an interesting *Nachlass* edition which sheds light on *Philosophical Investigations*.

Bernays also briefly discusses the controversy between the finitist-constructivists and the so-called classical mathematicians. Bernays himself was a Hilbertian classical mathematician and Wittgenstein a finitist one. According to Bernays, Wittgenstein adopted Brouwer's viewpoint and maintained everywhere a standpoint of strict finitism (*ibid.*, 11). Bernays naturally recoiled from this standpoint. For example, when Wittgenstein raised the following, admittedly somewhat strange question: "For how do we make use of the proposition: 'There is no greatest [finite] cardinal number'? [...] First and foremost, notice that we ask this question at all; this points to the fact that the answer is not ready to hand" (Wittgenstein 1967, II, § 5), Bernays was ready with an answer:

We might think that one need not spend long searching for the answer here. Our entire analysis with its applications in physics and technology rests on the infinity of the series of numbers. The theory of probability and statistics make continually implicit use of this infinity. Wittgenstein argues as though mathematics existed almost solely for the purposes of housekeeping. (Bernays 1959, 14)

This sounds rather amusing, but it should be kept in mind, as Bernays himself admitted, that Wittgenstein doubtless would have modified *RFM* had he had an opportunity to do so. However, it must also be accepted that *RFM* is just a collection of bits and pieces from Wittgenstein's literary remains and that it is not possible to explain away, with intertextual cross-references in Wittgenstein's extensive corpus all the blunders and silly remarks it admittedly also contains.

It is of course no surprise that Bernays disagreed with most of Wittgenstein philosophy of mathematics. For instance, as we know, finitists like Wittgenstein held the opinion that metamathematics is still mathematics and therefore unable to provide us with insight into the foundations of mathematics, whereas the Hilbertian formalists, as represented by Bernays, did not regard metamathematics as mathematics. Consequently, according to the formalists, even though metamathematics certainly had an epistemological status, mathematics itself was epistemologically and ontologically neutral. Moreover, Wittgenstein hardly accepted that Hilbertian formalism is not a philosophical position.

When it comes to the controversy between the finitist-constructivist view and the 'Platonic' existentialist view, I think that at least Bernays is correct when he points out that the latter view enables one to appreciate the investigations directed towards the establishment of elementary constructive methods, while for a radical constructivist a great part of classical mathematics simply does not exist.

It is also decisively important to see that as opposed to the Hilbertian formalists, Wittgenstein subscribed to a universalist conception of language which does not allow us, so to speak, to step outside our language and its conceptual system. According to this view, it is not possible to discuss the relations that connect our language with the world, i.e., the relations that constitute the meanings of the expressions of our language. In other words, it presupposes the ineffability of semantics. As Jaakko Hintikka has written: "a universalist is bound to believe in the ineffability of all conceptual truths" (Hintikka 1996, 25). Furthermore, a realistic metalanguage in which we could discuss the relations of our language to reality is, according to a universalist, absolutely impossible. This basic view was also Wittgenstein's fundamental reason for rejecting set theory.

All in all, Bernays' article and Wittgenstein's *RFM* contribute hardly anything essentially new to the earlier debate between the finitists and the classical mathematicians. Indeed, it seems to me that Bernays' article tells us more about his own approach to the philosophy of mathematics than it tells us about Wittgenstein's approach -- just as Michael Dummett's Frege: *The Philosophy of Language* (1973) tells us more

about Dummett than it tells us about Frege. When *RFM* came out, Bernays had already clarified his philosophy of mathematics in several articles and lectures and he was certainly aware of Wittgenstein's "heresy" with regard to the foundations of mathematics. It makes me wonder what made him react so strongly to *RFM*, making the resulting contribution more of a commentary than simply a review. I subscribe to Jaakko Hintikka's suggestion of stamping *RFM* -- as well as the rest of Wittgenstein's *Nachlass* -- with a "Handle with care!" label (Hintikka, Wittgenstein-lectures, October 1996, Helsinki).

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