Post-Classical Logic?

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> "[G]ood scientific theories, like good maps, can present the same 'domain' in a great variety of very different forms. But this theoretical pluralism is very disconcerting for the Legend of a unique scientific world picture. ... [T]he Legend insists that ... the diverse theories of today are merely provisional: in due course, so it is argued, they will be seen as different approximations to the 'theory of everything' that will eventually be completed. But any such 'theory of everything' ... is not merely hypothetical: it is not a meaningful concept." (Ziman 2000, 131).

The project of formalizing natural argumentation is an old one, and has long been dominated by classical logic (henceforth K). However, non-classical logics, which are non-conservatively revisionary of K, have increasingly come to threaten this hegemony. Non-conservative revisions of logic may proceed by several strategies. The least revisionary is simple restriction: adoption of a new logic which lacks previously valid inferences and theorems. However, circumstances which motivate restriction characteristically result in more wholesale revision: features of the logic beyond its formal calculus are exposed to criticism, and reformulated in response. Thus judicious restriction can initiate clarification and disambiguation of confused metalogical concepts, including the nature of consequence, and what it should preserve (the inferential goal). For example, relevant logic exposes the contrast between intensional and extensional constants, obscured in K, and permits a restatement of the consequence relation.

The most radical strategy is a non-conservative revision of the background theories behind the logic, precipitating a change of its inferential goal. This alters the motivation of the whole logical enterprise, moves the problem into a different area, and changes the subject matter of logic (*cf.* Haack 1978, 155; Beall and Restall 2000, 490). Thereafter, the question of which logic should be employed can no longer be addressed directly. It is superseded by the question of which background theories obtain, and thereby of which goal is being pursued. Such disputes can only be settled at the level at which the background theories conflict. Divergence amongst the different calculi is understandable but derivative: they have been designed to meet different specifications. Therefore the dispute is no longer in the discipline of logic, but rather in whatever discipline threw up the conflicting background theories. Amongst proposals of this character are accounts of logic as the science of information flow;[1] systematic

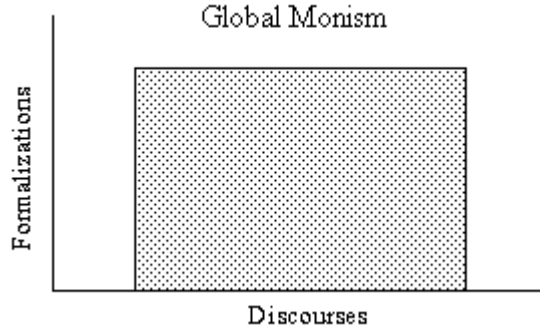
approaches to informal logic;[2] and perhaps some attempts at a 'feminist' logic.[3] Note that the non-conservative revision of background theories behind a change of subject matter need not entail the loss of key components of the formal system.[4]

The most important role that such a transition can play is to shift a programme onto new foundations offering higher standards of rigour and improved generality. Klein's *Erlanger Programm* may be understood as a move of this sort within geometry (Klein 1893). Klein's achievement was to found geometries not in more or less arbitrary lists of axioms, but in the invariants under groups of transformations, each group corresponding to a different geometry. Thus '_geometry' was reified from a subdiscipline of mathematics to an object of mathematical study, reconstructing an ancient subject on the modern foundations of group theory and linear algebra.

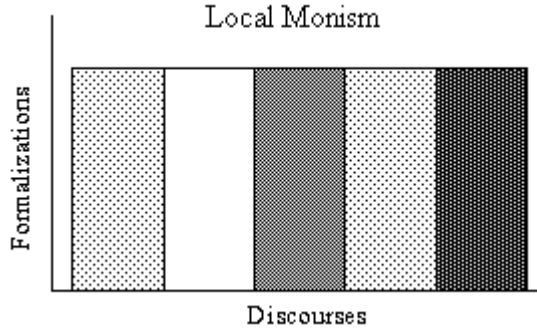
We may now discern two contrasting prognoses for the near future of research into the logic of natural argumentation. This is often portrayed as a continuing dispute amongst a proliferation of largely unrelated, competing non-classical programmes, each seeking the status of sole successor to K (Haack 1974; Sarkar 1990). However, within the context sketched above, this proliferation of logics may be understood to represent a refinement of logical method. The original quarry, the best logic for natural argumentation, has given way to something of higher generality: a structure which integrates the best features of a plurality of logics-an *Erlanger Programm* for logic. The articulation of such a structure as applied to natural argumentation is still in its earliest stages, but much recent work towards the provision of a general account of logical systems may lend itself to the advancement of this programme.[5]

In the remainder of this paper I shall seek to offer a glimpse of how such a structure may develop. First I must clarify some philosophically important distinctions, beginning with that between realist and anti-realist accounts of the nature of logic. Realists attribute irreducible factuality to judgements of logicality; anti-realists either seek to reduce facts about logic to facts about something else, such as the methodology of some formal system(s), linguistic conventions or cognitive characteristics, or they develop a non-factualist account of logic.[6] However, the questions with which I am most concerned-questions of how and why logics differ and change-are independent of this distinction. Both realists and anti-realists must concede that some systems of logic are better than others, on pain of retreat to the unreason of regarding all systems as equally tenable, including the trivial logic, in which all inferences are valid, and therefore that nothing can be said. Moreover, since neither realist nor anti-realist has access to any means of appraisal and comparison unavailable to the other, both must justify their preferences by appeal to the same features: simplicity, adequacy to data, non-*ad-hoc*ness, and so forth (Priest 199+, 24-25).

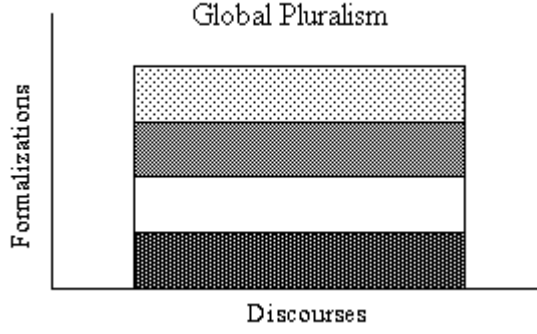
Two more pertinent distinctions with which the realism/anti-realism distinction is sometimes linked are that between monism and pluralism and that between localism and globalism. Monists believe that there can be at most one acceptable logic; pluralists believe that there can be several. Localists believe that the discourse of natural argumentation can be subdivided, and each subdivision formalized by a different logic; globalists insist that logic is topic-neutral. I shall argue that all three of these distinctions are mutually independent.[7] To see this, observe that the local/global distinction may be understood as a difference over how many natural argumentation discourses may receive distinct formalizations, and the monist/pluralist distinction may be understood as a difference over how many acceptable formalizations a given discourse may receive. Several different positions may be represented diagrammatically as bar charts, where the number of discourses is counted along the horizontal axis and the number of acceptable formalizations each may receive is counted up the vertical axis. I have assumed that division of natural argumentation into discourses precedes the formalization of these discourses; without this assumption a slightly more complicated picture would be required. The first position is global monism:



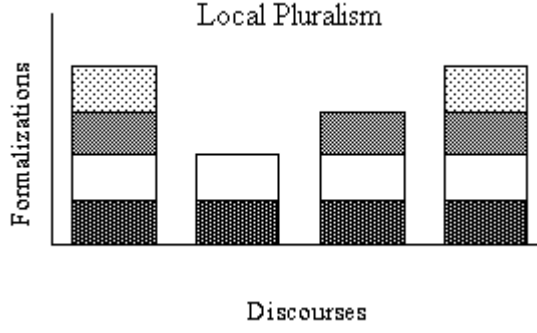
Global monists believe in the topic neutrality of logic and the uniqueness of an acceptable formalization. For realists this is the "one true logic", for anti-realists the one system that conforms to their standards. However, it is possible to reject topic neutrality, while retaining a commitment to the unique formalization of each discourse:[8]



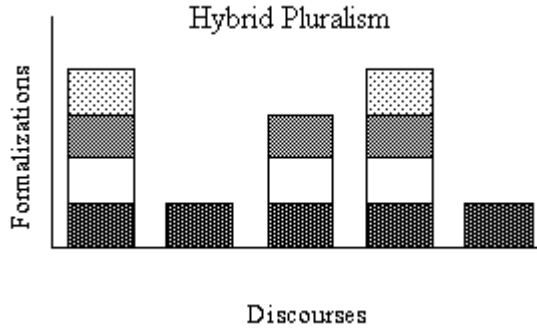
I call this position local monism.[9] The same realist and anti-realist attitudes are expressible here, relativized to each discourse. Alternatively, it is possible to retain topic neutrality while rejecting the uniqueness of formalization:



This position, global pluralism, is most familiar as a relativist, and therefore anti-realist, view of logic. However, it would also be tenable by a realist who supposed that reality underdetermined the choice of logic (Resnik 1996, 501). Finally, the local pluralist rejects both topic neutrality and uniqueness of formalization:



Here there are many different discourses, and no undisputed formalization of any of them. As a slight variation, one might admit that some discourses have a unique formalization, but that others do not::



I shall call this position hybrid pluralism. The local and hybrid pluralist positions are both arrived at by steps which I have shown to be available to realist and anti-realist alike. So not only are the local/global and monist/pluralist distinctions independent of each other, both are independent of the realist/anti-realist distinction.

Which of these five pictures best describes the logic of natural argumentation? Before asking how non-classical logics may be integrated into such a structure, I shall look at propositional K and its conservative extensions. The simplest picture is the first: global monism, with the single formal system understood to be first-order K. When classicists say that K is the one true logic, that is the natural understanding of their remark. However, although some classicists defend a restriction of logicality to first-order K (Quine 1953, Hazen 1999), most recognize a variety of quantified or modal extensions as equally logical. Taking this intuition seriously, while retaining global monism, would require the single formal system to somehow combine all the extensions of K which might be deployed in formalization of natural argumentation. Yet despite some naïvely misplaced optimism, the construction of such a compound system is a task of formidable technical difficulty if more than a small range of familiar extensions are to be used (Gabbay 1996). Furthermore, most conceivable applications would employ extensions containing only some of the extra constants rather than the unwieldy compound system containing them all. So local monism seems a closer approximation to the actual commitments of the classical programme (*cf*. Haack 1974, 44). The presence of the common fragment, K, in all of the systems used ensures the continuity of their application. As a further refinement, observe that most classicists acknowledge that some discourses lack an unambiguous choice of formalization. This suggests

monism about first-order K, and some of its extensions, and pluralism about some other extensions, such as modal systems, a perspective captured by hybrid pluralism.

If classicists are hybrid pluralists, might not a similar localism serve to integrate rival systems? Most important non-classical systems have a substantial common subsystem, K itself, which may serve as an analogue for the common fragment which motivated a sense of continuity between the various extended systems within the classical programme. A refinement of this picture may serve to provide philosophical motivation for the formal attempts at an Erlanger Programm for logic adumbrated above. K would be subsumed within such an approach as a key component, so it might best be regarded as a treatment not of non-classical logic but of post-classical logic.

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Endnotes

1 Typically by application of situation theory, as in Devlin 1991: particularly programmatic passages may be found at 10-11 and 295-298. But cf. Mares 1996, who assimilates situation theory to the less comprehensively revisionist relevant logic programme.

2 See Johnson & Blair (1997, 161), who 'distinguish informal logic from formal logic, not only by methodology but also by its focal point … the cogency of the support that reasons provide for the conclusions they are supposed to back up.'

3 Nye (1990, 175) concludes her indictment of 'masculine' logic with the claim that 'there can be no feminist logic', but her alternative could be seen as a change of subject matter-in which the word 'logic' itself would be jettisoned, despite the retention of some of its methods. Plumwood's (1993) feminist defence of relevant logic might appear to be a more conservative revision. However, her revision of classical background theories is substantial and her programme not necessarily continuous with that of more orthodox advocates of relevant logic.

4 For example, Devlin (1991, 10) is clear that he regards K as a special case, and Plumwood's preferred formal system, R, also recaptures K.

5 Promising leads include Belnap's display logic (Anderson et al. 1992 §62), Gabbay's labelled deductive systems (Gabbay 1994), Beall & Restall's logical pluralism (Beall and Restall 2000) and Sambin's basic logic (Sambin et al. 2000).

6 Haack (1974, 3; cf. 1978, 224) characterizes this distinction as one between realists and pragmatists, whereas Resnik (1996, 499-502) separates realism and six different varieties of anti-realism, without claiming to be exhaustive.

7 Contra Haack (1978, 225) for whom monism and pluralism are subdivisions of realism, and localism and globalism are subdivisions of pluralism.

8 The number of bars in this diagram is arbitrary, as is the number of bars and columns in all the subsequent diagrams, unless equal to one.

9 Misleadingly called local pluralism by Haack (1978, 223) and Resnik (1996, 499), who adopts her definition. This infelicity results from Haack's classification of localism and globalism as special cases of pluralism. Neither she nor Resnik considers the position which I call local pluralism.