Wittgenstein's Criticism of Russell's Distinction Between Pure and Applied Logic

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Sections 5.552-5.5521 and 5.557 of Wittgenstein's *Tractatus* contain some enigmatic remarks about the distinction between logic (presumably *pure* logic) and its application. I will argue here that these passages, like so much else in the *Tractatus*, are directed against Bertrand Russell's philosophy of logic. In particular, they constitute an important criticism, previously overlooked in the literature, that goes to the heart of Russell's philosophy of logic: Russell's logical Platonism cannot explain how the *applications* of logical propositions are necessarily true. If sound - and I will argue that it is - then this is a very serious objection to Russell's account of logic. For any philosophy of logic holding that logical propositions are necessarily true and can be applied must also be able to account for the necessary truth of the applications of these propositions.

It will first be necessary to sketch out - however briefly and dogmatically, given the time constraints - Russell's view during the period surrounding the publication of *Principia Mathematica* (1910-1913) of the distinction between pure and applied logic. For Russell, the axioms of *Principia Mathematica* constitute the basis of pure logic. The propositions of *pure* logic share four characteristics. First, they are *necessary*, i.e., true in all possible worlds (1912, 78; 1919/1920, 191f). Second, they can be known - and thus understood - *a priori*, i.e., independently of the experience of particular things (1912, 72-75, 84-90, 105f; 1918/1985, 108; 1919/1920, 204; 1914/1993, 66; cf. 1913, 490-492). Third, they contain no non-logical constants, but only logical connectives, quantifiers, etc. (1919/1920, 199-202, 205; 1918/1985, 43, 60f). This characteristic helps to satisfy Russell's demand that the propositions of pure logic be unrestrictedly general (1913, 487; 1919/1920, 197f; 1914/1993, 54, 66; cf. Hylton 1990, 200-205; and Griffin 1980, 122-128, 135-139). Fourth, they contain real (i.e., free) variables (1910/1962, 93; 1908/1956, 66f). This fourth feature ensures that the propositions of pure logic are "typically ambiguous" (1910/1962, 65), i.e., that their variables do not range over objects of any particular type. This, in turn, is Russell's means for preventing these propositions from violating the restrictions imposed by his theory of types (cf. 1910/1962, 5, 65, 92f, 95, 127-129).

Russell's view of the nature of the propositions of pure logic, I suggest, is an attempt to account for these four features within the constraints imposed by what he calls "the

fundamental epistemological principle in the analysis of propositions…: *Every proposition which we can understand must be composed wholly of constituents with which we are acquainted*" (1910/1917, 159; cf. 1912, 58). One consequence of this "principle of acquaintance" is that, since we can understand the propositions occurring in *Principia Mathematica*, they must be composed wholly of constituents with which our minds are directly acquainted. Indeed, it was for Russell a criterion of the adequacy of a logical symbolism that each of its simple symbols stands for objects with which our minds are acquainted (1918/1985, 58f). Russell's account of pure logic is thus an account of the nature of these objects referred to by the simple symbols of his logical notation.

Russell accounts for both the necessity and the *a priori* knowability of the propositions of pure logic by holding that the simple logical symbols occurring in their expression refer just to *universals*, and that these propositions are true just in virtue of relations holding among these universals (1912, 103). There are two basic sorts of universals that can occur as constituents of the propositions of pure logic. First, there are the referents of the *logical constants* (1903/1938, 106; 1913, 486, 492; 1910/1962, 8, 92). Second, there are the referents of real variables, which Russell calls *logical forms* (1913, 492; 1913/1992, 98; 1914/1993, 52-54, 67; cf. 1918/1985, 106; 1919/1920, 199f; 1903/1938, 45, 53, 106; Hylton 1990, 219f, 248-258). Logical forms include *monadic property, 2-place relation, 3-place relation, proposition*, etc. (cf. 1903/1938, xi; 1913, 488; 1914/1993, 53, 66; 1919/1920: 196-202). Together, the logical constants and logical forms constitute the *logical objects*. The propositions of pure logic are true in virtue of logical facts - i.e., relations among logical objects (1913, 486, 492; 1913/1992, 98; 1914/1993, 52-54, 66f; 1918/1985, 106; 1919/1920, 197-200) - and can be intuitively, or self-evidently, known though a direct mental inspection of logical facts.

So much for the propositions of pure logic. The *application* of pure logic consists of those propositions that result from substituting non-logical constants for the variables occurring in the propositions of pure logic (cf. 1910/1962, 93, 128; 1913, 488; 1912, 105; 1913/1992, 97f; 1914/1993, 66; 1918/1985, 60f; 1919/1920, 201). Such constants are either names (standing for particulars) or ordinary universals (i.e., predicates or relations). Conversely, the propositions of applied logic occur at the beginning of a process of "purification" (1913, 486), in which one substitutes variables for each of the non-logical constants of these propositions until one ends up with the propositions of pure logic, i.e., those in which no non-logical constants occur (1913, 485-489; 1914/1993, 62-67; 1918/1985, 103-108; 1919/1920, 196-202).

Although the propositions of pure and applied logic can be formed from each other in these ways, Russell insists that the former contain as constituents certain objects that

the latter do not. Naturally, there is *some* degree of overlap between the constituents of the propositions of pure and applied logic. For both contain logical constants, i.e., the objects to which the logical connectives refer. The propositions of pure logic, however, are alone in containing logical forms as their constituents (1913, 491f; 1913/1992, 98; 1914/1993, 63-67; 1918/1985, 106; 1919/1920, 199; cf. 1903/1938, 51). Russell argues that concrete propositions, whose constituents are ordinary names, predicates, and relations, cannot contain logical forms. For if one regards a logical form as something linking the constituents of a concrete proposition, then there would have to be a second form linking these constituents to the first form, and so on *ad infinitum*. This would have the absurd consequence that the simplest proposition would contain an infinite number of forms, and would thus be incomprehensible to a finite mind. Thus logical forms occur only in the *propositions* of pure logic, and (in one version of Russell's multiple-relation theory of *judgment*) in all ordinary judgments (1913/1992, 113-118; cf. 1903/1938, 51).

Let us now examine the *Tractatus* passages mentioned at the outset in the light of Russell's distinction between pure and applied logic. Section 5.552 begins with a head-on attack on Russell's Platonic Realism about pure logic:

> The "experience" that we need in order to understand logic is not that something or other is the state of things, but that something *is*: that, however, is *not* an experience. Logic is *prior* to every experience - that something *is so*. It is prior to the "how", not prior to the "what."

Here Wittgenstein makes it clear here that pure logic is *not*, as for Russell, about special objects: the logical forms. Wittgenstein's denial that understanding logic requires a special '"experience"' is a clear reference to Russell's notion of the mind's '"acquaintance with logical objects"' (1913/1992, 97; cf. 1912, 109), including logical forms. Wittgenstein is thus denying Russell's claim "that there certainly is such a thing as 'logical experience', by which I mean that kind of immediate knowledge, other than judgement, which is what enables us to understand logical terms" (1913/1992, 97). Instead, what makes *any* proposition logically true is simply that its logical connectives perform truth-operations that make it a tautology (6.1). And a tautology presupposes "that names have *Bedeutung* and elementary propositions sense; that is their connection with the world" (6.124). Thus a proposition of logic - i.e., a tautology - presupposes "that something *is*", i.e., the subsistence of the simple objects referred to by the names occurring in its elementary propositions. These objects are "the 'what'", i.e., the "substance" (2.021), of the world. Tautologies, however, do *not* presuppose the existence of special "logical facts" (cf. 6.111, 5.551) - i.e., "that something or other is the state of things… that something *is so*" (5.552). There are thus for Wittgenstein no "logical objects" for our minds to experience (5.4).

In 5.5521, Wittgenstein offers his reasons for rejecting Russell's view of pure logic: "And if this were not so, how could we apply logic? We might put it in this way: if there would be a logic even if there were no world, how then could there be a logic given that there is a world?" Wittgenstein in this passage is pointing out what he takes to be a fatal difficulty with Russell's distinction between pure and applied logic. Recall that Russell accounts for the necessity and unconditional generality of the propositions of pure logic by holding that they contain real variables referring to logical forms. In order to avoid a vicious regress, however, he insists that the applications of these propositions do not contain logical forms. Russell's view thus entails that the propositions of pure logic would be necessarily true (i.e., "there would be a logic" in 5.5521), even if none of the particular objects or concrete predicates or relations existed (i.e., "even if there were no world" in 5.5521). Nevertheless, the world of concrete facts *does* exist, and we can form propositions about such facts that are tautologous, and thus logically true. Wittgenstein's "it is either raining or not raining" (4.461) is an example of such a proposition. Nevertheless, since the very objects that Russell holds make the propositions of pure logic necessarily true are *not* among the constituents of their applications, Russell's view of logical necessity leaves entirely mysterious the necessity of the applications of logical propositions. Russell's view fails to explain how there could "be a logic given that there is a world" (5.5521), and is thus inadequate.

Section 5.557 takes Wittgenstein's implicit criticism of Russell's distinction between pure and applied logic one step further, exposing what are perhaps its deepest problematic commitments. In the first three sentences of 5.557, he expresses his agreement with Russell that there is a distinction between pure and applied logic, and that "logic must not clash with its application", i.e., that a proposition of pure logic and its applications must be necessarily true together. He then writes in the fourth sentence: "But logic has to be in contact with its application." The "contact" in question is presumably based in the fact that the *same* logical connectives account for the necessary truth of the propositions of pure logic and that of their applications. The "But" here presumably signals the fact that Wittgenstein is objecting to Russell's account of this "contact", in particular, his view that a proposition of pure logic and its applications contain logical constants *referring* to the very same *logical objects*. For Wittgenstein, on the other hand, the logical constants are simply expressions of the performance of truth-operations, i.e., ways of making the truth-conditions of one proposition depend on those of its bases (5.2-5.451).

In the fifth sentence of 5.557, Wittgenstein concludes: "Therefore logic and its application must not overlap." The "overlap" that Russell's theory posits is presumably the basis of Russell's account of the "contact" mentioned just above: that the propositions of logic and their applications refer to *some* of the same objects. Now the

particular overlap with respect to the logical connectives would not seem to pose any difficulties by itself. After all, as we just saw, Wittgenstein himself requires that there be "contact" between logic and its application. The difficulty with the overlap posited by Russell to which Wittgenstein alludes here thus presumably lies not with the overlap between the propositions of pure logic and their applications *per se*, but rather in a further assumption that *entails* this overlap. This assumption, I think, is none other than Russell's "principle of acquaintance", and his closely related view that every meaningful (simple) symbol in an adequate logical symbolism gets its meaning by referring to some object. Recall that it is this assumption that entails the highly problematic conclusion that only the propositions of pure logic are about logical forms. For if one assumes an ultimately referential theory of meaning and the four characteristics of Russell's propositions of pure logic enumerated at the beginning of this paper, then one is virtually compelled to hold that these propositions contain free variables that refer to logical forms. In 5.557 Wittgenstein thus argues plausibly that Russell's commitment to the principle of acquaintance, and the exclusively referential theory of meaning that it implies, is ultimately at the root of his inadequate distinction between pure and applied logic.

The fifth sentence of 5.557, then, alludes to one of the hallmarks of Wittgenstein's *Tractatus*: its novel doctrine of the meaning of an expression. Whereas Russell holds that all propositions can be analyzed into simple symbols that *refer* to objects (indeed, objects of acquaintance), Wittgenstein denies that all meaning is ultimately referential. Instead, for Wittgenstein, the meaning of an expression is just its contribution to the sense - i.e., the truth-conditions (2.221-2.222, 4.022) - of the propositions in which it can occur (3.31).

The passages we have examined make it clear that Wittgenstein does accept a distinction between pure and applied logic, but one quite different from Russell's. The exact nature of Wittgenstein's version of this distinction, and the role it plays in the *Tractatus* as a whole, is a topic seldom addressed in the literature. This, however, is a question for another time.

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Endnote

1 In the case of multiple editions Russell's works, I cite date of original work/date of edition used.