

Wittgenstein on Frege on Connectives

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When Wittgenstein says that "logical constants do not stand for anything", it is clear that his immediate target is the way Frege treated connectives and quantifiers. Quantifiers, as it is widely known, are defined by Wittgenstein in terms of simultaneous negation, and so he can consistently forget them when it comes to giving logical reasons to accept his claim about logical constants (5.3-5.4). Only connectives are then considered.

For Frege, connectives are not objects, but functions, and Wittgenstein is well aware of this. Wittgenstein is thinking of Frege when he opposes truth functions to material functions (5.44), saying that logical connectives like "not", "or", etc. should not be viewed as functions which take propositions as arguments. According to Wittgenstein, they belong to another logical category – they are operations. Even so, if they were functions (in the Fregean sense) they would be "objects" (in the Tractarian sense) – they would be entities for which certain symbols stand for. That's why it is legitimate to think that when Wittgenstein denies the existence of logical *objects*, he is also aiming at the Fregean treatment of negation, disjunction, and so on. Frege thinks that connectives "stand for something", and that's the very idea Wittgenstein is trying to avoid.

None of Wittgenstein's arguments against Frege seems more intuitive than the one we find on aphorism 5.44:

If there was an object called '~', then '~~p' would have to say something other than 'p'. For the one proposition would then treat of ~, the other would not. [5.44]

Apparently, Wittgenstein is saying something directly applicable to the context of Frege's semantics of sense and reference. If we take negation as being a concept, we have to deal with the negation sign in the same way as we deal with any functional symbol. It will have a sense and a reference. Its reference will be the concept of negation, and any proposition containing a negation sign will have to be "about" this concept (among other things). As a consequence, "p" and "~p" will deal with different things, since "~p" would be "about" the concept of negation, while "p" would not, and that is plainly absurd. "The proposition '~p' does not deal with denial as an object", according to Wittgenstein (5.44).

As I have just said, in these contexts the word "object" should be taken in the Tractarian, not in the Fregean sense. The reference of a conceptual term is certainly not a Fregean object, but (for the sake of argument) it can be imagined as a Tractarian one. According to this reading, Wittgenstein would be attacking Frege on the flank of reference. The argument would have the following layout: (i) The negation sign has its own reference. (ii) A proposition is about (deals with) the references of its terms. (iii) "p" and "~p" are about (deal with) different entities. As (iii) seems to be absurd and (ii) seems to be evident, we are bound to deny (i). The negation sign is not a "logical constant"; it does not name a "logical object"; it has no reference.

Thus reconstructed, the argument rests on the slippery notion of "aboutness". Wittgenstein's text itself suggests this reading by the use of the verb "handeln" (to deal with). It seems that all depends on what the sentences "p" and "~p" deal with, that is to say, "what they are about". But so interpreted the argument will have a weakness which make it

completely ineffective as far as Fregean semantics is concerned.

The problem is that, if we do not want to rely on intuitive claims, we have to define "aboutness" within Fregean theory. We have basically two options. We could say that a sentence is about any reference of any of its proper names, provided that the proper name is not itself a sentence. ("The cat is on the mat" would be about the cat, the mat, but not about the truth-value named by the sentence which is being denied.) But this move would be of little help to Wittgenstein. If we are using the word "name" in such a way that no functional expression is a name, then "~" is not a name, and "~p" is not about denial.

Finally, we could take functional expressions as special kinds of names. In this case, "~p" would be about a truth-function, but there would be nothing strange in saying that. We would only be saying that the sentence contains a functional expression that has its own reference. From the point of view of the theory, that is not absurd. On the contrary, it is quite trivial.

So it is better to abandon any reading involving the notion of "aboutness". Perhaps Wittgenstein is not talking about reference. He could be talking about sense. The textual evidence, in this case, would be the use of the verb "sagen" (to say). Let us examine Wittgenstein's phrasing once more. He says that "if there was an object called '~', then '~p' would have to say something other than 'p'." We could rephrase it as follows: "if there was an object called '~', then *the sense* of '~p' would have to be different from *the sense* of 'p'." That seems to be a quite natural reading, since what a proposition says is the sense it expresses, and not the truth-value it names. Now as the word "object" is obviously being used with a Tractarian tone, it is better to translate this into the Fregean dialect. In Fregean terms, the sentence would say something like this:

If there was a concept called "~", then the sense of "~p" would have to be different from the sense of "p".

That is not simply nonsense, though I don't think it is a fatal objection to Frege. Let us examine the situation in some detail.

If there was a concept called "~", this concept would have to be the reference of the negation sign (in the usual contexts, at least). If the negation sign has a reference, it must have a sense as well. The principle of compositionality must be at work on both levels. The reference of the sentence – its truth-value – must be a function of the partial references involved, and the sentential sense must be completely determined by the corresponding partial senses.

At the level of references, it would be a mistake to understand compositionality in terms of parts and wholes. Socrates was Plato's teacher, but Plato is not a "part" of Socrates in the same sense as the first chapter of a book is part of the whole book. Compositionality does not mean juxtaposition. At the level of reference, it only means functional determination: given this argument (Plato) and this function (the teacher of x), we get exactly this value (Socrates). In a certain sense both function and argument "disappear" in the value. They are not inscribed in it. Given a value (Socrates) and a function (the teacher of x), the argument is not given as yet. After all, Socrates was also Theaetetus's teacher.

The situation is even clearer when we are dealing with truth-values. If the sentence

The teacher of Plato was put to death.

is a name of the True, then if the partial references are conserved, the total reference will be conserved as well.

Socrates was put to death.

keeps being a name of the True. But it would be absurd to think that the True and Socrates unequivocally determine a certain propositional function. The truth-value is determined by the function and argument involved, but function and argument are not "parts" of the truth-value. They *produce* the value, and (so to speak) disappear leaving no trace behind them.

The same reasoning could not be repeated at the level of senses. If we proceed within the compositional paradigm, we will be inclined to think of sentential senses as having constituent parts. We will also be inclined to think that we could distinguish each of these partial senses "inside" the sentence. The sentential sense wouldn't seem to be an entity quite different of, and only functionally associated with the partial senses. The sentential sense seems to be "formed" by them, to be something that could not be given without them. Partial senses would, so to speak, be the very stuff sentential senses are made of. In a word, the compositional paradigm seems to be much more demanding at the level of senses than it was at the level of references.

Now it is possible to give strictly logical reasons in support of these intuitions. Consider, for instance, the proposition " $5 > 2$ ". We cannot change the reference of the whole sentence (its truth-value) without changing the reference of one of its parts – that's the meaning of the "functional dependence" in this case. But we could easily change the reference of a part without changing the reference of the whole: " $7 > 2$ ", for instance, keeps being a name of the True. But how could we alter the sense of a part without altering the sense of the whole? Just compare the sense of

The number of fingers in my hand is greater than two.

with the sense of an arithmetical proposition like " $5 > 2$ ". A change in the parts is, so to speak, a guarantee of change on the whole. We are clearly dealing in this case with conditions which are not only necessary but also sufficient – no changes in the whole without a change in the parts, and no change in the parts without an immediate echo in the whole. The sense seems to be given *with* the parts, and not merely *by means of* them.

If that is true, Wittgenstein seems to have a point. If the sign " \sim " is the name of a concept, then it must have a sense, that's to say, it must be linked to a special way of presenting the concept of denial. But if the sense of " \sim " is a component part effectively present into the sense of " $\sim\sim p$ ", then we must admit that " p " and " $\sim\sim p$ " have *different* senses.

Now that is not an admission entirely free of undesirable consequences. First of all, if " p " and " $\sim\sim p$ " do not have the same sense, it is difficult to imagine in what conditions sameness of sense should be admitted. All examples given by Frege (passive voice, for instance) seems to involve, so to speak, a greater distance from one sense to another. What could be closer in sense to " p " than " $\sim\sim p$ "? Moreover, if " $\sim p$ " and " $\sim\sim\sim p$ " do not have the same sense, then we would have an infinite number of negations introduced in language. We could define " $\sim\sim\sim p$ " as " $\sim p$ ", define " $\sim\sim\sim\sim p$ " as " $\sim p$ ", and it is obvious that " $\sim p$ ", " $\sim\sim p$ " and " $\sim\sim\sim p$ " will have the same truth-value, but different senses.

In order to avoid this infinite multiplication of senses, we can postulate a sense for " \sim " which makes it work approximately as a Tractarian operation. Our aim is to find a sense for the negation sign such that, applied once, it produces the "opposite" sense; applied twice, it cancels out. I think the only way to conceive of the sense of the negation sign in these terms is to take it as *the empty presentation of the True, when applied to a presentation of the False, and an empty presentation of the False when applied to a presentation of the True*. Being an "empty presentation", the sense of " \sim " does not add any sense-feature to the proposition which it belongs to, so that now " p " and " $\sim\sim p$ " will have exactly the same sense. The trick is achieved through the use of the word "empty" in the definition we gave. Suppose that " p " is a name of the True. Both its sense and the sense of " $\sim\sim p$ " will be *the same* non-empty presentation of the True. On the other hand, " p " and " $\sim p$ " will have opposite senses: the sense of " p " will be a non-empty presentation of the True, while " $\sim p$ " will be a non-empty presentation of the False. They would agree as to the "non-empty presentation" involved, but not as to the logical object presented in each case.

The notion of an "empty presentation" is certainly not an easy one to swallow. If we take the Fregean notion of sense as being equivalent to the notion of "cognitive content", for instance, the idea of an "empty presentation" seems to be too close to the contradictory idea of a "contentless content". We can avoid this consequence by saying that any logical connective is to be taken as a "contentless contextual pointing". It contentlessly points to a truth-value which varies according to the truth-value which the rest of the context is pointing to. Even so we are left with the impression that a "contentless pointing" is a function that the sign itself could carry out alone. Why would it need the help of an absolutely transparent sense?

Be that as it may, the fact is that although the doctrine works reasonably well with negation, it does not work with other truth-functions. Suppose both " p " and " q " are names of the True. How could we deal with the obvious difference of sense between " $p \& q$ " and " $p \supset q$ "? According to our approach, both sentences would be non-empty presentations of the True, and would agree on each point of their non-empty parts. We would have to say that they have the same sense, and that is simply absurd. A parallel absurdity would emerge in the analysis of the sense of the connectives in this context. Both would be empty presentations of the True. The doctrine is clearly an untenable one.

In a desperate move, we could try to take the sense of a connective as being a *presentation of a determined way of associating truth-values to truth-values*. Conjunction and material implication are different ways of doing this kind of association. Although they may produce identical results sometimes, the procedures are different. Even though " $p \& q$ " and " $p \supset q$ " are names of the same truth-value, " $\&$ " and " \supset " have different senses. They introduce different ways of associating truth-values with truth-values. A truth-table is a good picture of the kind of content the sense of a connective amounts to. The price to be paid is obvious: " p " and " $\sim\sim p$ " will not have the same sense. The sense of the second sentence involves the presentation of a kind of truth-table, while the sense of the first sentence does not have any presentation of this kind of thing. To repeat Wittgenstein, " p " would have to say something different from " $\sim\sim p$ ". I think Frege would be forced to concede this extreme consequence of his doctrines. But this admission would amount to a *reductio ad absurdum* of the whole theory.