A Note on *Tractatus* 5.521

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Introduction

Wittgenstein scholars have long been puzzled by §5.521 of the *Tractatus*. It reads as follows:

> I separate the concept *all* from the truth-function. Frege and Russell have introduced generality in connexion with the logical product or the logical sum. Then it would be difficult to understand the propositions "$(\exists x).fx$" and "$(x).fx$" in which both ideas lie concealed.

We shall best get to the heart of the puzzle by considering the way Bertrand Russell treated the matter in his "Introduction" to the *Tractatus*. Indeed, Russell made a particular assumption which seems on the face of it to be incoherent. He alludes to "Mr Wittgenstein's theory of the derivation of general propositions from conjunctions and disjunctions" (TLP, 15), a perspective Wittgenstein ascribes to Frege and Russell himself. In fact Wittgenstein explicitly says that both "Frege and Russell have introduced generality in connexion with the logical product or the logical sum", thus deriving "$(x).fx$" from "$fa.fb.fc.$ …" and "$(\exists x).fx$" from "$fa \vee fb \vee fc \vee$ …", a procedure he rejects.

However, astonishingly as it may seem, after 1929 Wittgenstein criticized his earlier conception of generality, which took "$(x).fx$" to be a "logical product" and "$(\exists x).fx$" to be a "logical sum". In his record of Wittgenstein's lectures of 1930-33, G.E. Moore writes that "[h]e said that there was a temptation, to which he had yielded in the *Tractatus*, to say that $(x).fx$ is identical with the logical product '$fa.fb.fc.$ ...', and $(\exists x).fx$ identical with the logical sum '$fa \vee fb \vee fc \vee$ …'", assuming that "this was in both cases a mistake" (MWL, 89). Further, Moore notes that "[h]e said that, when he wrote the *Tractatus*, he had supposed that *all* such general propositions were 'truth-functions'", recognizing that "in supposing this he was committing a fallacy, which is common in the case of Mathematics, *e.g.* the fallacy of supposing that 1+1+1 ... is a sum, whereas it is only a *limit*" (ibid.). And, in the same vein, G.H. von Wright reports that "[i]n one of the first conversations [he] ever had with Wittgenstein (in 1939), he said the biggest mistake he had made in the *Tractatus* was that he had identified general propositions with infinite conjunctions or disjunctions of singular propositions" (von Wright 1982, 151, n.28).

It looks, therefore, as though there is a contradiction, even if Wittgenstein's remark at §5.521 of the *Tractatus* clearly suggests that Russell's interpretation cannot be right. Thus H.O. Mounce, who is one of the most lucid interpreters of the generality issue in the *Tractatus*, even arguing that Russell misunderstood it, emphasizes that "we can be certain, from Wittgenstein's own remarks on the subject, that he was confused on this matter at the time of the *Tractatus*"; and he goes on saying that "[w]hat is not at all easy to determine, however, is where precisely his confusion lies" (Mounce 1981, 67). In this paper, following the lead of Mounce, I shall try to make clear that Wittgenstein's criticism in the 1930s is directed at his earlier view that the content of general propositions can be *enumerated*, not at the way in which he *introduced* such propositions. But, on the basis of the third of the wartime notebooks that survived and the so-called *Prototractatus*, I go deeper into Wittgenstein's alleged "confusion", analysing some hitherto neglected aspects.

I

Section 73 of the "Generality" chapter of the *Big Typescript*, which bears as title "Criticism of my Earlier Understanding of Generality", begins with a remark deriving from the first entry of 1 August 1931 in MS111. Wittgenstein says:

> My understanding of the general proposition was that $(\exists x).fx$ is a logical sum, and that although its terms weren't enumerated *there*, they could be enumerated (from the dictionary and the grammar of language). (…) (TS213, 326: BT, 249e)

And the next remark of the section, deriving from the first entry of 1 December 1931 in MS113, runs as follows:

> Of course, the explanation of $(\exists x).\phi x$ as a logical sum and of $(x).\phi x$ as a logical product cannot be maintained. It was linked to a false view of logical analysis, with my thinking, for instance, that the logical product for a particular $(x).\phi x$ would most likely be found some day. – Of course it's correct that $(\exists x).\phi x$ functions in some way as a logical sum, and that $(x).\phi x$ functions in some way as a product; indeed for *one* use of the words "all" and "some" my old explanation is correct, namely, in a case like "All the primary colours can be found in this picture", or "All the notes of the C major scale occur in this theme". But in cases like "All people die before they are 200" my explanation is not correct. (…) (TS213, 326-7: BT, 249e)

The examples given by Wittgenstein illustrate the *singular* character of our universal statements, that some refer to a simple *set*, which is thinkable as belonging to a totality that is presupposed (all colours, all musical notes), and that others refer to a whole whose *particularity* is manifest. In this latter case, there is not, therefore, any logical product, the dots in "$fa.fb.fc.$ ..." being dots of innumerability, not "dots of laziness", as Wittgenstein called them in his lectures, the sort we use to speak, for example, of the alphabet in terms of "A, B, C …" – that is, when the enumeration, though possible, is not carried out (cf. MWL, 90; also AWL, 6).

It is noteworthy, however, that the reason why the enumeration of an "infinite series", say "1, 2, 3 ...", is impossible is not our inability to complete it; it is rather that it belongs to the concept of *infinite* its innumerability (cf. MWL, 90; also LWL, 90). In a conversation on 22 December 1929 with Moritz Schlick and Friedrich Waismann recorded by the latter, Wittgenstein actually alludes to the whole that is made up by the four primary colours as a "finite conjunction", constituting it a "*finite* logical product" - a *contradictio in adjecto*, since there can be no infinite logical products (cf. WVC, 45). What is projected in such cases is simply a horizon of vagueness.

Let us consider the example Wittgenstein gives in the opening of the conversation alluded to above, which is: "All men in this room are wearing trousers" (WVC, 38).

What is at stake here? That "Professor Schlick is wearing trousers, Waismann is wearing trousers, Wittgenstein is wearing trousers, and no-one else is present", that is to say, that "Mr. Carnap is not in this room, Mr...., etc." (ibid.). But do we really think of an infinite number of propositions about what is *not* the case? Wittgenstein now holds, contrary to his original idea, that this constitutes, rather, an "incomplete picture", which the symbolism "must show [to be] *incomplete*" (WVC, 39-40). The question is: how to do that? How can we represent in a propositional scheme that "There is no man is this room" except by means of "~(∃*x*).*fx*", which, being equivalent to "(*x*).~*fx*", immediately yields a logical product "*~fa.~fb.~fc.* ...", thus requiring an enumeration? Wittgenstein's suggestion is that we should translate existential propositions such as "*x* is in the room" or "There is someone in the room" by means of "*fx*", corresponding its negation solely to "*~fx*" (cf. WVC, 40, 44). What he claims is that the variable at issue is not an "apparent variable" but a "real variable", one that does not require *individual constants* (cf. WVC, 39). Note also that in the case of the existential proposition "(∃*x*).*fx*" we have a similar case of enumeration, since it corresponds to the logical sum "There is in the room either this person or that person or that person, etc.", an expression that, according to Wittgenstein, is nonsensical. Obviously, this does not happen with propositions like "In this square there is one of the primary colours", to use one of the examples Moore mentions, because there the expression "*fa* ∨ *fb* ∨ *fc* ∨ …" is *conclusive*, being equivalent to "In this square there is either red or green or blue or yellow" (MWL, 89). But in all the other cases, including of course the negative ones, viz. "(∃*x*).~*fx*", we would have infinite remissions. Now, in rejecting (Frege's and) Russell's notation, which he had previously adopted, Wittgenstein not only avoids the indefinite enumerability problem, but also the "twofold negation" problem, i.e. that "(∃*x*).*fx*" does not have the "right multiplicity" (WVC, 39-40). This shows, in truth, that "There is no man who is not in the room" is nonsensical and that "~(∃*x*).~*fx*" cancels the meaningfulness of "(∃*x*).*fx*".

Wittgenstein's conclusion is that, as Moore reports, "the cases to which the *Principia* notations (*x*).ϕx and (∃*x*).ϕx apply [...] are comparatively rare", given that "oftener we have propositions, such as 'I met a man', which do not 'presuppose any totality'"; moreover, Wittgenstein goes as far as to argue that "the cases to which the *Principia* notation apply are only those in which we could give proper names to the entities in question", something that "is only possible in very special cases" (MWL, 91). All the others require, in effect, a concrete grammatical analysis. They cannot be seen in the light of a predefined scheme but in what they really involve.

II

We are now in a position to reconsider Wittgenstein's "confusion". The problems identified appear to conflict with important Tractarian themes. However, Wittgenstein's early view actually withstands the criticisms that he himself later identifies. In fact, what he contests at §5.521 of the *Tractatus* is merely the *extralogical* way in which Frege and Russell "have introduced generality".

Let us briefly examine Wittgenstein's procedure for deriving general propositions, which is presented at §5.52. He writes:

> If the values of ξ are the total values of a function *fx* for all values of *x*, then N($\bar{\xi}$) = ~(∃*x*).*fx*.

His idea is that the N operator can be applied to "*fx*", an existential proposition, which can be written in the form of "(∃*x*).*fx*", obtaining "~(∃*x*).*fx*", i.e., "*~fa.~fb.~fc.* …", that is to say, *all* the propositions of ξ being false, which results in "(*x*).~*fx*". The application of N to this gives us "~(*x*).~*fx*", which in turn is equivalent to "(∃*x*).*fx*". If we then apply N to "*~fx*", we get "~(∃*x*).~*fx*", that is, "(*x*).*fx*", and by the same operation again we obtain "~(*x*).*fx*", which is equivalent to "(∃*x*).~*fx*". According to this proposal, the universality is, paradoxically enough, derived from existentiality, from what we do have indeed, even if it is also true that we do have an original relation to the idea of "all". Still, to derive "(*x*).*fx*" from "*fa.fb.fc.* …" is a big step, one that can only be taken *extralogically*.

This became apparent to Wittgenstein at the time of composing the third of the surviving notebooks from the First World War. The opening entry of 13 July 1916 provides a clue:

> One keeps on feeling that even in the elementary proposition mention is made of all objects. (MS103, 23r: NB, 76e)

And in an entry from 20 July, omitted in the *Notebooks 1914-1916* along with quite a few remarks on the same subject sketched in the previous days (cf. MS103, 25r-27r), Wittgenstein observes:

> ~~The~~ My old division of all propositional forms was fundamentally correct, only another mode of generality will be required. (MS103, 27r: my translation)

This is the reason why, as we read in the *Prototractatus* manuscript, where the original version of §5.521 of the *Tractatus* was formulated, Wittgenstein "separate[s] the concept all from ~~the logical product.~~ the truth-function" (MS104, 87, §5.3201: my translation, adapted to Ogden's). It is true all the same that he holds it as a "logical product" – and this is the core of his later criticism. But it is one thing to hold it and another to derive it.

The way Russell understood "Mr Wittgenstein's theory of the derivation of general propositions", while proceeding, *in fine*, as his own, "from conjunctions and disjunctions", is thus incomprehensible, all the more since a few lines above he had written that "Wittgenstein's method of dealing with general propositions […] differs from previous methods by the fact that the generality comes only in specifying the set of propositions concerned", so that "when this has been done the building up of truth-functions proceeds exactly as it would in the case of a finite number of enumerated arguments *p*, *q*, *r*…" (TLP, 14). Russell will have not noticed, therefore, the real innovation of such a methodology, which lies, precisely, in the specification of "the set of propositions concerned", not being needed an enumeration of them.

This, as a matter of fact, had already been pointed out by Wittgenstein to Russell in the postscript to a letter dated 19 August 1919. There, replying to a number of questions raised by Russell in a letter from 13 August (cf. CL, 121-3), Wittgenstein states:

> I suppose you didn't understand the way, how I separate in the old notation of generality what is in it truth-function and what is purely generality. A general prop[osition] is A truth-function of *all* PROP[OSITION]S of a certain form. (CL, 126)

And he goes on saying, referring to the symbol "N($\bar{\xi}$)" and to Russell's feeling that "the duality of generality and existence persisted covertly in [his] system" (CL, 122):

> You are quite right in saying that "N($\bar{\xi}$)" may also be made to mean $\sim p \vee \sim q \vee \sim r \vee \sim$... But this doesn't matter! I suppose you don't understand the notation of "$\bar{\xi}$". It does not mean "for all values of ξ...". But all is said in my book about it and I feel unable to write it again. (CL, 126)

In short, Russell will have seen in Wittgenstein's method simply another way of obtaining all the quantifiers, not a truly alternative way of deriving generality, avoiding the "old" recourse to "*fa.fb.fc.* …" and "*fa* ∨ *fb* ∨ *fc* ∨ …". Yet, only by means of an *extralogical* procedure we may turn round the *singular* nature of our point of view. It is the N operator that makes it possible to realize that our relation to the universal is *constitutive*, even though the *epistemological* status of that relation is problematic, amounting propositions such as "All men are mortal" to mere "variable hypotheticals", in the phrase of F.P. Ramsey (1931, 237). Wittgenstein's refusal of an *inductive logic*, expressed at §6.31 of the *Tractatus*, turns out, in this light, to be clearer. What is not at all clear is why Wittgenstein followed §5.3201 in the *Prototractatus* notebook by a remark, which he crossed out, saying that "[e]thics is not one of the natural sciences" (MS104, 87: my translation). This, however, I cannot go into.[1]

Literature

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