Some Remarks on Wittgenstein and Type Theory in the Light of Ramsey

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Paradoxes, Logicism and the Theory of Types

Russell developed his Theory of Types as an answer to a range of paradoxes he saw his logicist project confronted with. One of these paradoxes is Russell's well-known paradox about the set of all sets not containing themselves, others are such famous paradoxes as the liar, Berry's paradox and the Grelling/Nelson paradox. Henri Poincaré coined the term ›vicious-circle fallacies‹ for all of these: what they seem to have in common is, in Russell's words, that

> [i]n each contradiction something is said about *all* cases of some kind, and from what is said a new case seems to be generated, which both is and is not of the same kind as the case of which *all* were concerned in what was said. (Russell 1908, 224)

Russell presented the first draft of a solution to this class of paradoxes as an appendix to his *Principles of Mathematics* in 1903 and a full-blown solution in his ›Mathematical Logic as based on the Theory of Types‹ in 1908. The core idea behind type theory is that each propositional function has a ›range of significance‹, *i.e.*, a set of possible arguments, and the following limitation:

> This leads us to the rule: ›Whatever involves *all* of a collection, must not be one of the collection‹; or, conversely: ›If, provided a certain collection had a total, it would have members only definable in terms of that total, then the said collection has no total‹. (Russell 1908, 225)

The 1908 version of the theory played a key role in the monumental logicist project Russell and Whitehead were working on then, the *Principia Mathematica*. One serious flaw of the theory, however, was the need for the *Axiom of Reducibility*, which is everything but a *prima facie* plausible axiom of logic.

Ramsey's classification of the paradoxes

Around 1925, F. P. Ramsey, a logicist as well, was trying to find a way to dispense with the Axiom of Reducibility and to that avail examined Russell's reasons for introducing the Theory of Types in the first place: the various vicious-circle paradoxes. He introduced a nowadays generally accepted distinction between them:

> We can easily divide the contradictions according to which part of the theory is required for their solution, and when we have done this we find that these two sets of contradictions are distinguished in another way also. The ones solved by the first part of the theory [*i.e.*, the Simplified Theory of Types] are all purely logical; they involve no ideas but those of class, relation and number, could be stated in logical symbolism, and occur in the actual development of mathematics … Such are the contradictions of the greatest ordinal, and that of the class of classes which are not members of themselves. With regard to these Mr. Russell's solution seems inevitable.
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> On the other hand, the second set of contradictions are none of them purely logical or mathematical, but all involve some psychological term, such as meaning, defining, naming or asserting. … [I]t is possible that they arise … from ambiguity in the psychological or epistemological notions of meaning and asserting. Indeed, it seems that this must be the case, because examination soon convinces one that the psychological term is in every case essential to the contradiction, which could not be constructed without introducing the relation of words to their meaning or some equivalent. (Ramsey 1926, 192)

Ramsey classified the first type as *logical*, the second type as *psychological paradoxes*, though the term ›semantical paradoxes‹ for the latter is more common today. Regarding these two types of paradoxes, different parts of Russell's 1908 type theory are responsible for their solution. For the simpler logical paradoxes, the part of Russell's theory which is akin to his first proposal from 1903 was sufficient. Ramsey distilled that part from Russell's more complex 1908 theory and coined the term *Simplified Theory of Types* (STT) for the result. The semantical paradoxes, on the other hand, proved to be consistent regarding the STT and remained a problem requiring the full-blown theory, the *Ramified Theory of Types* (RTT), for its solution. Now, according to Ramsey, the logicist project was not at all confronted with the semantical paradoxes—he claimed that those paradoxes were problems of language, not of mathematics, and so it was not mathematics' job to deal with them. If, following Ramsey, the STT was indeed sufficient for the goals of mathematics, there was no need for using the RTT—and since the STT was lacking the negative side effects of the RTT that lead to the necessity of introducing the axiom of reducibility, Ramsey's modification made the logicist project much more acceptable.

Wittgenstein's critique of the Theory of Types

In the *Tractatus*' 3.33 ff., Wittgenstein presents his arguments against Russell's Theory of Types:

> In logical syntax the meaning of a sign should never play a role. It must be possible to establish logical syntax without mentioning the *meaning* of a sign: *only* the description of expressions may be presupposed. (Wittgenstein 1921, 3.33)
> From this observation we turn to Russell's ›theory of types‹. It can be seen that Russell must be wrong, because he had to mention the meaning of signs when establishing the rules for them. (Wittgenstein 1921, 3.331)

The core element of Wittgenstein's criticism can be understood in at least three ways: (1) In formulating the Theory of Types, Russell uses terms (›truth‹, ›meaning‹, ›type‹, …) that are, according to Wittgenstein, meaningless. If one understands Wittgenstein in this way, an alternative type theory, formulated on a purely syntactical level, could escape his criticism. Church's Theory of Types (Church

1940) is constructed in such a way and, given that this is the crucial point in Wittgenstein's criticism, could be regarded as a valid alternative to Wittgenstein's approach. James Davant discussed this option in his (Davant 1975) and came to the conclusion that *any* version of type theory is incompatible with Wittgenstein's system in the *Tractatus*; I will not here repeat his arguments. (2) Wittgenstein's criticism is directed at Russell's talking about the meaning of the symbols of the ›object language‹. (3) Wittgenstein's criticism must be understood as a combination of (1) and (2)—this is the way I understand Wittgenstein.

Indeed, Russell has to classify symbols according to their type: When he says that, *e.g.*, some symbols stand for individuals of type 0 or propositional functions of type 2, Russell is *in some sense* talking about the meanings of the respective symbols. This sense is a very basic one, no more problematic than saying that the relation ›is larger than‹ has to be accompanied by exactly two terms to add up to a meaningful sentence. Nonetheless, this *is* talking about the meaning of symbols and one may very well buy Wittgenstein's arguments against this if one likes.

In my opinion, this rather fundamental difference between Russell and Wittgenstein is grounded in their different approaches to language: Wittgenstein's ideal language in the *Tractatus* is no purely artificial language but the end point of an actual analysis of ordinary language, and thus somewhere between an ideal and an ordinary language. Though we do not use the *Tractatus'* language for actual communication, according to Wittgenstein we use the language on a very fundamental level of our thinking. Its names do refer directly to the *objects* (*Gegenstände*) of the world: in the *Tractatus*, there is a very close-knit connection between language, thinking, and ontology. As a consequence of this, Wittgenstein cannot state the meanings of names, of the symbols of his language, *in* his language: The meanings only *show themselves* through their use. Russell, on the other hand, is free to do this; he may very well use a metalanguage or a hierarchy inside his language to assign meanings to his symbols, since his (much more artificial) language does not necessarily stand in a fixed relation to our thinking and hence is not subject to the restrictions holding for Wittgenstein's language.[1]

Wittgenstein's way to avoid the logical paradoxes

Since Wittgenstein has to dispense with type theory, he has to put forth an alternative way to escape the problems associated with the paradoxes mentioned above. Moreover, type theory may very well have it's origin in the solution of the paradoxes, but its benefits surpass the simple fact that it can deal with them: the theory offers some deep insights into the nature of language, *e.g.*, into ambiguity, which is a crucial element in the logical paradoxes. Wittgenstein was very well aware of that and saw the need to give an explanation of these phenomena, too:

> In order to avoid such errors [resulting from ambiguity] we must make use of a sign-language that excludes them by not using the same sign for different symbols and by not using in a superficially similar way signs that have different modes of signification: that is to say, a sign-language that is governed by *logical* grammar—by logical syntax. (Wittgenstein 1921, 3.325)

Wittgenstein's symbol/sign distinction reminds one very much of Peirce's more familiar type/token distinction. Instead of saying that the word ›count‹ has two meanings, it could be said with Wittgenstein that there are two different *symbols* (types) which have the one *sign* (token) ›count‹ in common. The connection between the symbol and its meaning is constant; it is necessary to refer to the context of the sign—its position in the sentence—to ascertain what its correct symbol is, since a sign in isolation cannot have a meaning. Hence, the analysis of the use of signs in sentences reveals their corresponding symbols and thereby their logical form (see (Wittgenstein 1921), 3.326 ff.). The first step is a kind of optional disambiguation from sign to symbol; the second step the recognition of the symbol's logical form.

Once this is established, syntactical mistakes can be recognised. This does apply to more ordinary syntactical mistakes (»table chair« is not a meaningful combination of names) as well as to the not-so-obvious logical paradoxes: In ordinary language, some sentences do occur in which there seems to be a combination of symbols leading to a kind of vicious circle. In analysis, however, these problems disappear: by regarding the sign's context one can get from the sign to the correct symbol; disambiguation takes place. Wittgenstein gives an example:

> The reason why a function cannot be its own argument is that the sign for a function already contains the prototype of its argument, and it cannot contain itself.
> For let us suppose that the function F(fx) could be its own argument: in that case there would be a proposition »F(F(fx))«, in which the outer function F and the inner function F must have different meanings, since the inner one has the form φ(fx) and the outer one has the form ψ(φ(fx)). Only the letter »F« is common to the two functions, but the letter by itself signifies nothing.
> This immediately becomes clear if instead of
> »F(Fu)« we write »(∃φ):F(φu) . φu=Fu«.
> That disposes of Russell's paradox. (Wittgenstein 1921, 3.333)

One might have strings of growing complexity, *Fu*, *F*(*Fu*), *F*(*F*(*Fu*)), … in which similar signs ›F‹ occur in different positions. Analysis reveals that, though the different symbols' signs ›F‹ are identical, every sign belongs to a different symbol. This is exactly the approach that can be found in the STT. There, similar symbols (not to be understood in Wittgenstein's sense)—*e.g.*, the relation of identity—do appear on different types, *i.e.*, are systematically ambiguous. In the example above, each step to a more complex string can be regarded as a step from one type to the next in Russell's STT. Without some explicit indicator, *e.g.*, its type attached as an index to the symbol (which would be nothing but a disambiguation of the symbol, of course), Russell would have to resort to the context of the symbol, *i.e.*, its arguments, as well, to get to know its specific type. The last sentence in the previous citation makes the whole matter clear: this kind of disambiguation is the key to the solution of the logical paradoxes (of which Russell's paradox is the most well-known and explicitly mentioned by Wittgenstein), and both Russell and Wittgenstein offer means to solve the logical paradoxes by disambiguation. In Russell's as well as in Wittgenstein's ideal language there is exactly one name for each object. So, on the most

1 I have argued for this approach in my (Leerhoff 2008).

fundamental level, when analysis is done, there is no room for ambiguities nor, as a consequence, for the logical paradoxes, which can no longer be formulated.

The semantical paradoxes

Both the STT and Wittgenstein have similar techniques to avoid the kind of systematic ambiguity involved in the logical paradoxes. The semantical paradoxes, on the other hand, are *much* more complicated to avoid. They can still be formulated, and they are still paradoxical in an ideal language of Russell's kind with only STT-restrictions. Hence Russell developed the RTT to guard his language against them. In a nutshell, the states of affairs described in the semantical paradoxes can still be expressed in the ideal language, but the RTT enforces a non-paradoxical ›translation‹ for them. As I have stated above, there is a high price to pay for this: the RTT is extremely complicated and, at least for some areas of application, further axioms have to be postulated.

How does Wittgenstein's solution of the semantical paradoxes fare in this respect? In all these paradoxes some semantical (or, in Ramsey's word, ›psychological‹) terms play a crucial role, *e.g.*, ›truth‹, ›naming‹, ›lying‹, etc. In Wittgenstein's ideal language, there are no and can be no expressions for these ordinary-language terms, so the whole question of semantical paradoxes is a non-issue for Wittgenstein. This, of course, is a high price to pay as well, since it sets definite limits to the areas of application for the language. In Wittgenstein, these limits do not result from the threat posed by the semantical paradoxes; their ›solution‹ has to be regarded as a kind of side effect of limits that are grounded in the *Tractatus'* concept of language.

Conclusion

Ramsey's distinction of the paradoxes in logical ones on the one hand and psychological (or semantical) ones on the other proves to be valuable for an examination of Wittgenstein's alternative to Russell's Theory of Types. The logical paradoxes pose a threat for Wittgenstein's system as well as for Russell's. Since Wittgenstein cannot integrate a type theory in his system, he offers an alternative approach to the disambiguation of terms, which is the key to the solution of those paradoxes. His way of solving these problems has striking similarities to Russell's STT. The semantical paradoxes, however, do pose a threat for Russell's system, but not for Wittgenstein's. This difference is due to the diverging concepts of language in their respective variants of logical atomism.

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