The Determination of Form by Syntactic Employment: a Model and a Difficulty

Colin Johnston, London, England, UK

1.

An entity's logical character, for Russell and Wittgenstein, is a matter of the ways in which it may combine with other entities to form atomic facts. Where Russell gives a theory of the logical constitution of atomic facts, however, Wittgenstein asserts that the ways in which entities combine in facts can be known only *a posteriori* through the process of analysis.[1] Russell was thus mistaken in Wittgenstein's eyes in laying out as he did his logical variety of particulars and the various kinds of universal. Pressing the Tractarian position, Ramsey claims that we know "nothing whatever about the forms of atomic propositions". We do not know, for example, "that there are not atomic facts consisting of two terms of the same type" (Ramsey 1990 p29).

I shall suggest that this Tractarian agnosticism is in tension with the Tractarian doctrine that the logico-syntactic use of a sign determines a logical form. Imagine a 'world' in which there are only two forms (that is, logical types) of object and only one mode of combination, a mode in which a single object of each form is combined. The symmetry of this world is such that the two object forms are internally indistinguishable. The internal character of each form is exhausted by its being the form of an object whose only possibility for combination is in a certain mode with an object of the other form, and the internal character of the mode of combination is exhausted by its being a mode of combination of one object of each form. Wittgenstein's agnosticism regarding the forms of reality means that he cannot say in advance that reality does not, like our imagined 'world', include distinct but internally indistinguishable forms. A logico-syntactic use, however, is to determine a logical form by virtue of determining the internal nature of that form only. If reality turns out to include internally indistinguishable forms it follows that the determination as envisaged of logical form by logico-syntactic use will not everywhere be possible.

To bring this concern into focus I want to develop a simple, semi-formal account of syntactic use, of form, and of the place of syntactic use in the determination of form. The account will be appropriately general to accommodate Wittgenstein's ignorance of the nature of the forms of reality. I do not claim that the semi-formal work is at every point implicit in the Tractatus. Rather, the work is intended as an elucidatory model of certain Tractarian ideas.

2.

The notion to be developed is of an *atomic syntactic system*. An atomic syntactic system S has:

a vocabulary V of signs, and
a set $T = \{M_j : j \in J\}$ of sign types

where each $M_j \subseteq V$ and J is an indexing set. Signs here are typographically identified marks. Further, the system S has:

a set C of manners of sign combination.

A manner of sign combination $c \in C$ will be a manner of combination of a determinate, finite number of ordered signs. The combination in mode c of the signs $s_1, s_2, \ldots, s_n$ so ordered is denoted by $c(s_1, s_2, \ldots, s_n)$. Finally for S there is, for each manner of combination $c \in C$, a rule of the form:

$$x_1 \in M_{f(c,1)}, x_2 \in M_{f(c,2)}, \ldots, x_n \in M_{f(c,n)} \Leftrightarrow c(x_1, x_2, \ldots, x_n) \in F$$

where f is some (appropriately partial) function from $C \times \mathbb{N}$ to J. Set F is the set of formulae of S; it contains no members besides those provided by the system's rules of combination. Note that the rules for membership of F have the form of equivalences. What is not allowed in a syntactic system is, say, $c(s,t) \in F$ and $c(u,v) \in F$, but $c(s,v) \notin F$. Each position in each manner of combination determines a set of signs which figure in that position in a formula, and whether or not a combination in a mode of C of signs from V is a formula of the system depends on the signs' positions in the combination and their membership of such sets only.

Next we want to reach an idea of the structure of a syntactic system, abstracting away from the signs and manners of combination deployed in any particular system instantiating that structure. The thought here is that what is of structural interest is simply the number of positions belonging to each sign type in each manner of combination. Thus let's say:

$X \in T$ occurs n (≥ 0) times in combination c if, and only if, $X = M_j$ and exactly n of f(c,i) are equal to j

And with this we make the definition:

Two atomic syntactic systems S1 and S2 with manners of combination C1 and C2 and sets of syntactic mark-types T1 and T2, are *isomorphic* if, and only if, there exists a bijection $\alpha: C1 \to C2$ and a bijection $\beta: T1 \to T2$ such that, for all $c \in C1$ and $X \in T1$, (X occurs n times in c) $\Leftrightarrow$ ($\beta(X)$ occurs n times in $\alpha(c)$).

Such a bijection $(\alpha,\beta): C1 \times T1 \to C2 \times T2$ is an isomorphism from S1 to S2.

The notion of an atomic syntactic system and its structure is now given. Let's take a look at what its interest might be.

1 See Wittgenstein 1961 5.55 – 5.5571. See also Wittgenstein 1993 pp. 29-30 and Wittgenstein 1979 p. 42.)

3.

An atomic syntactic system is a system of combinations of marks. Whether and how certain marks in a system's vocabulary may combine with each other to make formulae of the system depends on what types of marks they are. With this in view we could say: a *syntactic use* of a sign is a role that sign has in some syntactic system S as a possible element of members of F, the set of formulae of S, by virtue of its membership of a particular sign type of S. Of course, such a use is bound to the modes of combination and sign types of S, but this tie is something we can abstract away from. If S1 and S2 are isomorphic systems with isomorphism (α,β), then the role a mark has in S1 by virtue of its membership of a sign type X of S1 is structurally equivalent to the role in S2 had by a member of β(X) by virtue of its membership of that set. And at first glance one might think to say here: the two syntactic uses determine the same *form*. With its syntactic use in a certain system, a sign determines a place in the abstract combinatorial structure instantiated by that system – it determines a form.

On closer inspection, these last two sentences will be seen to be slightly hasty. But let's not worry about that right away. Rather, let's run with them and look instead at a few concrete examples of atomic syntactic systems, beginning with the case of a Russellian system. A Russellian atomic system has:

$$F_R = \cup_{n\geq 2} \{c_n(x_1, x_2, \ldots, x_n): x_1 \in U_{n-1}, x_2, \ldots, x_n \in P\}$$

U_n here is the set of universal signs of degree n, and P is the set of particular signs. In line with traditional scripts, one might use P = {'a_i'}, U_n = {'$R^n{}_i$'} and set $c_n(x_1, x_2, \ldots, x_n)$ to be the combination that the x_i are written in order. (Thus F_R would contain such formulae as '$R^1{}_1a_1$', '$R^2{}_4a_1a_2$', '$R^3{}_2a_5a_1a_6$'.) Of course, many other sign types and combinatorial modes could be used; the resulting systems would, however, all bear the same structure.

Systems with structures quite different from the Russellian structure can of course be readily concocted. Ramsey envisages the possibility of atomic facts consisting of two entities of the same type. Forms answering to this description would arise within such (non-isomorphic) systems as S1, S2 and S3 defined by:

F1 = {c1(x, y): x, y∈A},
F2 = {c2(x, y): x, y∈B} ∪ {c3(x, y): x∈C, y∈D }, and
F3 = {c4(x, y): x, y∈E} ∪ {c5(x, y, z): x∈E, y∈F, z∈F}

Ramsey's claim against Russell is that we have no more reason to believe that *logical* forms – the forms of reality – are those generated in F_R any more than they are those generated by such entirely different systems as F1, F2 and F3.

4.

Pausing on the system S2, an interesting possibility may come into view. An atomic syntactic system, one will notice, can be non-trivially self-isomorphic. A mapping (α,β):{c2, c3}×{B, C, D}→{c2, c3}×{B, C, D} set to identity other than β(C) = D and β(D) = C is an isomorphism from S2 to itself. Similarly we might consider a system S4 defined by:

F4 = {c6(x, y): x, y∈G} ∪ {c7(x, y): x, y∈G}

This system is again non-trivially self-isomorphic with a non-trivial isomorphism taking G to G, c6 to c7, and c7 to c6.

With such possibilities in mind, let's make a few further definitions. Consider a system S with manners of combination C and set of sign types T. Then for each t∈T and c∈C let

Λ_t = {x∈T: there is an isomorphism (α,β):C×T→C×T such that β(t)=x}
Γ_c = {x∈C: there is an isomorphism (α,β):C×T→C×T such that α(c)=x}

From this we may say that a system S with manners of combination set C and set of syntactic mark-types T is *symmetrical* with respect to K⊆T if, and only if, there exists t∈T such that K=Λ_t≠{t}. Similarly S is symmetrical with respect to L⊆C if, and only if, there exists c∈C such that L=Γ_c≠{c}. If S is not symmetrical with respect to any set then S is asymmetrical.[2]

How are we to place the possibility of symmetry within atomic syntactic systems? Well, Wittgenstein envisages the possibility of distinct objects which are internally indistinguishable.[3] In a similar vein we imagined above a 'world' (call it W1) in which there are only two forms of object and only one mode of combination, a mode in which one object of either form is combined. The two object forms of this world are distinct but the symmetry of the combinatorial situation is such that they are internally indistinguishable. Alternatively we could imagine a world W2 in which there is a single form of objects and two modes of combination, each mode being a mode of combination of two objects. Here the two modes are distinct but internally indistinguishable. And what is in general being imagined with such indistinguishabilities, we can see, are precisely worlds whose structures are instantiated by symmetric syntactic systems. S4 above, for instance, instantiates the structure of W2 and is symmetrical with respect to {c6, c7}. The structure of W1 is instantiated by a system S5 defined by

F5 = {c8(x, y): x∈H, y∈I}

which is symmetrical with respect to {H, I}.

5.

It would appear that we should revise the general thought above that a sign in use in an atomic syntactic system determines a place in the abstract combinatorial structure instantiated by that system, that is that it determines a form. Take the system S2. This system has a structure with three forms; two of these three forms are, however, internally indistinguishable. A sign of S2 which is a member of B determines as such the distinguishable of these three forms; in use as a member of B the sign has that form. Members of C and D, however, determine as such only the class of the two indistinguishable forms: their syntactic use gives the shared nature of the two forms but

2 Note that the Λt and Γc partition T and C respectively. They cover T and C, for t∈Λt and c∈Γc (put (α,β) to identity). Next, if sign type q∈Λr∩Λs then there exist isomorphisms (α1,β1) and (α2,β2) on S such that β1(r)=β2(s)=q. Then (α2-1.α1, β2-1.β1) is an isomorphism on S such that β2-1.β1 (r)=s. (The inverse of an isomorphism is an isomorphism (as defined), and the composition of isomorphisms is an isomorphism.) Now take some u∈Λs. There exists an isomorphism (α3,β3) on S such that β3(s)=u. But then (α3.α2-1.α1, β3.β2-1.β1) is an isomorphism on S such that β3.β2-1.β1(r)=u. Thus u∈Λr and so Λs⊆Λr. Similarly Λr⊆Λs, and so Λr=Λs. In the same way, if there is a mode of combination d∈Γe∩Γf then Γe=Γf.

3 See Wittgenstein 1961 §2.0233. Indeed, he envisages the possibility of two entities which are externally as well as internally indistinguishable (Wittgenstein 1961 §§2.02331, 5.5302).

does not select between them. Noting the possibility of such a situation one might move to say that a syntactic use determines not a form but a form type. Taking up this description of the matter one needs, however, to bear in mind that the number of ‘tokens’ had by a particular ‘form type’ is internal to the type. Where the type has only one token, then, the determination is of nothing less than the token.

In whatever terms one chooses to weaken the general claim that syntactic uses determine forms, the Tractarian position that a sign in *logico*-syntactic use determines a *logical* form comes under threat. Wittgenstein does not know what the logical forms are; he does not know the logical structure of reality. Therefore he does not know that the structure of reality is not symmetrical with regard to certain object forms. But if reality is so symmetrical, a logico-syntactic employment of a sign – that is, a syntactic employment of a sign in a system instantiating the structure of reality – will not always determine a unique logical form.

The point might be thought to be somewhat nitpicking. A logico-syntactic use is guaranteed to determine, as said, a ‘form type’, even if it is not certain that all logico-syntactic uses will determine a single form. Is this not good enough for Wittgenstein? Well I cannot here follow through what all the repercussions might be for his system if the thesis of the determination of logical form by logico-syntactic use is relaxed as mooted. We can quickly note, however, that on pain of the possibility of nonsense Wittgenstein will have to allow that what one symbol – that is a sign in logico-syntactic use – can mean might depend on what other symbols of the language actually mean. To see this note first that two signs in the same use may not refer to entities of distinct types: two signs in the same use will be intersubstitutable in propositions, and so their reference to entities of distinct types would entail the possibility of nonsense propositions. Now suppose that reality has two internally indistinguishable forms. In a language instantiating the structure of reality there will, under this supposition, be a logico-syntactic use u which determines the type of these indistinguishable forms but does not select between them (in fact there will be two such uses). A sign in use u will not, however, be free to refer to an object of either of these two forms: it will, on pain of the possibility of nonsense, be constrained to refer only to objects of the same form as those referred to by other signs in the same use.

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