“Reality” and “Construction” as Equivalent Evaluation-Functions in Algebra of Formal Axiology: A New Attitude to the Controversy between Logicism-Formalism and Intuitionism-Constructivism

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In general, the formal-logical equivalence between “existence of a” and “existence of the algorithm of (construction of) a” is not true. Nevertheless, terms “formal-logical equivalence” and “formal equivalence (equivalence of forms)” are not synonyms. Consequently, there is a possibility of existence of such a formal equivalence of “being” and “being of the algorithm of constructing”, which does not imply (logically) their formal-logical equivalence. The article demonstrates just such an unusual (hitherto unknown) formal equivalence of “existence of a” and “existence of the algorithm of construction of a”. This result is obtained within two-valued algebra of formal axiology. In this algebra formal-axiological meanings of the words “existence” and “algorithm” are considered as formal-axiological operations. The evaluation-functional sense of these operations is defined by tables. A formal-axiological equivalence relation is defined strictly. By means of these definitions it is demonstrated that there is the formal-axiological equivalence between axiological forms of “being” and “being of the algorithm”. As, in general, there is no logical identity between the notions “axiological forms” and “logical forms” (of “being” and “being of the algorithm”), there is no logical contradiction between the above-affirmed hitherto unknown formal-axiological equivalence and the famous mathematical facts underlying the controversy between formalism-logicism and intuitionism-constructivism. The submitted result is elementary from the proper mathematical point of view as the technical aspect of it is basic one, but the result is very important for illuminating hitherto unknown (ignored on principle) properly philosophical (axiological) grounds of the controversy between the two kinds of philosophies of logic and mathematics.

According to one of the most influential traditions in studying philosophical foundations of mathematics, there is the following triple of intellectually respectable trends in this studying: the formalism; the logicism and the intuitionism-constructivism. However there are serious problems in the mentioned three-sided tradition. For example, reducing L.E.J. Brouwer’s intuitionism (Brouwer 1913) to A. Heyting’s one (Heyting 1975) is a very strong simplification: probably, Brouwer himself would like to protest against this reducing. Moreover, the constructivists used to manifest and emphasize the existence of significant distinctions between their philosophy of mathematics and the intuitionists’ one.

In spite of the expectations, the submitted paper is not devoted to direct discussing the mentioned three-sided tradition. The paper presents an attempt to jump out from the dominating paradigm by means of concentration on “ethicism” – the “well-forgotten-old” (ancient) aspect of investigating the properly philosophical grounds of human creative work in sphere of mathematics. Probably, the word “ethicism” (in philosophy of mathematics) is a hitherto unknown (not used) one. Nevertheless the direction of research it stands for exists from Pythagoras and Plato to A.N. Whitehead. I mean investigating the properly ethical aspect of mathematical activity which deals with the good and the bad (evil) sides of it. (The words “good” and “bad (evil)” are used in their moral meanings.) The Pythagorean Union insisted upon the existence of not only logical and aesthetical but also ethical foundations of mathematics. Being under the strong influence of Pythagoreans, Plato tried necessarily to combine notions “mathematics” and “the good”. According to A.N. Whitehead, Plato’s attempt was not successful. The attempt of B. Spinoza was not successful too. Nevertheless a small finite set of not successful attempts is not a strict proof of the impossibility on principle. In XX century the relevance (and even indispensability) of a fundamental uniting “mathematics” and “the good” was substantiated by A.N. Whitehead. He insisted upon the relevance of continuing the attempts to unite the two. However he did not submit a concrete variant of such uniting. He suggested the realization of the mentioned idea to other researchers. Being inspired by the above-indicated reasons, in present paper I submit a concrete variant of moving forward in direction of combining “mathematics” and “the good”. First of all it is necessary to make clear that I imply transition from the ethics to a formal one, and then from the formal ethics to a mathematical (mathematized) formal one. At the end of this transition I am to apply the mathematical (mathematized) formal ethics to philosophical foundations of mathematics and to study results of this application. From the history viewpoint, the logicism emerged in the same (analogous) way.

Now let us make agreements about meanings (rules of using) the words involved in our discourse. Let the term “formal ethics” stand for such a branch of ethics, which study moral forms of (any) free human activity deprived of their specific contents. Thus the abstraction from specific contents of moral forms (of activity) is accepted and used systematically. Let the term “mathematical (mathematized) ethics” stand for such a branch of formal ethics, which study mathematical simulations of formal ethics. The present paper exploits two-valued algebra of (moral) actions – the most elementary discrete mathematical simulation of formal ethics. (It is a simulation of the moral rigor, which is the most primitive moral attitude. However this basic attitude does exist in reality.)

Let us define basic notions of two-valued algebra of formal ethics. This algebra is based upon the set of actions (moral ones) and their moral forms deprived of the contents. (Subjects of actions are reduced to their actions.) By definition, actions are such operations, which are either good or bad (in moral sense). (Subjects of actions are also either good or bad.) Elements of the set {g (good), b (bad)} are called moral values of actions (and of action subjects). As subjects of actions can be reduced to their actions, for the sake of simplicity, below we shall talk only about actions. Let symbols *x*, *y* stand for moral forms of actions deprived of their contents. Moral forms of simple actions play the role of independent axiological (evaluative) variables. Axiological variables take their values from the above-mentioned set {g (good), b (bad)}. Moral forms of compound actions represent moral evaluation-functions. These functions take their values from the set {g (good), b (bad)} as well. Complex moral action forms (compound moral evaluation functions) are obtained by applying formal-axiological connectives to the axiological variables. Below we introduce only such formal-axiological connectives, which are relevant to the theme of the paper, namely, only such, which are necessary for the explication of moral (formal-ethical) foundations of mathematical activity. Symbols standing for the unary moral operations under discussion are introduced by means of the following glossary.

Glossary for the below given table 1. The symbol *Bx* stands for the moral evaluation function determined by one variable “being (existence) of *x*”. *Nx* – “non-being (non-existence) of x”. *Cx* – “construction (production, creation) of *x*”. *Dx* – “deconstruction (destruction, extermination) of *x*”. .*Ax* – “algorithm of (what, whom) *x*”. *Mx* – “machine of (what, whom) *x*”. $A^F x$ – “algorithm for (instead of) *x*”. $M^F x$ – “machine for (instead of) *x*”. *Ox* – “opposite (opposition) for *x*”. *Px* – “process of *x*”. *Rx* – “reality (actuality) of *x*”. *Fx* – “completeness (fullness) of *x*”. *Ux* – “incompleteness of *x*”. *Ix* – “contradiction in *x*, i.e. inconsistency (contradictoriness) of *x*”. *Gx* – “consistency (non-contradictoriness) of *x*”. The moral-evaluation-functional sense of these unary formal-ethical operations is defined by the following table 1.

x	Bx	Nx	Cx	Dx	Ax	Mx	A^Fx	M^Fx
g	g	b	g	b	g	g	b	b
b	b	g	b	g	b	b	g	g

x	Ox	Px	Rx	Fx	Ux	Ix	Gx
g	b	g	g	g	b	b	G
b	g	b	b	b	g	g	B

In the two-valued formal-ethics algebra, by definition, moral action forms (x and y) are called formally-ethically equivalent if and only if they (x and y) acquire identical moral values under any possible combination of moral values of variables occurring in x and y. Let the symbol “x=+=y” stand for the formal-ethical equivalence of action forms x and y. By means of the above definitions it is easy to demonstrate the following formal-ethical equations. To the right from each equation I have placed its translation from the symbolic language into the natural one. In these translations the word-homonym “is” stands not for the formal-logical connective but for the above-defined equivalence relation “=+=”.

1) *Ix=+=Nx*: contradiction in *x* is nonbeing of *x* (D. Hilbert).

2) *Rx=+=Bx=+=NIx=+=Gx*: reality (being) of *x* is nonbeing of contradiction in *x* (D. Hilbert).

3) *Bx=+=Cx*: being of *x* is construction of *x* (intuitionists-constructivists).

4) Cx=+=Ax: construction of *x* is algorithm of *x* (constructivists).

5) Bx=+=Ax: being of *x* is algorithm of *x* (constructivists).

6) *Bx=+=BCx*: being of *x* is being of construction of *x* (constructivists).

7) *Bx=+=BAx*: being of *x* is being of algorithm of *x* (constructivists).

8) *Bx=+=BACx*: being of *x* is being of algorithm of construction of *x* (constructivists).

9) *NACx=+=Nx*: nonbeing of algorithm of construction of *x* is nonbeing of *x* (constructivists).

10) *Rx=+=PCx*: reality of *x* is process of construction of *x*.

11) *Rx=+=Cx*: reality of *x* is construction of *x*. (This statement is directly relevant to the theme of the symposium section which I have submitted the paper to.)

12) *Gx=+= Fx*: consistency (non-contradictoriness) of *x* is equivalent to completeness of *x*.

At first glance many of the above equations seem extremely paradoxical (even crazy). For instance, being formulated in general, the last equivalence seems to be an evident absurdity – a logical contradiction with the obvious (well-established) facts – K. Gödel’s famous meta-theorems about the formal arithmetic. However this “contradiction” is nothing but a logic-linguistic illusion, as the equation 12 means not the formal-logical equivalence of the fact of non-contradictoriness and the fact of completeness, but the formal-ethical (formal-axilogical) equivalence of the value of non-contradictoriness and the value of completeness. One commits a strictly forbidden blunder when he/she replaces the term “formal-ethical equivalence of values” by the term “formal-logical equivalence of propositions affirming that the values are realized”. Committing this blunder necessarily results in the impression that the equation 12 logically contradicts to the meta-theorems of K. Gödel. But the rule A—D, precisely formulated below prohibits committing this blunder. From *x=+=y* it does not follow logically that the proposition informing that *x* is real, and the proposition informing that *y* is real, are logically equivalent. Truth of the universal statement of formal-ethical equivalence of moral-evaluation-functions “consistency” and “completeness” is logically compatible with falsity of the universal statement of formal-logical equivalence between affirming that consistency is real and affirming that completeness is real.

Another strong illusion of an evident paradox concerns the above equations 3-11 *establishing a fundamental formal unity (even identity) of reality and construction*. In respect to this formal identification there was the famous psychological explosion (intuition-language one) in philosophy of mathematics. The paradox impression has caused the famous sharp conflicts between the formalists-logicists and the intuitionalists-constructivists. However, from the viewpoint of above-submitted algebra this famous controversy is a result of logic-linguistic confusion. I repeat that in the above translations of the equations into the natural language the word-homonym “is” stands for the relation “=+=”. Chaotic mixing and substituting (for each other) the formal-logical and the formal-ethical meanings of the word “is” is strictly forbidden by the principle of formal-logical autonomy (i.e. nonbeing of valid formal-logical inferences) between corresponding facts and evaluations. The formal-logical gap between them is absolutely unbridgeable. In algebra of formal ethics this autonomy principle is mathematically represented by the following rule.

Let *Ex* stand for an act of informing (true or false affirming) that *x* takes place in reality. The above-said (about “=+=” and the formal-logical connectives) may be formulated as the following rule A—D. (A) From the truth of *x=+=y* it does not follow logically that the logical equivalence of *Ex* and *Ey* is true. (B) From the truth of the logical equivalence of *Ex* and *Ey* it does not follow logically that *x*=+=*y* is true. (C) From the truth of *x*=+=*y* it does not follow logically that [either (*Ex* logically entails *Ey*, or (*Ey* logically entails *Ex*)] is true. (D) From the fact that [either (*Ex* logically entails *Ey*), or (*Ey* logically entails *Ex*)] is true, it does not follow logically that *x*=+=*y* is true.

This rule is an effective remedy for the naturally emerging impression (illusion) that the above-listed formal-ethical sentences are paradoxical. To produce and use this remedy the above observation recognizing the homonymy of “is” is indispensable.

By means of the submitted discrete mathematical simulation of formal ethics it is easy to see that the above list of equations is logically consistent. In particular, even generally speaking, the equations 2 and 8 are logically compatible. The first impression of their incompatibility (in general) is a logic-linguistic illusion generated by violating the above-formulated rule of formal-logical autonomy of facts and evaluations. Hence, according to the present paper, in relation to the moral ideal of creative work in mathematics, the formalists and the constructivists are together: their distinctions are not significant. Consequently, the equations 1-11 mathematically represent important ethical foundations of mathematics as creative activity – one and the same moral ideal of mathematicians belonging to both parties: to the one of Hilbert-Russel and to the one of Brouwer–Heyting. Thus after the split mathematicians are united again.

Literature

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