

Is 'a = a' Apriori?

Wolfgang Freitag, Konstanz, Germany

wolfgang.freitag@uni-konstanz.de

In the beginning of his 'On Sense and Reference', Frege states the following, claiming it to give rise to the puzzle of informative identity:

$a = a$ and $a = b$ are obviously statements of different epistemic value: $a = a$ is valid apriori and has to be called, according to Kant, analytic, while statements of the form $a = b$ often contain very valuable extensions of our cognitions and are not always justifiable apriori. (Frege 1994 [1892], p. 40)

Frege states that 'a = a' is apriori, while 'a = b' is not. Hence there are informative identity statements. His problem is then to explain how informative identity is possible. Frege's solution is notorious; he proposes a form of semantic dualism via the distinction between sense and reference. The *reference* of a name is the object referred to. A *sense* is the mode of presentation of the reference. It has at least two functions: (i) it determines the reference of the name, and (ii) it contributes to the cognitive value of any expression in which the name occurs. The informativity problem is solved with respect to (ii). Co-referring names may have different senses and hence different cognitive values, which makes the resulting identity informative.

We may draw two different conclusions from the Frege quotation:

(F1) There are identity statements with different epistemic values, some being apriori, others being aposteriori.

(F2) Identity statements of the form 'a = a' are apriori and identity statements of the form 'a = b' are aposteriori.

(F1), if true, causes the trouble for identity. (F2), if true, transfers the trouble to the difference in names; the use of different names makes for informativity; the use of the same name twice makes for apriority. Though the dispute around informativity concerns (F1), namely, whether it is true and, if so, what consequences to take, the discussion is usually led with respect to the stronger claim (F2).¹ I wish to remain neutral about (F1) and claim with respect to (F2) that, given there are aposteriori identities of the form 'a = b', there are aposteriori identities of the form 'a = a'. After analysing the general reason for this in terms of type conditions for name tokens, I will show that sense alone does not fulfil its function (i): it does not furnish the reference for a name.

1. Arguments for the apriority of 'a = a'

Let me begin with the arguments for the apriority of 'a = a'. Such arguments are scant; usually, they are alluded to only perfunctorily, as if to satisfy the *formal* requirement of giving some argumentation where the conclusion of the argument is not in need of any support. Two such arguments can be identified.

¹ (F2) is stronger only if there are identity statements of the two forms. Grant me this presupposition.

First argument: 'a = a' is a truth of logic in the sense of being valid in all logical calculi. Truths of logic are apriori. Hence 'a = a' is apriori.

Replies: Firstly, 'a = a' can be a truth only in logical systems with identity. If there is no identity sign in the system, there will not be an identity truth. Secondly, even considering only logical systems with identity, there may be logical systems in which 'a = a' is not apriori. These will be systems which, e.g., allow different tokens of 'a' to refer to different objects. Thirdly, and most importantly, Frege's thesis is, I think, not a thesis about identity in logical systems. It is a thesis about identity in *ordinary* language. At present, we want to know whether 'Hesperus = Hesperus' or 'Fred is identical to Fred', as statements of ordinary English, are apriori.

Second argument: 'a is self-identical' is true because of the meaning of the self-identity predicate alone. A sentence of the form 'a is F' which is true because of the meaning of the predicate 'F' alone is apriori. Hence 'a is self-identical' is apriori.²

Reply: The argument seems cogent to me, but it is not relevant for our discussion. Even if 'a is self-identical' is apriori, the apriority of 'a = a' does not follow: 'a = a' does not state that a is identical to itself. It says that a is identical to a. There isn't any reflexive term like 'itself' involved in the identity statement. There is only the identity sign and two tokens of the graphic type 'a'. Differently put, 'self-identical' is a monadic predicate, while '=' is a dyadic predicate.³ Of course, one might *define* self-identity in terms of identity. With the help of, e.g., lambda-abstraction we can define 'a is self-identical' as '(λx) (x = x) (a)' from which lambda-conversion gives us 'a = a'. But given that the epistemic status of a defined statement is derived from that of the defining statement, the apriority of 'a is self-identical' would have to be grounded in, and could not itself ground, the apriority of 'a = a'.

2. Arguments against the apriority of 'a = a'

Perhaps there are, as yet, no good arguments for the apriority of 'a = a'. The *onus probandi*, however, is on the opponent of the apriority of 'a = a'. So what are the arguments?

First argument: 'a = a' might be false. Suppose that the 'a' is multi-referential,⁴ i.e., is a name of several objects. Suppose further that the left occurrence of 'a' refers to a different object than the right occurrence of 'a'. Then 'a = a' is false. Thus it is not true and therefore not apriori true. Identities of the form 'a = a' need not be apriori.

² Arguments of this type can be derived from, e.g., Barcan Marcus 1981, pp. 505–506, and Tichy 1983, p. 232. They even claim that 'a = b' is no more than stating the self-identity of an object.

³ A similar point is made by Salmon 1991, who furthermore claims the propositions to be expressed to contain two constituents in the self-identity case, and three in the identity case.

⁴ I hesitate to call such names 'ambiguous', in order not to provoke discussion on points tangential to my thesis.

There is an obvious reply: Since different objects may have the same name in ordinary languages, this argument is certainly cogent, but equally certainly Frege's intentions are based on the exclusion of aposteriority due to such possible falsity. We may simply conditionalise (F2) on *true* identities and obtain

(F2*) 'a = a' is – if true – apriori and 'a = b' is – if true – aposteriori.

This weakening of (F2) is not vulnerable to the first argument.

Second argument: Is (F2*) indisputable? I do not think so – at least not if we accept that 'a = b' can be aposteriori (something I assume for the sake of the argument). Consider the following scenario:

A philosophy student hears about Saul Kripke twice a week, once in his seminar on modal logic and once in his class on Wittgenstein's *Philosophical Investigations*. He rightly wonders whether it is one and the same person having such extraordinary thoughts. Upon looking in the Web, he finds out that this is indeed the case. 'Kripke = Kripke' has been found out aposteriori.⁵

So 'a = a' is, at least in some cases, aposteriori. In claiming that it is (always) apriori, Frege must assume that different tokens of the same name have the same sense. But as our example shows, this is in general not true. If one accepts the Fregean Hesperus–Phosphorus identity as informative, then one must accept that there are informative identities of the form 'Phosphorus = Phosphorus'.⁶

I have modelled a case of 'a = a' to one of 'a = b'. But we may also make the reverse move, by claiming that 'a = b' can be as apriori as 'a = a' can be. Suppose that the philosophy professor introduces not only the name 'Saul Kripke', but also – in the same breath, as it were – the name 'Pierre' for our genius. The student, possibly confused by his professor's using two names for the same individual, but not about the co-referentiality of these names, will find 'Pierre = Saul Kripke' to be a noninformative statement with respect to nonlinguistic facts. A Fregean will have to say the following: 'a = b' is apriori, when 'a' and 'b' have the same sense.⁷

3. Name tokens and their types

Let me discuss why

(1) a = a and

(2) a = b

may both be aposteriori or, in general, can take the same epistemic values. Identity statements combine three different elements. They are composed of the identity sign, flanked by two different name tokens. Let 't₁', ..., 't_n' stand for name tokens. An identity statement has the form

t_x = t_y, with x, y ∈ {1, ..., n} and x ≠ y.

Let 't₁', ..., 't₄' stand for the name tokens in (1) and (2), then we get

(1*) t₁ = t₂,

(2*) t₃ = t₄.

What is the difference between (1) and (2)? One answer would be to say that, in (1), the tokens are, while, in (2), the tokens aren't of the same name type. The truth of this depends of course on one's theory of name types. But let us ignore this question. More generally, we may say that there is a *graphic* type φ, such that both, t₁ and t₂ are of the type φ. t₃ and t₄ are of different graphic types. In short, we may say that 'a = a' is a *homographical* identity, while 'a = b' is a *heterographical* one.

Identities can be classified not only with respect to graphic types. Because of this, I shall make the general distinction between *homotypical* and *heterotypical* identities. (1) is homo- and (2) is heterotypical with respect to sorts of graphic type. With respect to a given sort of type, any identity is either homotypical or heterotypical.⁸ An identity statement may also be homotypical with respect to one sort of type and heterotypical with respect to another. E.g., 'colour = color' is homotypical with respect to phonetic types, but heterotypical with respect to spelling types.

There are many different sorts of type. Besides graphic and phonetic types, there may be speaker-relative types, positional types, etc. These all form different sorts of *non-semantic* type. But there are also sorts of *semantic* type: Semantic types subsume their tokens via their semantic values. We may, e.g., speak of the *reference* types: t_x and t_y are of the same reference type if and only if they refer to the same object. Reference-homotypical identities are true, reference-heterotypical identities are false. For Fregeans, there are also *sense* types. Since, according to Frege, sense determines reference, sense-homotypical identities are always true. Sense-heterotypical identities may be true or false.

Presumably, the following covers Frege's intentions with (F2*):

(F3) There is a sort of non-semantic type τ such that the τ-homotypical identities – if true – are apriori and the τ-heterotypical identities are – if true – aposteriori.

If we were to allow 'τ' to range also over sorts of semantic type, then the corresponding version of (F3) could be true: If τ is the sort of sense type, then the τ-homotypical identities are apriori⁹ and the τ-heterotypical identities are aposteriori. Irrespective of theory, however, allowing sorts of semantic type would make Frege's examples irrelevant as support for (F3). Whether 'Hesperus = Hesperus' is sense-homotypical and 'Hesperus = Phosphorus' is not cannot be determined from the identity statements alone. We must therefore restrict the possible values for τ to sorts of non-semantic type. But, since we may construct Kripke–Kripke examples for all sorts of non-semantic type τ, (F3) is false. τ-homotypical and τ-heterotypical identities can both be equally informative and noninformative. One may therefore introduce Frege's puzzle not only with cases like 'Hesperus = Phosphorus', but also with cases like 'Phosphorus = Phosphorus'.

(F3) is motivated with the common presumption that there are sorts of non-semantic type which are neverthe-

⁵ The case is analogous to the famous Paderewski case of Kripke 1979, pp. 265 f.

⁶ Wettstein 1989 also argues for the informativity of 'a = a'. He claims all identities to be informative in the sense that one need not know that the respective terms are co-referential. This, according to Wettstein, dissolves the Fregean puzzle. I agree with the point that all identities may be informative in the sense discussed by Wettstein, but think, with Salmon 1991, that this notion of informativity is not the one intended in Frege's discussion. Frege's problem arises from the (alleged) fact that there are identities which are nonlinguistically, say astronomically, geographically etc., informative. This type of informativity cannot be accounted for by Wettstein. The question is, of course, whether there is such informativity, a matter not to be decided here.

⁷ A similar point is made already in Kripke 1979, p. 245.

⁸ Ignore the complication introduced by vagueness.

⁹ Actually, this statement would have to be qualified in view of the later discussion.

less semantically authoritative in that types of such sort are able to provide constraints on the semantic values of their tokens. Most philosophers, Fregean and non-Fregean alike, seem to think that there is a sort of non-semantic type such that all tokens of a type of this sort have – in virtue of this very type! – the same semantic value, so that homotypical identities with respect to this sort of type are apriori indeed. But this is mistaken. Different tokens of non-semantic types always might (but need not) have different semantic values. In this way, homographic identities like 'a = a' may be as aposteriori as heterographic identities like 'a = b'

4. Sense and reference determination

Frege's senses have two different functions. They are supposed to account for informativity of (true) identities and to determine the reference of names. It will now be argued that a name's sense does not determine its reference. The sense determines *some* reference. But the sense does not determine the reference of a name token, since it does not determine *itself* to be the sense of this name token. Different tokens of the same non-semantic type may be of a different sense type. Sense-homotypicality does not supervene on non-semantic homotypicality. This alone does not preclude that sense determines reference for a name; different name tokens of a certain nonsemantic type may be of different sense types but still of the same reference type. So, even given different senses, the reference for a name might be determined.¹⁰ But as I have argued, in the same way as all homotypical identities with respect to a sort of non-semantic type may be reference-homotypical, i.e., true, they may be reference-heterotypical, i.e., false. Therefore, although the sense determines *some* reference, the sense does not determine the reference for a given name.¹¹ Different name tokens may have different senses determining different reference objects.

The sense of a name, the mode of presentation, is – according to Frege – an objective thing, denizen of an abstract realm. The semantic gap between a name and its sense surely is as big as that between a name and its reference. The question therefore is: what determines the sense of a name? There are two possibilities for the Fregean: (a) He might claim that the sense of a name is determined by another sense, a sense of second order.¹² But since the second-order sense type of a token will also not supervene on its non-semantic type, this idea, repeated for the determination of sense of whatever level, will lead to an infinite, vicious regress of senses. (b) He might refer to Millianism for the determination of sense, the theory of an unmediated link between a name and its semantic value. In neither case, appeal to a second semantic dimension, that of sense, is sufficient to explain the reference determination for names. Hence senses cannot fulfil one crucial task assigned to them by Frege: The theory of sense is insufficient to explain the relation between a name and its reference. The *semantic* motivation for introducing senses is misguided; at some point, even the Fregean needs to take a Millian step.¹³

Conclusion

In the discussion of Frege's puzzle it is often assumed that different tokens of a non-semantic type are of the same semantic type, have the same reference and/or the same sense. But neither is necessarily true. This leads directly to the demonstration that senses cannot fulfil one important function that Frege assigns to them, namely that of determining the reference of a name. What are the consequences for the problem of informative identity? To deny (F-2) is not to deny (F-1). So my claim does not count against the possibility of informative identity statements and hence not against Frege's puzzle. Even worse, my considerations may be seen as a first step to generalise the problem; perhaps *all* identities, homotypical and heterotypical (with respect to some sort of non-semantic type), are informative alike. This would make a solution to Frege's puzzle an even more urgent matter. A Fregean might claim that *each* name token has a sense of its own. But we might also develop a different perspective on the puzzle. If informativity is not linked to *heterotypical* identities, specifically, one might consider the problem of informativity as unrelated to 'ways of referring' and, therefore, as independent of 'modes of presentation'; it might be seen as bound up with reference *simpliciter*. The Fregean claim that the informativity contained in identities is non-linguistic – something that I have granted here for the sake of my limited purposes – may exert less power over our minds, so that the various non-Fregean accounts of names may appear more plausible.

Literature

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¹⁰ I owe this point to Alexandra Zinke.

¹¹ Note that even if the non-semantic type were to determine that all its tokens are of the *same* sense type, it would not determine of *which* sense type they are.

¹² Let me remark that the introduction of senses of second order would allow for the possibility of nonlinguistic informativity of identities which are homotypical with respect to first-order sense types.

¹³ Analogous considerations apply to the description theory of reference.