Tractatus 6.3751

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In the *Tractatus*, ascriptions of colour were *prima facie* threats to the independence of elementary propositions. If it is logically impossible for two colours to be at the same place in the visual field (6.3751), then the ascription of different colours to the same place must take the form of a contradiction. But if "*a* is red" and "*a* is green" were both elementary, they could not contradict each other due to the independence thesis. There are at least three possible escapes from this predicament: to deny that there is a "logical" (and not for instance a "merely psychological") exclusion involved here; to abandon the independence thesis; or to deny that colour ascriptions are elementary. Wittgenstein clearly chose this last option, but left no more than a hint about the line of analysis to be pursued in this case.

The hint is given in the second paragraph of 6.3751. In physics, he says, a contradiction like "*a* is both red and green" presents itself as the impossibility for a particle of having two different velocities. Taken in physical (and not phenomenal) terms, the statement "*a* is both red and green" would appear as an everyday-language translation of a statement like "the velocity of *p* is both *m* and *n*", where *p* is a particle, and both *m* and *n* are numbers. So the logical exclusion of colours appears in physics as a particular case of the logical exclusion of diverging numerical ascriptions. And this is plainly reasonable, since the mutual exclusions of numbers and colours display exactly the same logical structure. In Aristotelian terms, we could say that we are dealing with cases of contrariety, not of contradiction. Diverging ascriptions of number — like incompatible ascriptions of colour — can always be both false, but never simultaneously true. It is possible that the number of men in this room is neither 5 nor 7, as it is possible that the colour of a shirt is neither red nor green. But it can't be both. Conjunction should necessarily give rise to logical contradiction.

Something like this should be true of chromatic phenomena. Phenomenal ascriptions of colour certainly exclude each other, says Wittgenstein, and exclude logically: "the assertion that a point in the visual field [*ein Punkt des Gesichtsfelds*] has two different colours at the same time is a contradiction [*ist eine Kontradiktion*]" (6.3751 again). The statement is inequivocal enough, and the suggestion is clear. As physical colour appears as a special case of number ascription (regarding velocity of particles), phenomenal colour must also involve numerical ascriptions somehow. Statements describing perceptions, like "*a* is red", are not elementary, but highly complex — at least as complex as a statement like "there are three circles in my visual field". They involve nested quantifiers, and the logical behaviour of these quantifiers should explain the logical relations between ascriptions of colour.

I agree that this is more a horizon than a trail, but the fact is that we have to go along with it. It would be useless to look for something more specific in the book. So let us explore this horizon a little bit more remembering how Wittgenstein imagined he could deal with numbers when he wrote the *Tractatus*. Number is defined as the "exponent of an operation" (6.021). The import of this definition is very simple. It implies that any ascription of number should be seen as a member of a formal series of propositions — a series generated by a formal procedure of producing a new proposition out of a given one. This kind of procedure is what Wittgenstein calls an "operation". As a matter of fact, there are operations applicable to more than one proposition — disjunction, for instance. But only operations applicable to just one proposition can generate what Wittgenstein calls "formal series", since the basic requirement of a formal series is uniform progression from term to term — from one proposition to its "successor" in the series.

It would be a gross mistake to imagine that every operation in the *Tractatus* must be a *truth*-operation. Simultaneous negation *is* a truth-operation in the sense that the truth-value of the proposition obtained as a result is *completely* determined by the truth-value of the propositions we began with. But consider this formal series of propositions:

There is no dog in this room.

There is just one dog in this room.

There are exactly two dogs in this room.

and so on

Clearly there is a formal process of transformation involved in the construction of the series. We know how to add any member to the series after the last given one. We just use the next member of the series of natural numbers. But this is so to speak the "macroscopic" aspect of the succession law. The use of logical lens would reveal a much more complicated process, involving the use of nested quantifiers, each one of which could by its turn be reduced to applications of simultaneous denial to formally selected groups of propositions. But no "microscopic" detail would deprive the process of its formal deteminateness. Quite the opposite. The logical microscope of analysis would simply give us more evidence of the completely formal nature of the whole process.

Let us insist on a fundamental point. That series is formal because it has a "basis" (i.e., a proposition we "begin with"), and it is generated by a constant formal procedure of obtaining a new proposition out of a given one. The *same* transformation (in terms of nested quantifiers) that makes me advance from the first proposition to the second one will make me advance from the 57th to the 58th. It is always a question of introducing new existential quantifiers at the same places. Let us call the first proposition "*p*", and let us make a capital "O" indicate the logical operation involved in this case. Now the whole series could be represented this way:

p, O'p, O'O'p, and so on.

Or, using the tractarian notation,

O^{0}'p, O^{1}'p, O^{2}'p, and so on.

That is the way Wittgenstein explain the role that the word "two" plays in a sentence like "there are exactly two people in this room". It marks the place of this proposition within a formal series of propositions. Within each proposition of the series we do not find numbers, but only nested quantifiers. Numbers are not part of the basic tools of language. After analysis, they simply disappear, leaving no traces behind.

Every context involving numbers should be analysed along similar lines. Measuring contexts should be no exception. "This table is 3 meters long" should be a proposition of the form $O^{3}{}'p$ for some operation O and some proposition *p*. The same could be said of a proposition ascribing a certain velocity to a particle, or a certain colour to a place in my visual field. If our analysis is right, a proposition such as "This is red" should be seen as the n^{th} member of a formal series whose first member is a certain proposition *p*. Now two related questions naturally arise: (i) Which kind of proposition could play the role of a "basis" in order to generate the whole set of chromatic ascriptions arranged in a formal series of propositions? (ii) How to build this formal series without using anything but logical tools (like nested quantifiers)? What would these quantifiers range over? The *Tractatus* is absolutely silent about these questins. As interpreters, we are condemned to overinterpret the text, trying to imagine different kinds of solutions that would be compatible with the tractarian point of view.

If we examine the texts he wrote in the early 30's, we come up with an interesting suggestion. Systems of representation of the so-called "space of colors" are presented and evaluated as to their ability to depict the logical relations governing that space. Colours may be displayed for instance in a circle, and also in an octahedrom. Wittgenstein says that the octahedric representation is "more perspicuous" for it is capable to depict directly, in a purely geometrical way, certain fine grammatical distinctions that the circular representation does not grasp. For instance, the unmixed character of the phenomanally pure colors — green, red, yellow, blue, white and black. In a circle, they are on the same level as any other, while in the octahedron they occupy a distinguished position on the six vertices of the solid. There is a clear suggestion that we could associate a system of coordinates to the octahedron in order to determine each color by means of three numbers. One of these coordinates would range from apex to the bottom vertex of the octahedron, having white and black at the extremities, and a neutral grey right in the middle of the whole solid. The second coordinate would go from the blue vertex to the yellow one. Accompaning this coordinate, we would see the pure blue progressively loosing its hue, turning into gray, and then progressively acquiring a more and more yellow tone. The third coordinate would make a similar trajetory from red to green. If we associate numbers to these coordinates, any color of the visual spectrum can be associated to a triple of numbers ranging, let us say, from -1 to 1.

Using this system of representation amounts to analysing any ascription of color as the conjunction of three different statements, each one expressing a different kind of chromatic property. The first property is expressed in English by means of two different the verbs: to darken and to whiten. In the octahedric representation, we would say that something is "darkenning" by means of a number ascription closer and closer to -1; and we would say that it is "whitenning" by means of ascriptions approaching 1. Similarly, we would have numerical expressions corresponding to expressions like "to become redder", "more yellow", "bluer" and "greener". Saying that something is grey would amount to say that it does not have any degree of white-black, nor any degree of red-green, nor any degree of yoellow-blue. And now we are as close as possible to a metric of colors — a system of representation in which ascriptions of colors could be analysed as ascriptions of numbers, ascriptions of numbers could be analysed as quantified propositions, and quantified propositions could be analysed as truth-functions of elementary ones.

I won't push the analysis further, since we are stepping in a purely hipothetical territory. There are many possible ways of representing the space of colors by means of geometrical figures, and it would be possible to associate a coordinate system to each one of these figures. I just want to stress that Wittgenstein had good reasons to believe that it was perfectly feasible to give numerical expression to the logical multiplicity we find in our visual space as far as colors are concerned.