A Digitalized Tractarian World

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1.

There is a widely accepted opinion that Wittgenstein abandoned the logical atomism in TLP because of the color exclusion problem. But some interpreters insist that such kind of judgment isn't enough to evaluate his whole program of the Tractarian world. The real reason, why they couldn't accept his abandonment, lies in the interest of the philosophy of information which is recently developed. In anyway it isn't fully appreciated that Wittgensteins's Tractarian world can be interpreted on behalf of the philosophy of information, even the basic concept of the digitalized world is given by him in TLP.

My aim is to give some constructive suggestions about the digital conception of a Tractarian world through the analysis of the logical space in which the logical scheme of truth functional propositions can be digital calculated, when the truth values F and T are to replace as 0 and 1. In that context it is necessary to explicate the binary arithmetiziation of 16 logical connectives in TLP. Further more I will try to show that a syntactical sameness exists between a digitalized Tractarian world and the digitalized conception of I Ching's symbolic system. The proof can be arrived if the same numerical symbol of yin and yang in I Ching are interpreted as 0 and 1, and the digitalized proposition system can be transformed into the graphic figures, because the whole system of I Ching consists of 64 graphic figures. The Tractarian world and I Ching's system base on the binary arithmetic, as Leibniz already had grounded its arithmetical sameness before 300 years ago.

2.

Recently regarding to the development of philosophy of information rises an interesting question in the milieu of studies on Wittgenstein's Tractatus world, whether the world of the logical atomism could be digitalized. Logic gates, for example, OR gate, AND gate, etc. are the logical construction of the Karnaugh map in the electronic computer technology. The model of the Karnaugh map in which 16 electronic circuits are presented is borrowed from 16 logical connectives of the propositional logic. The most interpreters on the Karnaugh map believe that Boole has invented 16 connectives and are unnoticing the fact that Wittgenstein also has introduced it in TLP.

However, the misleading idea that Wittgenstein might have abandoned the project of the logical atomism goes back to the logical impossibility which only depends on the relationship between tautology and contradiction. Namely, the famous colure exclusion problem formulates that nothing could be appeared as two colures in a same place and time. No one can say, something is red and green at the same time in a given place. But properly to say, the colure exclusion problem doesn't belong to the logical problem in its own way. The logical truth doesn't have to do with the problem of time. It also isn't related to the matter of an un-expressible world, as Wittgenstein says in the last sentence in TLP.

In different times you can say, it is red in t1, and it is green in t2. The real issue of the colure exclusion problem in TLP goes back to the strong beliefs on the world of the logical atomism in terms of an atomist like Boltzmann. But that kind of atomistic world doesn't fit for the logical program in TLP which expresses the totality of the existing atomic facts which turns out to be all that is the case. But, as it is well known, the problem of the totality of facts is the reduction which demands two responses about the state of affaires, logically either yes or no, and ontologically either being or not being. The logical problem is connected in an ontological label with everything and nothing. In case of Leibniz two principles, the law of identity and the law of contradiction are each other complementary for the totality of monadic world. For Wittgenstein the principle of contradiction and the principle of tautology are reduced in a full sense to the logical space for the atomistic world. The contradiction encompasses the whole of logical space, where the tautology encompasses none of logical space. For Boole's algebra 0 means Nothing and 1 means Universe, where the language of yin and yang in I Ching's system also can be interpreted as Boolean algebra 0 and 1. If o and 1 are represented as ● and ○, and ● and ○ are complementary, we can visualize it as follows. When ● enters into ○, then it will be changed into ◐, ◑ and ○. It will get on the same result, when ○ enters into ●, as what Wittgenstein means about the encompassment of the logical space through contradiction and tautology.

Let's say, that the truth possibility of p is presented by two labels, for example tautology or contradiction in a logical space (4.31). But, p and ~p don't have the same truth value in terms of the logical space. That is the logical impossibility on facts of p and ~p. However we know that p can be assigned to the truth values T or F. That is, they only have opposite senses, as Frege emphasizes that meaning of the object is same, but senses are different. We only have the True or the False in reality. I think Frege's project of an ideal language bases on the ontological states which is explained only by the notion of the logical impossibility, where we need to know the truth condition of propositions on affaires of states. So J. L. Zalabardo proposes a proposition p is a logical consequence of a set of propositions Γ which appeals to the logical impossibility that a set of truth combination for the elements of Γ is true and p is false.

Wittgenstein also agrees that all the truth grounds are truth grounds of a certain proposition, and the truth of that proposition follows from the truth of the others. (5.11). The basic idea of the logical atomism is that all propositions can be analyzed as truth functions of elementary propositions, when all elementary propositions are logically independent from one another and their components refer to simple subjects. The logical impossibility of p and ~p doesn't have the same truth value, but there is the same reality in the logical space, even no one can know what the sign p means. We can only suggest that p is only a nominal giving name which can not be defined. Sheffer is already aware of that kind of problem. His idea is that all the quantifier free formulas of sentential calculus in *Principia* can be expressed per one logical connective which is calked the Sheffer Stroke "|".

However, that we have one proposition of p means we only have two possibilities to express the truth function about p.

P	P
T	1
F	0

Fig. 1

If we read the letters from the bottom to the top, we get the number 0 and 1 at the first label. For two propositions of p and q, we only have 4 combinatory possibilities T and F. In truth table it can be diagramed as follows.

P	Q	P	Q
T	T	1	1
T	F	1	0
F	T	0	1
F	F	0	0

Fig. 2

Here we get the order of the binary numbers (0, 0), (0, 1), (1, 0), (1, 1). In 5.11 the account of the logical consequence is clearly expressed that the truth possibility of p and q can be presented by (T, T), (T, F), (F, T) and (F, F) at the second label. And the truth possibility of p, q and r can be presented by (T, T, T), (F, T, T), (T, F, T), (T, T, F), (F, F, T), (F, T, F), (T, F, F) and (F, F, F) at the third label.

P	Q	R	P	Q	R
T	T	T	1	1	1
T	T	F	1	1	0
T	F	T	1	0	1
T	F	F	1	0	0
F	T	T	0	1	1
F	T	F	0	1	0
F	F	T	0	0	1
F	F	F	0	0	0

Fig. 3

Here we get the binary order as follows (0, 0, 0), (0, 0, 1), (0, 1, 0), (0, 1, 1), (1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1).

Now we are arrived on the progression of binary number from the one-place at first label to the three places at the third label. The first label is 1 x 2 = 2, the second label 2 x 2 = 4, the third label 2 x 2 x 2 = 8.

Wittgenstein says in 5.1 that every number of elementary propositions can be listed by the 16 truth functional connectives which are called *syncategoremata* in the mediaeval age. 16 truth functional connectives come from the operation of 4 basic elements which are operated by 0 and 1. The order of T F letters goes as follows. (TTTT), (FTTT), (TFTT), (TTFT), (TTTF), (FFTT), (FTFT), (FTTF), (TFFT), (TFTF), (TTFF), (FFFT), (FFTF), (FTFF), (TFFF), (FFFF). The order of the binary number of 16 truth functional connectives goes on as follows.

[See Figure 4 at the end of the paper]

If the world consists of each other independent atomic facts and could be conceived of as digitalized bits in logical space, that kind of the logical connective is already involved in Leibniz's understanding of the monadic world in terms of the law of identity and the law of contradiction, where Wittgenstein uses tautology and contradiction in the logical space for the same context.

Against R. A. Young who interprets that the atomic facts would be bits, I propose that these atomic facts can be operated by bits which are deposited as the truth values 0 and 1 in the logical space. Wittgenstein's proposal that a-pole depicts truth value and b-pole depicts false value in a b notation goes to the same direction for the operation of the truth values in logical space. In the logical space logical truths are each other dependent. No one proposition could be defined without knowing the truth ground of another proposition, because all propositions are each other dependent.

Wittgenstein says in 5.123, if a god creates a world in which certain propositions are true, then by that very act he also creates a world in which all the propositions that follow from them come true. And similarly he didn't create a world in which the proposition p was true without creating all its objects.

3.

D. Miller, D. E. Knuth and others show that the set of 16 connectives of two variables Boolean algebra and the set of 16 subsets of a four element set (a 4-set) have the same graphic structure, i.e. a Boolean lattice, without acknowledging Wittgenstein's achievement in TLP. So it is interesting to prove the structural sameness between binary expression of 16 connectives or Boolean algebra and Wittgenstein's 16 truth functional connectives through graphic diagrams. One important point for the visualization of graphic diagrams is the contrast between dark and bright, as we often observe it in the constellation of earth, moon and sun. We only introduce here two symbols ● ○ as graphic diagram. At first the series of 16 truth functional connectives come from 4 combinatory possibilities of p and q and at second 4 elements come from bi polarity of ○ ●. Fig. 2 is as follows.

	●	○
●	●●	●○
○	○●	○○

Fig. 5

These symbols ○ ● are independent, but complementary. They encompass each other in the logical space. The combinatory order of 4 elements ●●, ●○, ○●, ○○ are corresponding to the binary numerical order 00, 01, 10, 11. If we have two propositions p and q, we can get totally 16 combinatory possibilities of truth values on p and q. The binary order starts from ●●●● to ○○○○. It means they start from 0000 to 1111.

1	2	3	4	5	6	7	8
0000	0001	0010	0011	0100	0101	0110	0111
9	10	11	12	13	14	15	16
1000	1001	1010	1011	1100	1101	1110	1111

Fig. 6

[See Figure 7 at the end of the paper]

16 truth functional connectives are only results of 4 elements, i.e. the combination of truth values in p and q.

4.

I Ching which consists of symbolic order of yin and yang (陰陽) is the oldest book in ancient China. Leibniz had invented as first European its mathematical meaning of 64 codes with the Chinese missionary J. Bouvet. The language of I Ching only is yin and yang. Two principles (duro rerum principium) of I Ching are yin symbol - - and yang symbol —. This order has one place in which the binary numbers 0 and 1 are corresponding to - - and —. From the two principles come 4 elements which are produced by truth variables of p and q. The symbolic order of 4 elements is ●, ◐, ◑, ○ (四象). We read from the bottom to the top, the binary number of these symbols are (0, 0), (0, 1), (1, 0), (1, 1) which are called ‘Four great images’ or ‘quatuor imagines’ and mean 1, 2, 3, 4 according the order of natural number. This order has 2 places. For example OR gate can be presented in truth table as follows.

	OR
○	—
◑	—
◐	—
●	- -

Fig. 8

[See Figure 9 at the end of the paper]

[See Figure 10 at the end of the paper.]

The truth functional propositions with 3 places go to the following order, ☷, ☳, ☵, ☱, ☶, ☲, ☴, ☰. This is hexameter of 8 Gue (八卦). The natural order of the symbolism is corresponding to 0, 1, 2, 3, 4, 5, 6, 7. The whole construction of I Ching system consists of double order of 8 Gue. So 8 x 8 = 64.

[See Figure 11 at the end of the paper]

What Wittgenstein operates with “T” and “F” for the truth table of elementary propositions, is that all possible truth grounds of elementary propositions fully can be transformed through the binary arithmetization of functional connectives. When the truth functional connectives purely depends on a digitalization of the Propositional logic, the logical space of possible atomic facts also can be digitalized, even possible atomic facts need not exist. The main idea of the digital culture through the modern computer can be successful achieved in TLP, where Wittgenstein needs the logical space of possible atomic facts. I think the last sentence of TLP also doesn’t relate to the meaning of un-expressible world, because a logical space includes an element of the space of possible affaires. (3.4-3.42). That kind of idea can be easily linked to graphic diagram i. e. Venn diagram and visualized.

5. Conclusion

Wittgenstein’ program of the logical atomism isn’t ended. A new beginning of his project can be grounded on the philosophy of information which operates only binary number 0 and 1, where Wittgenstein uses its truth variable as T and F. The binary progression of 0 and 1 opens new horizon that all kind of truth functional connectives can be digitalized. In fact Wittgenstein has developed 16 truth functional connectives for the construction of the world of the logical atomism. In I Ching it is explained that 0 and 1 produce 4 elements (0, 0), (0, 1), (1, 0), and (1, 1), and 4 elements produce 8 hexagrams (0, 0, 0), (0, 0, 1), (0, 1, 0), (0, 1, 1), (1, 0, 0), (1, 0, 1), (1, 1, 0) and (1, 1, 1). The whole 64 hexagrams consist of the double 8 trigrams which begin from (0, 0, 0, 0, 0, 0) and ends (1, 1, 1, 1, 1, 1).

Regarding to these facts about the parallel structure can be drawn that he basic structure of the modern digital is programmed through the binary numbers 0 and 1, and dyadic values yin(● or - -) and yang (○ or —) in Wittgenstein’s Tractarian world and the I Ching’s symbolic world.

	Contradiction															Tautology
	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

Fig. 4

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
●	●	●	●	●	●	●	●	○	○	○	○	○	○	○	○
●	●	●	●	○	○	○	○	●	●	●	●	○	○	○	○
●	●	○	○	●	●	○	○	●	●	○	○	●	●	○	○
●	○	●	○	●	○	●	○	●	○	●	○	●	○	●	○

Fig. 7

	Tautology If p then p, and if q then q.	Not both p and q.	If q then p.	If p then q.	p or q.	Not q.	Not p.	P or q, but not both.
○	- -	- -	- -	- -	- -	- -	- -	- -
◑	- -	- -	- -	- -	—	—	—	—
◐	- -	- -	—	—	- -	- -	—	—
●	- -	—	- -	—	- -	—	- -	—

Fig. 9

If p then p, and if q then p.	p.	q.	Neither p nor q.	p and not q.	q and not p.	q and p.	Contradiction p and not p, and q and not q.
—	—	—	—	—	—	—	—
- -	- -	- -	- -	—	—	—	—
- -	- -	—	—	- -	- -	—	—
- -	—	- -	—	- -	—	- -	—

Fig. 10

	☷	☳	☵	☱	☶	☲	☴	☰
☷	☷ ☷	☳ ☷	☵ ☷	☱ ☷	☶ ☷	☲ ☷	☴ ☷	☰ ☷
☳	☷ ☳	☳ ☳	☵ ☳	☱ ☳	☶ ☳	☲ ☳	☴ ☳	☰ ☳
☵	☷ ☵	☳ ☵	☵ ☵	☱ ☵	☶ ☵	☲ ☵	☴ ☵	☰ ☵
☱	☷ ☱	☳ ☱	☵ ☱	☱ ☱	☶ ☱	☲ ☱	☴ ☱	☰ ☱
☶	☷ ☶	☳ ☶	☵ ☶	☱ ☶	☶ ☶	☲ ☶	☴ ☶	☰ ☶
☲	☷ ☲	☳ ☲	☵ ☲	☱ ☲	☶ ☲	☲ ☲	☴ ☲	☰ ☲
☴	☷ ☴	☳ ☴	☵ ☴	☱ ☴	☶ ☴	☲ ☴	☴ ☴	☰ ☴
☰	☷ ☰	☳ ☰	☵ ☰	☱ ☰	☶ ☰	☲ ☰	☴ ☰	☰ ☰

Fig. 11