Model-Theoretic Languages as Formal Ontologies

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The success of ontological engineering using logical methods in the construction of pragmatically oriented domain ontologies revived interest to the old problem of the relations of logic and ontology. On the one hand, ontology extends its scopes and takes back its intellectual respectability. On the other hand, logical pluralism makes logic to take care of its own basis and bounds. One of the attempts to give an exact definition of the concept of logic is a definition of abstract logic in generalized (abstract) model theory. The concept of abstract logic is a generalization of the concept of truth as relation between structures and sentences. An abstract logic consists of (1) a collection of structures closed under isomorphism, (2) a collection of formal expressions, and (3) a relation of satisfaction between the two. This definition does not include any conditions concerning rules of inference. Hence in seems more appropriate to use the term 'model-theoretic *language*' instead of the term 'abstract *logic*'. Even though the generalized model theorists use the term 'abstract logic' they do it frequently only by pragmatic reasons of simplicity and brevity. My purpose is to interpret abstract logics as formal ontologies, i.e. as genuine logics at least in phenomenological sense.

The interpretation of logic as formal ontology, an a priori science of objects in general, goes back to Edmund Husserl. Although the truths of logic apply to all regions of reality, Husserl believed it to be possible to give its transcendental justification only if we postulate a special region of abstract categorical objects. If we want to save logic from the specific relativism of Kant's interpretation of logical structures in terms of universal human abilities, we should, Husserl believed, consider them as structures of some objective area of abstract higher-level objects. What is the nature of these objects? The answer to this question is crucial for all the phenomenological project of justification of logic.

In my view, the model-theoretic analogues of categorical objects of Husserl's formal region are classes (types) of isomorphism considered as abstract individuals of higher order. Any two isomorphic structures represent the same abstract system. A system is considered to be abstract, if we do not know anything of its objects except the relations existing between them in the system. Formal ontologies viewed as abstract logics are formal theories of relations quite similar to indivisible species *(automon eide)* of Aristotle's ontology. They do not distinguish between specific individuals in the domain, but are not 'empty' in Kant's sense, since they deal with individuals of higher order, i.e. classes of isomorphic structures.

At the same time classes of structures closed under isomorphism may be viewed as generalized quantifiers. Generalized quantifiers express Husserl's mental properties and relations which, unlike physical, do not influence on other properties and relations, but exist by virtue of other properties and relations. For example, Mostowski's generalized quantifiers interpreted by classes of subsets of the universe attribute cardinality properties to the extensions of first-level unary predicates. More precisely, a Mostowski's generalized quantifier is a function Q associating with every structure A a family Q (A) of subsets of the universe of A closed under permutations of the universe of A. Thus Mostowski's quantifiers perfectly satisfy the permutation invariance criterion by Alfred Tarski.

In his famous lecture "What are Logical Notions?" delivered in London in 1966 and published posthumously in 1986 Tarski proposed to call a notion logical if and only if "it is invariant under all possible one-one transformations of the world onto itself" (Tarski 1986, 149). Tarski's informal definition of logical notions was an extension to the domain of logic of Klein's Erlanger Program for the classification of various geometries according to invariants under suitable groups of transformations. Tarski characterized logic as a science of all notions invariant under one-one transformations (permutations) of the universe. He gave several examples of logical notions. Among individuals there are no such examples, among classes the logical notions are the universal class and the empty class. It is remarkable that the only properties of classes of individuals which we can call 'logical' are "properties concerning the number of elements in these classes" (ibid, 151). What does cardinality have to do with logicality? Tarski proposed the following general philosophical interpretation of his invariance criterion, "our logic is logic of cardinality" (ibid.). In fact Mostowski's quantifiers nicely satisfy this criterion. But Mostowski's definition is not sufficiently general even to cover Aristotle's quantifiers. There is, however, no conceptual necessity to consider quantifiers as second-order *properties*. The obvious challenge here is to generalize this understanding on second-order *relations*. This generalization of quantifiers was proposed by Per Lindström (1966). His quantifiers interpreted as second-order relations between first-order relations on the universe are polyadic. Binary examples of Lindström's quantifiers are syllogistics quantifiers, e.g. «all … are…» = $\{\langle X,Y\rangle: X,Y\subseteq U \text{ and } X\subseteq Y\}$, Resher's quantifiers $Q^R = \{\langle X, Y\rangle: X, Y\subseteq U \text{ and } \mathrm{card}(X) < \mathrm{card}(Y)\}$, Hartig's quantifiers $Q^H = \{\langle X,Y\rangle: X,Y\subseteq U \text{ and } \mathrm{card}(X)= \mathrm{card}(Y)\}$.

Polyadic quantifiers go back to scholastic 'multiple quantifiers'. However in standard logical notation they are not to be regarded as having an independent value, but interpreted as iterated unary quantifiers. On the other hand, any iterated quantifier prefix may be viewed as a polyadic quantifier. Polyadic interpretation is especially important in the case of heterogeneous quantifier prefixes. The point is that heterogeneous quantifier prefixes, expressing properties of classes of pairs of individuals, i.e. binary relations, distinguish equicardinal relations. Let us consider a simple model with the universe U= {a,b,c} (Микеладзе 1979, 296). Let set two binary relations on U, F1= {(a,a), (a,b), (a,c)} and F2= {(a,a), (b,b), (c,c)}. These relations have an identical number of elements. However $\exists x\forall y F_1(x,y)$ is not equivalent to $\exists x\forall y F_2(x,y)$, and $\forall x\exists y F_1(x,y)$ is not equivalent to $\forall x\exists y F_2(x,y)$. In other words, binary quantifiers $\exists x\forall y$ and $\forall x\exists y$ distinguish equicardinal relations F1 and F2. Thus Tarski's thesis of 'our logic' as 'logic of cardinality' may be fair for the theory of monadic quantification (logic of properties of classes of individuals), but not for the theory of binary quantification (logic of properties of classes of pairs of individuals). Polyadic quantifiers take into account not only cardinalities, but more refined formal features of the universe. Not only cardinalities, but also patterns of ordering of the universe have to be taken into account by logical conceptualization. Logic with polyadic quantifiers is not ontology of cardinality but formal ontology of structures, types of ordering of the universe.

In general, the permutation invariance criterion assimilates logic to set theory. It is not unexpected in the context of the model-theoretical reconstruction of Husserl's idea of formal ontology as "an a priori discipline that investigates all truths belonging to the essence of objectivity in general in formal universality" (Husserl 2008, 54). Husserl emphasizes the 'inseparable unity' of logic and mathematics. "People are", in his view, "in the habit (a habit thousands of years old) of keeping the two bodies of knowledge in drawers far apart from one another. For thousands of years, mathematics has been considered a unique, special science, self-contained and independent like natural science and psychology, but logic, on the other hand, an art of thinking related to all special sciences in equal measure, or even as a science of forms of thinking not related any differently to mathematics than to other special sciences and not having any more to do with it than they" (ibid). Thus the unity of logic and mathematics had not been realized because of a normative interpretation of logic as a technical adjunct of psychology and metaphysics. For Husserl, pure logic as *Mathesis Universalis* embraces logic and mathematics: "the whole of pure logic is to be understood as a formal ontology. The lowest level, apophantic logic, investigates what can be stated in possible form a priori on the first level about objects in general. The higher ontologies are concerned with purely formally determined higher-level object formations like set, cardinal number, quantity, ordinal number, ordered magnitude, etc." (ibid, 76). On the other hand, according to Tarski's definition, as Gila Sher remarks, "any mathematical property can be seen as logical when construed as higher-order. Thus, as a science of individuals, mathematics is different from logic, but as a science of higher-order structures, mathematics is logic" (Sher 1991, 63). As it was shown by Vann McGee an operation is logical according to Tarski's permutation invariance criterion if and only if it is definable in the infinitary language $L_{\infty, \infty}$ (McGee 1996). $L_{\infty, \infty}$ is the language which allows conjunctions and disjunctions of any cardinality together with universal and existential quantification over sequences of variables of any cardinality.

This assimilation of logic to mathematics contradicts W. V. O. Quine's thesis of ontological neutrality of logic. For Quine, logic cannot assume any special entities as existing ones. Thus if logic is supposed to be independent of ontology, not only set theory but also second-order logic as "set theory in sheep's clothing" go beyond the bounds of logic. In my view, the reason of this collision of two classical tests for logicality is the possibility of various interpretations of the formality of logic. Logic distinguishes formal, metaphysically unchanging features of reality. But what does it mean precisely? If we interpret formality of a theory as its invariance under permutations of the universe it means that the theory does not distinguish individual objects and characterizes only those properties of model which do not depend on its nonstructural transformations. This formality of a theory does not imply its ontological neutrality. Expressive power of a formal (in the permutation invariance sense) logic may be sufficient for the distinction of abstract mathematical objects.

Thus metaphysical considerations become a factor in choosing logical framework for formalizing theories. It seems worth trying to examine how more sophisticated models of reality can affect the choice of logical constants. For example, the permutation invariance criterion may be viewed as "only one extreme in a spectrum of invariance, involving various kinds of automorphisms on the individual domain" (van Benthem 1989, 320). The invariance criterion generalized this way is wide enough to include logics of abstract objects, for example, 'logic of colour'. As Ludwig Wittgenstein assumes in his Tractatus, "the simultaneous presence of two colours at the same place in the visual field is impossible, in fact logically impossible, since it is ruled out by the logical structure of colour" (Wittgenstein 2004, 6.3751). For Wittgenstein, as Jaakko Hintikka pointed out, "the conceptual incompatibility of color terms can be turned into a logical truth simply by conceptualizing the concept of color as a function mapping points in a visual space into color space" (Hintikka 2009, 52). Thus "nonlogical analytical truths sometimes turn out to be logical ones when their structure is analyzed properly" (ibid.). If we accept as a test for logicality the invariance not only for isomorphism but also for automorphism, namely, for all permutations of individuals which respect an additional structure of chromaticity, 'logic of colour' becomes possible. In the context of Klein's Erlanger Program this logic may be considered as a member of a family of logics which are in their turn 'geometries' whose notions are invariant for permutations respecting some additional structures. Thus abstract logics become logics of abstract objects quite similar to domain ontologies of ontological engineering.

However these liberal principles of the demarcation of the bounds of logic may seem too exotic. But as John Barwise remarks, the ideology of abstract logics does not contradict even the person-in-the-street notion of logic. "On the common sense view on logic", he believes, "all the concepts we use to cope with and organize our world have their own logic" (Barwise 1985, 4). The principles of the demarcation of the bounds of logic may have proof-theoretical or model-theoretical character. The first approach is the traditional one that characterizes logic as a theory of valid inferences. The second is the one that understands logic as a theory of specific classes of structures. Abstract logics or logics with generalized quantifiers assume liberalization of metalogical requirements to logical systems and lead to their interpretation as formal ontologies, i.e. theories of formal structures of the universe. Even though the abstract logics are model-theoretic *languages* they belong to the tradition of *Mathesis Universalis* presupposing the understanding of logic as *calculus ratiocinator*, but not as *lingua characteristica*.[*]

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