Logic and its Application in Ludwig Wittgenstein's Early Philosophy

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1. Logical notation and natural language

Logical necessity (*logische Notwendigkeit*) is a distinguished notion in Wittgenstein's early works. One can even say: Wittgenstein's entire early philosophy is the theory of logical necessity.

This point of view is consistent with the idealistic (or non-realistic) interpretation of *Tractatus*. This interpretation allows to state, as Jerzy Perzanowski writes, that Wittgenstein's thesis about logic and language determinate Wittgenstein's thesis about the world and reality (Perzanowski 1984, p. 224).

The starting point of Wittgenstein's early philosophy is the assumption that logical necessity is the basis of any representation of reality (McGuinness 2002, pp. 85-86). The main aim of Wittgenstein's investigations is to reveal this basis, which is concealed beneath the surface of any natural language.

Therefore, logical analysis must ignore the contingent shapes of natural language and express the logical necessity in the proper *logical notation* (*Begriffsschrift, Zeichensprache*) (TLP 3.325), which reveals universal *logical syntax* (*logische Syntax*) (TLP 3.325, 3.33, 3.334, 3.344, 6.124). Finally, one can read in *Tractatus* that *All philosophy is 'critique of language'* (TLP 4.0031).

It is very important to grasp that Wittgenstein's investigations on logical necessity apply not only to the artificial logical notation but, as Max Black remarks, to any possible representation of reality – any natural language (Black 1964, p. 24).

Wittgenstein is not interested in perfect language. Elizabeth Anscombe points out that this very important trait of Wittgensteins's early philosophy was missed in Bertrand Russell's *Introduction* (Anscombe 1959, p. 91). Wittgenstein writes: *all the propositions of our everyday language, just as they stand, are in perfect logical order* (TLP 5.5563).

According to this interpretation, logical notation is, as Peter M.S. Hacker writes, not a perfect language (Hacker 1979, p. 231) – it is only a tool of logical analysis. In natural language the necessary basis cannot be shown directly, but in logical notation everything that is logically necessary can be expressed *on one occasion* (TLP 5.47).

One should emphasizes that Wittgenstein's early investigations concern the necessary essence of symbolism – not the contingent way of material form of the symbol, i.e. the *sign* (*Zeichen*) (TLP 3.11, 3.32, 3.321, 3.325, 3.326). Symbol is the sign used *in its projective relation to the world* (TLP 3.12). However, the sign in itself is not a possible representation of a part of reality (Glock 1996, pp. 315-316).

The essence of symbolism is expressed only by logic and *it is not arbitrary* (TLP 3.342, 6.124). The distinction between arbitrary (accidental) and not arbitrary (essential) features of symbolism is crucial to properly understand Wittgenstein's philosophy of logic (Black 1964, p. 150). One can say that every symbolism or arbitrary notation can be analyzed with non-arbitrary notation, i.e. logical notation.

To sum up this section, the multiplicity of natural languages is the multiplicity of arbitrary notations. However, if one subtracts everything that depends on conventions, it will remain the indivisible core of the logical necessity, manifesting itself especially in the logical propositions of the classical propositional calculus.

2. Logical necessity in logical notation

Since substitutions of the thesis of the classical propositional calculus, i.e. substitutions of the propositions of logic, are necessarily true propositions, propositions of logic must express the logical necessity with no reference to anything that is contingent. In other words, the classical propositional calculus must be an effective method for solving which formulas of the classical propositional calculus are the propositions of logic. Logic must maintain its autonomy, as Wittgenstein writes: *Logic must take care of itself* (NB 22.08.1914; TLP 5.473).

The claim that logic is autonomous means that the necessary truth of the substitutions of the propositions of logic must be recognized from the symbol alone (RUL Nov., 1913, Norway, 1913; TLP 6.113, 6.126) (Glock 1996, pp. 200-201). Finally, the propositions of logic must be considered as tautologies (TLP 6.1; NM p. 114). Tautologies like denied tautologies, i.e. contradictions, *say nothing* (NB 3.10.1914; NM p. 108; TLP 4.461, 4.462, 4.463, 5.43, 6.11, 6.124) and *are not pictures of reality* (TLP 4.462, 6.1, 6.11, 6.111) (Glock 1996, p. 355; Link 2009, p. 45).

The necessary condition for expressing the propositions of logic as tautologies is the existence of *elementary propositions* (*Elementarsätze*). Elementary propositions are the simplest propositions that do not consist of any other propositions and have only one determinant of their truth – reality (TLP 4.01, 4.05, 4.06, 4.21, 4.25) (Glock 1996, pp. 102-103). Furthermore, elementary propositions are logically independent of each other, i.e. neither truth nor falsehood of another elementary proposition can be inferred from a truth or falsehood of one elementary proposition (TLP 4.211, 5.134) (Baker 1988, p. 95; Cheung 2004, pp. 97-98; Fogelin 2006, p. 35).

Wittgenstein also adds that elementary propositions assert the existence of a contingent *state of affairs* (*Sachverhalt*) (TLP 4.21); that if it exists as a *positive fact* (*positive Tatsache*), it is a reference of the true elementary proposition (TLP 2.06, 4.25). For every state of affairs is contingent, the truth of every elementary proposition is contingent, too. One can say that every elementary proposition and every proposition consisting of elementary propositions, which is not the substitution of tautology or contradiction, is *bipolar* – possibly true and possibly false (Baker 1988, pp. 39, 54, 93; Cheung 2004, p. 97; Glock 1996, pp. 63-64; Wright 1982, p. 193).

Thanks to the existence of elementary propositions it is possible to present in the truth-tables the *truth-possibilities* (*Wahrheitsmöglichkeiten*) (TLP 4.4, 4.41) of elementary propositions, and finally, the conditions of truth

and falsehood of all propositions, including propositions consisting of elementary propositions (TLP 4.41, 4.431). Wittgenstein establishes that the elementary proposition is a *truth-function* (*Wahrheitsfunktion*) of itself and every proposition consisting of elementary propositions is a truth-function of elementary propositions (TLP 5).
Now Wittgenstein can present all propositions of logic as tautologies, i.e. formulas in the classical propositional calculus distinguished by an effective method that expresses the necessary truth of the substitutions of the propositions of logic and the necessary falsehood of the substitutions of the denied propositions of logic. As Wittgenstein writes: tautologies are true and contradictions are false *for all the truth-possibilities of the elementary propositions* (TLP 4.46).

It is worth emphasizing that elementary propositions are logically independent of each other if, and only if, elementary propositions are bipolar. Although it may seem that substitutions of propositional variables are also non-bipolar propositions, from the logical point of view, every substitution of propositional variables are only bipolar propositions. For example, one can say that in formula Φ□¬Φ variable Φ can represent all formulas, including tautologies and contradictions. Notwithstanding variable Φ is meta-linguistic and does not belong to the formal language of the classical propositional calculus. One can also add that the idea of the truth-table in the classical propositional calculus presumes that the truth-table can be applied to all their substitutions.

Now it is clear why Wittgenstein states that *The only necessity that exists is* logical *necessity* (TLP 6.37, 6.375). Only substitutions of tautologies are necessarily true and only substitutions of contradictions are necessarily false (TLP 5.525). Others propositions are contingently true, i.e. possibly true and possibly false.

To sum up this section, an effective method for solving which classical propositional formulas are the propositions of logic must guarantee the existence of its basis. This basis can only consist of elementary propositions. Thus, the logical notation must contain propositional variables that represent only bipolar elementary propositions. As Wittgenstein writes, there must be elementary propositions *on purely logical grounds* (TLP 5.5562).

3. Contingent applicaton of logic

According to Wittgenstein, logical notation must express only what is *not arbitrary* (TLP 3.342, 6.124). Since logical notation gives a definitive way to show which formulas are its propositions, it expresses only what is not contingent. Even a determination of the scope of arbitrary issues is itself arbitrary, and thus, cannot be expressed by logical notation.

Wittgenstein distinguishes between logic and its *application* (*Anwendung der Logik*); he asserts that logic cannot anticipate its application (TLP 5.557). Application of logic seems to be a matter of arbitrary and contingent decisions. If logic anticipates its application, the application would belong to logic, and thereby, would not be the application of logic, but logic itself or logic would not be logic, but only a matter of contingent elements. Contingency must be excluded from logic. Therefore, logical notation cannot express anything that belongs to application of logic.

How can one grasp the relation between necessary logic and the contingent application of logic? The best way is to get this picture: logic is like a stencil which is put on the surface of the natural language. Through this stencil one can see the natural expressions as elementary propositions or the propositions consisting of elementary propositions.
One can deal with the great importance of the distinction between logic and its application in virtue of Wittgenstein's investigations on the redundancy of the identity sign '=' in logical notation. Wittgenstein writes: *It is impossible to* assert *the identity of meaning of two expressions* (TLP 6.2322).

Wittgenstein explains that to say of two things that they are identical is nonsense, on the other hand – to say of one thing that it is identical with itself is to say nothing at all (TLP 5.5303). Thus Identity of object I express by identity of sign, and not by using a sign for identity. Difference of objects I express by difference of signs (TLP 5.53). Wittgenstein concludes: The identity-sign, therefore, is not an essential constituent of conceptual notation (TLP 5.533; NB 29.11.1914).

The identity of the meaning – or better say, the identity of reference of the names and identity of a possible reference of the propositions – is the identity of the symbols: names or propositions. All expressions of the identity are trivial and redundant. Thus, in the logical notation the same symbols should be represented by the same signs (Fogelin 2006, pp. 73-74).

A notation with different symbols having the same reference (or the same possible reference) must be supplemented by an arbitrary interpretation in which every reference of any symbol should be established by the identity sign. However, a such notation would not be the logical one – the ultimate mean expressing the logical necessity.

Since the symbols having the same reference (or the same possible reference) are the same symbols, they are also represented in logical notation by the same signs. When different symbols have a different reference, the arbitrary interpretation of the symbols' reference is redundant, and thereby, the identity sign expressing such interpretation is also redundant.

In logical notation the only thing that is expressed by signs is the identity of a representation of the identical signs. The same signs represent the same symbols (names or propositions) and the same symbols have the same reference. Signs, as it was pointed out, do not depict reality, but they can be used as depictions, i.e. propositional symbols.

Wittgenstein's investigations on the material shape of logical notation lead to the following conclusions: logical notation is only a tool for logical analysis, and thus, it is only the system of signs which cannot depict reality, but which can be used to analyze propositions depicting reality. For example, propositional variables are not elementary propositions, but they are only signs which represent elementary propositions.

Now it is also clear that for Wittgenstein, logical notation is not a perfect language. One can even say: for the author of *Tractatus* logical notation is not a language at all.

It is only a scheme that can be used to show which expressions are elementary propositions and which are propositions consisting of elementary propositions.

To sum up this section, the perspective from the viewpoint of necessary logic and the perspective from the viewpoint of the contingent application of logic in natural language are not mutually exclusive. They complete each other and create a picture of any language on which there is a place for both what is logically, non-arbitrary and for what is contingent, arbitrary. Finally, one shouldn't reject Wittgenstein's early philosophy as the doctrine of the artificial or perfect language. Wittgenstein presents not a language, but merely a stencil which one can use without any restraint in analyzing any representation of reality.

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RUL= 'Extracts from Wittgenstein's letters to Russell' [1912-1920], in NB, p. 120-132.

TLP = *Tractatus Logico-Philosophicus* [German-English parallel text], tr. D. F. Pears, B. F. McGuinness, (London: Routledge & Kegan Paul, 1961). References are to numbered sections.