A Proposed Solution to Two Puzzles in Mathematical Mapping

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1. Equals

The “equals” function in mathematics identifies different things as the same thing. Being different, however, they are not the same. Mapping different thing to different thing “proves” sameness. Still not the same thing, mapping is ascription of sameness.

Reconsidering the “equals” function, one source indicates, “Two quantities are said to be equal if they are, in some well-defined sense, equivalent.” (Weisstein http://mathworld.wolfram.com/Equal.html.) Thus is the, “**equal sign** … **:** a sign = indicating mathematical or logical equivalence.” (Mish 391) As to the nature of “equivalent,” the initial source provides, “If A=>B and B=>A … , then A and B are said to be equivalent.” (Weisstein http://math world.wolfram.com/Equivalent.html) If true, however, then if Jill is married to Jack, and if Jack is married to Jill, “then [Jill] and [Jack] are … equivalent,” when they are not.

Error occurs assuming, “‘Implies’ is the connective in propositional calculus which has the meaning ‘if A is true, then B is also true.’” (Weisstein http://mathworld.wolfram. com/Implies.html) Identifying sequential indexing of exclusively disjunctive elements, implication distinguishes elements which are not the same. Identifying sequential indexing of inclusively disjunctive elements, equivalence distinguishes elements which are the same. Implication concerns form, not content. Equivalence concerns content, not form. Misconception focuses on indexing, not indexed.

Reexamination begins with, “[1]**equal** … equivalent … ***syn*** see same.” (Mish 391) Developing this is, “**equivalent** … **2a:** like in signification or import … ***syn*** see same.” (Mish 392-393) Evident is “**equal**” is “**equivalent**” because both mean “same.” Relevant, then, is,

> [1]**same** …**:** identical …. same may imply and self-same always implies that the things under consideration are one thing and not two or more things …. identical may imply selfsameness or suggest absolute agreement in all details. (Mish 1035-1036)

Elements are equal when identical because the same, and unequal when similar because not the same. Same elements are identical because associated. Like elements are similar because disassociated.

2. Puzzles

Introduced is, “**Frege’s Puzzle:** how, if true, can ‘A=B’ differ in cognitive significance from ‘A=A’?” (Bealer http://www.yale.edu/philos/grad3.html) “A=A” being indistinguishable from “A=B,” however, introduced is Wittgenstein’s Puzzle:

> The formula ‘*a*=*a*’ uses the identity sign in a special way: for one would not say that *a* may be substituted for *a*. Yet we do start in inductions with something like *a*=*a*. … Does … it [make] sense to write ‘*x*=*x*’? (Ambrose 208)

Now, (A=A)∧¬(A=B) on Frege’s Puzzle, and (A=B)∧¬(A=A) on Wittgenstein’s Puzzle.

Accepting both puzzles, (A=A) and (A=B) are mutually defeating. Significance occurs considering, “It is true that a=a and a=b are used at the start of proofs by induction.” (Ambrose 208) Introduced is the first fundamental condition,

> which a deductive presentation must satisfy if it is to be fully rigorous: 1. Explicit enumeration of the primitive terms for subsequent use in definitions. (Blanche 21-22.)

Initiated is a set, significance of which is indicated considering, “Without [set theory], not only can we not *do* modern mathematics, we can’t even say what we are talking about.” (Stewart 43) A=A and A=B introducing an initial element in any well-ordered set, if mutually defeating, then sets are impossible. Sets being impossible, when modern mathematics is impossible without sets, then modern mathematics is impossible if A=A and A=B are mutually defeating.

3. Proof

Presented is a mapping problem introduced by David Hilbert’s “distinction … between a subject matter under study and discourse about the subject matter.” (Nagel 31) Linkage is by means of a substitution whereof, “One must at all times be able to replace ‘points, lines, planes’ by ‘tables, chairs, beermugs.’” (Boyer 610.) Implementing this function, Kurt “Gödel first showed that it is possible to assign a *unique number* to each elementary sign, to each formula (or sequence of signs), and each proof (or finite sequence of formulas).” (Nagel 69)

As to the nature of proof, nothing in Hilbert and Gödel’s mapping scheme is inconsistent with pairing every element of “the subject matter” ***D*** with one element of the “subject matter understudy” ***T***, and every element of ***T*** with one element of ***D***, and no element of ***D*** or ***T*** with more or less than one element of the other, establishing a bijection of “the subject matter” and the “subject matter understudy.” Hereby, “the situation is perfectly symmetrical; and if we turn all the arrows round we define another function … in the opposite direction.” (Nagel 71) Proceeding thus, “the subject matter” is converted into the “subject matter understudy.”

Achieving this, “the subject matter” $\boldsymbol{D}^1$ of which the current “subject matter” ***D*** constituted the “subject matter understudy,” can be converted into the current “subject matter understudy” ***T***, and so on until the universe is converted into the current “subject matter understudy” ***T***. Now every ***D*** is converted into ***T***. Being so, the process can be reversed “in the opposite direction,” converting every ***T*** into ***D***.

Now whether the current “subject matter understudy” ***T*** is “the [current] subject matter” or not is ambiguous. Thus it is impossible in the process of mapping “the subject matter” ***D*** to the “subject matter understudy” ***T***, to specify “a *unique* element *f*(*x*) of ***T*** … . so that there is no ambiguity attached to it.” (Stewart 67-68) Proof by mapping as Hilbert and Gödel propose is unnecessary, then. Ambiguity is resolved only by embracing it in circularity, conjoining otherwise separate linear conversions of “the subject matter” and the “subject matter understudy.”

Implemented is proof by mathematical induction. Like a rational number, engendered is an indefinitely repeating sequence of consecutive digits. Sought by marginally transitioning from “individual in the universe of the discourse” to “individual in the universe of the discourse” by the degrees of similarity represented by the fuzzy set, is an indexical conversion of individual into individual.

4. Recursion and Iteration

Constituting mathematical identity is location within a set. Separating the accidental and essential theories of mathematical identity is the nature of the set within which location constitutes identity. There are two kinds of sets considering this, cardinal and ordinal, which are distinguished by the mechanism generating them. Cardinality is generated recursively, and ordinality is generated iteratively. Functionally the essential theory of identity is recursive, location within a cardinal set. Functionally the accidental theory of identity is iterative, location within an ordinal set.

Recursion is the means by which identity occurs in a domain—indeed it is what constitutes a domain. Iteration is the means by which identity occurs in co-domains—indeed it is what constitutes co-domains. Mathematical identity is neither essential nor nominal, then, it is both. It is essential in a domain, and nominal across domains. Additionally, it is nominal across domains because appearance of a thing in different domains is different. Appearance differs at least concerning those things to which something is related.

Both recursion and iteration are mechanisms of identity, determining the membership of a set. This occurs by a process of sequencing. Common to both recursion and iteration is identity of sequence members by analogy with an archetype, this archetype constituting the intensional criterion of sequence membership. Distinguishing recursion and iteration is the nature of the identifying archetype.

Each initiates with identification of a base case, but differs according to the nature of the base case. Recursion is essential identity from a constant base case, and iteration is accidental identity from an inconstant base case. An identity function continuously applied to a constant analogical archetype constitutes a recursive sequence. An identity function continuously applied to an analogical archetype or archetypes in a transitive sequence constitutes an iterative sequence.

Recursion is the means by which identity occurs within a domain—indeed it is what constitutes a domain. Iteration is the means by which identity occurs in co-domains—indeed it is what constitutes co-domains. Identity is neither essential nor nominal, then, it is both. It is essential in a domain, and nominal across domains. And it is nominal across domains because appearance of a thing in different domains is different. If nothing else, they differ by to what they are related.

Distinguishing the forms of sets are the mathematical concepts of “field,” “commutative ring with unity,” and “corecursive hyperset.” A recursive set determines a field, composing an unlimited set. Constituent are conjoined elements without disjoined elements. Set identity is constant determined in any sequential order.

An inductive iterative set determines a ring, composing a limited set with subsets. Constituent are conjoined and disjoined elements, with disjoined elements defining set limits. Set identity is inconstant determined in any sequential order from one limit to the other limit. Different resolution being possible at each disjunctive, set limits are inconstant.

A deductive iterative set determines a corecursive hyperset, composing a limited set without subsets. Constituent are conjoined elements with disjoined elements defining set limits. Set identity is constant, determined in any sequential order from one limit to the other limit. Different resolution being impossible at each conjunctive, set limits are constant.

Both a ring and hyperset integrate induction and deduction into a Platonic dialectic. The set of all analytic proofs can be proven only synthetically. The set of all synthetic proofs can be proven only analytically. Therefore, the set of all proofs can be proven only circularly, reciprocally synthetically and analytically. Analytic proof being a priori identity, identified is the synthetic a priori.

This can be recursive in the form of a field, or iterative, whether parallel in the form of a ring, or sequential in the form of a corecursive hyperset. Whether parallel or sequential, reciprocal analytic and synthetic identity constitutes self-identity. Self-defining, composed is a self-contained system. Converging onto alternate limits, conjoined reciprocal proofs are mutually verifying.

5. Hyperset and Ring

Resolved are both Frege’s puzzle and Wittgenstein’s puzzle. Differentiating them is the means by which “the subject matter” and the “subject matter understudy” are mutually mapped. A=A identifies a corecursive hyperset, which from alternate limits reciprocally generates the same transmutative sequence in inverse order. Distinguished are aspects of the same thing. A=B identifies an iterative ring, which from alternate limits reciprocally generates the same transmutative sequence in variable order. Distinguished are instances of the same thing.

Relevantly, a set whose constituents are diffused does not contain itself, and a set whose constituents are fused does contain itself. Elements of a diffused set are conjoined; elements of a fused set are implicated. A conjunctive diffused set is a cardinal set, constituting the same set—having the same identity—in any sequence of elements. An implicative fused set is an ordinal set, constituting a different set—having different identity—in different sequences of elements.

Considering the limit of the power set symbolized by ℘, whether this limit is real or nominal, shows this. Identifying every element between the limits of a set, the power set of any set is ambiguous. Distinguishing each member of a set is understandable in contradictory ways, as generating both one indivisible thing and infinite indivisible things. Either every element is fused into one with nothing separating one from another, or every element is diffused into infinity with nothing linking one to another.

Identifying every element between the elements of a set when, “The rational fractions are so dense that between any two of them, no matter how close, there always will be another,” the power set is understandable as separating or integrating “the rational fractions.” (Boyer 566) Neither there is a subset, nor there is no subset, between any two subsets, is inconsistent with the power set. The first identifying a dense set, and the second identifying a discrete set, the power set is consistent with dense and discrete set.

Proceeding thus, movement is from identity of all the elements of the set in an all encompassing ordinal sequence, to all the elements of the set in an all encompassing cardinal sequence. So doing, different cardinal sequences of all system elements can be analogically identified by transposing elements within a sequence according to corresponding elements within another system until converting the former into the latter.

Implemented is a translation function transforming systemization of elements into systemization of elements. Implicatively integrated by a sequential conversion of systemization into systemization, constituted is a coherent whole. Systemization seamlessly transforming into systemization, the whole is consistent.

As consistent, the transformative set is well-ordered. It is so because different encompassed systemizations are understood as aspects of one another. Being mutual aspects, systemization is indistinguishable from systemization. Because indistinguishable, such systemizations compose a single transformative systemization.

Being so, exhibited is a function whereby constituents reciprocally fuse into the same indistinguishable whole by repetitive iterative application of the "+" conjunctive function in any sequential order. Alternately, they diffuse into the same distinguishable parts by repetitive iterative application of the "." disjunctive function in any sequential order. Constituents as fused whole are an object, and as diffused parts are objects.

Manifest is the conjunctive relation of identifiable particulars. Relation is an unbroken path between two elements within a domain. If a broken path, how is an element prior to the break known to be the same element subsequent to the break? Elements in different domains are proven related by tracing an unbroken path between them, incorporating both into a common domain. Proof is tracing such an unbroken path, mathematically constituting identifying a dense set. It is material when physical, an unbroken path of matter between limits. It is mental when phenomenal or conceptual.

Relation being a continuum between particulars, a particular is identifiable within a continuum by alternate identity as constituent and non-constituent of the continuum. Fusion of constituents renders a particular distinguishable from the continuum. Diffusion of constituents renders a particular indistinguishable from the continuum. Cyclic transitive marginal conversion of constituents from fusion to diffusion and diffusion to fusion renders a particular distinguishable and indistinguishable from the continuum.

Thus, form is not independent of substance. It has no independent ontological status. Linguistically, it is a verb, identifying a substantive state of being or substantive states of being. As a substantive state, it is contained within a recursive field. As substantive states, it contains within itself an iterative sequence. Mapping identifies an iterative sequence. Mathematical equality identifies iterative sequences.

Reciprocal transitions from limiting states are equated, distinguished by the means constituting the same continuum. Alternating ordinal succession identifies A=A, the same sequence distinguished by reverse order. Alternating cardinal succession identifies A=B, the same sequence distinguished by differential order. Thus, A=A identifying aspects, and A=B identifying instances, they are mutually consistent. This is because they designate different things. Being the means by which sets are composed, when sets constitute modern mathematics, modern mathematics is possible.

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