THIN OBJECTS

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1 INTRODUCTION

Kant famously argued that all existence claims are synthetic. An existence claim can never be established by conceptual analysis alone but will always require some appeal to intuition or perception, thus making the claim synthetic. This view is rejected in Frege's *Foundations of Arithmetic*, where Frege defends an account of arithmetic which combines both platonism and logicism. Frege's platonism consists in taking arithmetic to be about real and independently existing objects. And his logicism consists in taking the truths of pure arithmetic to rest on just logic and definitions and thus be analytic. Most philosophers now probably agree with Kant in this debate: the existence claims of Frege's platonism cannot be established on the basis of logic and conceptual analysis alone.

However, the disagreement between Kant and Frege is alive and well in a somewhat different form. Forget the problematic analytic-synthetic distinction. Can there be objects which are "thin" in the sense that very little is required for their existence? A classic example is the view in the philosophy of mathematics that the mere consistency or coherence of a mathematical theory suffices for the existence of the objects that the theory purports to describe. This view has been held by many leading mathematicians and continues to exert a strong influence on contemporary philosophers of mathematics. A more recent example is the neo-Fregean view that the equinumerosity of two concepts is conceptually sufficient for the existence of the number that specifies the cardinality of both concepts. For instance, the fact that the knives on the table can be one-to-one correlated with the forks on the table is said to be conceptually sufficient for the existence of a number that specifies the cardinality both of the knives and of the forks.[1]

If the defenders of thin objects are regarded as heirs to the Fregean view that there are analytic existence claims, then there are also lots of heirs to the contrasting Kantian view. For instance, Hartry Field has at-

[1] See for instance Wright (1983) and the essays collected in Hale & Wright (2001).

tacked views according to which mathematical objects are thin, sometimes mentioning the Kantian origin of his criticism.[2] Like the original Kantian view on analytic existence claims, the contemporary view that there can be no thin objects strikes many people as highly plausible. Appeals to thin objects often come across as attempts to pull rabbits out of a hat.

The goal of this article is to explain, defend, and demystify the idea of thin objects. I will refer to the view that there are thin objects as *meta-ontological minimalism* or *minimalism* for short. Let me explain the label. Ontology is of course the study of what there is. Meta-ontology, on the other hand, is the study of these key concepts of ontology, such as the concepts of existence and objecthood. Meta-ontological minimalism is accordingly the view that the key concepts of ontology have a minimal character. Not surprisingly, this view tends to result in very generous ontologies. For the less that is required for existence, the more objects there will be.

However, it is important to note that minimalists do not claim that *all* objects are thin and that their existence thus makes only some minimal demand on reality. Their claim is that the notion of an object itself is thin and thus *allows for* thin objects. But they happily admit that many kinds of objects are thick. For instance, elementary particles are thick because their existence makes some substantive demand on reality. But the minimalists insist that the thickness of elementary particles derives from what it is to be an elementary particle, not from what it is to be an object.

This paper is structured as follows. First I review some considerations in favor of thin objects. Next I describe the neo-Fregean approach to thin objects and outline some problems that this approach faces. Then I outline my own version of this approach, which is better equipped to answer the problems. This approach ties the notion of an object to that of a semantic value and make crucial use of a principle of compositionality for semantic values.

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[2] See Field (1989), pp.5 and 79–80.

Meta-ontological minimalism appears to enjoy a number of appealing features. For instance, it promises a way to accept face value readings of discourses whose ontologies would otherwise be philosophically problematic.

Arithmetic provides a good example. The language of arithmetic contains a variety of proper names which (it seems) are supposed to refer to certain abstract objects, namely the natural numbers. The language also contains quantifier phrases which (it seems) are supposed to range over the natural numbers. Moreover, a great variety of theorems expressed in this language appear to be true. For a lot of such theorems are asserted in full earnest by educated lay people as well as professional mathematicians. And since the arithmetical competence of these people is beyond question, there is reason to believe that most of their arithmetical assertions are true. But if these theorems are true, then their various subexpressions must succeed in doing what they are supposed to do. In particular, their singular terms and quantifiers must succeed in referring to and ranging over natural numbers. And for this kind of success to be possible, there must exist abstract mathematical objects.

This is a powerful argument. But is it sound? Since all I did was to observe that the premises *appear* to be true, they can of course be challenged. However, it is *prima facie* attractive to take these appearances at face value, since this will save us the difficult task of showing how both lay people and experts can be deceived about something they take to be obviously true. And when the premises are taken at face value, the argument shows that there must exist abstract mathematical objects.

However, this ontology of abstract objects is often found to be philosophically problematic. One well-known worry concerns epistemic "access" to such objects. Since perception and all forms of instrumental detection are based on causal processes, these methods cannot give us access to abstract objects such as the natural numbers. How then can we acquire knowledge of them?[3] Another worry is the sheer extravagance of postulating such huge ontologies. How can we postulate an infinity of new objects with such a light heart? No physicist would so unscrupulously postulate an infinity of new physical objects. Why then should mathematicians get away with it? Philosophers are notoriously divided over how serious these

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3 This worry was made famous by Benacerraf (1973). For discussion and improvements, see Field (1989) and Linnebo (2006).

worries are. But any successful account of mathematical objects needs to have some response to the worries, even if only to explain why they are misguided.

Thin objects offer an extremely promising strategy for responding to the worries. The vast ontology of mathematics may well be problematic when it is understood in a thick sense. If mathematical objects were pretty much like elementary particles except for being abstract, then there would indeed be good reason to worry about epistemic access and ontological extravagance. But perhaps mathematical objects need not be understood in this way. If there are such things as thin objects, then the existence of mathematical objects need not make much of a demand on the world. It may for instance suffice that the theory purporting to describe the relevant mathematical objects is coherent. And although facts about the coherence of mathematical theories are still inadequately understood, they are less problematic than thick mathematical objects would be. It is at least not a complete mystery how we can have epistemic access to facts about the coherence of mathematical theories. And since this account of mathematical objects sets the bar to existence extremely low, it is not at all surprising that an extravagant ontology should result.

3 An Abstractionist Approach to Thin Objects

One approach to thin objects is found in the neo-Fregean philosophy of mathematics developed by Hale and Wright. The neo-Fregeans seek to provide a logical and philosophical foundation for classical mathematics on the basis of so-called *abstraction principles*. These are principles of the form

$$(*)\ \sum(\alpha) = \sum(\beta) \leftrightarrow \alpha \sim \beta$$

where α and β range over items of some sort, where $\sim$ is an equivalence relation on such items, and where $\sum$ an operator that maps such items to objects. The neo-Fregeans are particularly fond of *Hume's Principle*, which says that the number of Fs (symbolized as $\#F$) is identical to the number of G just in case the F and the Gs can be one-to-one correlated (symbolized as $F \approx G$):

$$(\text{HP})\ \#F = \#G \leftrightarrow F \approx G$$

This principle has the amazing mathematical property that, when added to second-order logic along with some definitions, we are able to derive all of ordinary (second-order Peano-Dedekind) arithmetic. Abstraction principles are available for many other kinds of abstract object as well, for instance directions, geometrical shapes, and linguistic types.

If true, an abstraction principle will provide unproblematic epistemic and semantic access to the objects invoked on the left-hand side of an abstraction principle: we simply proceed via its unproblematic right-hand side. However, what reason do nominalists have to accept Hume's Principle and other abstraction principles as true? The neo-Fregean response to this challenge turns on regarding the objects invoked on the left-hand side as thin.

This response can be developed in the form of a view of what is required for a singular term t to refer (Hale & Wright 2009a). At the very least the term must have sense. But more is presumably required. What is this further requirement? The question can be put in terms of the following equation:

(E) t has sense + $X \Leftrightarrow t$ has reference

where '$\Leftrightarrow$' means something like mutual conceptual entailment. That is, what requirement X do we have to add to the claim that t has sense to get something that is conceptually equivalent to the claim that t has reference?

When t is an *abstraction term* – that is, a term of the form '$\sum(\alpha)$' – then Hale and Wright claim that the further requirement X that is needed to advance from sense to reference is just that the item α associated with t be equivalent to itself: $\alpha \sim \alpha$. That is, they propose the following abstractionist solution to the equation (E):

(A) '$\sum(\alpha)$' has sense + $(\alpha \sim \alpha) \Leftrightarrow$ '$\sum(\alpha)$' has reference

According to this view, the left-hand side of (A) conceptually entails the right-hand side and its claim that the abstraction term '$\sum(\alpha)$' refers. The notions of reference and objecthood have been "scaled" so as to ensure that (A) comes out right (Hale & Wright 2009b). This abstractionist approach to thin objects will serve as the starting point for my own approach to be outlined below.

Although the idea of thin objects holds great promise, it also faces a number of problems which any successful version of minimalism must be capable of addressing.

> **The problem of existence**. Why should we believe that there are such things as thin objects in the first place? Why should certain innocent facts suffice for the existence of certain controversial objects? What prevents someone from accepting the innocent facts while denying that this suffices for the existence of the objects?
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> **The problem of overgeneration**. Assume that the problem of existence can be solved. Then the question arises just how thin various kinds of object actually are. Few people would want to claim that elementary particles are thin. But once we open the door to thin objects, what right do we have to deny that other objects, such as elementary particles, are thin as well?
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> **The problem of lack of uniformity**. Assume that the problem of overgeneration can be solved. Then we still face the question how much thin and thick objects have in common. Do these kinds of item belong under the same rubric at all? Perhaps the words 'existence' and 'object' are being used ambiguously.
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> **The problem of consistency**. The last and potentially most fatal problem is the threat of inconsistency. The acceptable abstraction principles turn out to be surrounded by unacceptable ones, which are incoherent or downright inconsistent, or which conflict with one another. This is known as *the bad company problem*.[4]

In what follows I develop a version of minimalism which I believe will allow us to answer all of these problems. This version is inspired by the abstractionist approach described above. My argument proceeds in three steps. The first step glosses the notion of object as a possible referent of a singular term. The second step glosses the notion of a referent of a singular term in terms of the notion of a semantic value. The third step offers a minimalist account of what it takes for a singular term to possess a seman-

[4] See Linnebo (2009) for an introduction and further references.

tic value. Taken together, the three steps provide a minimalist account of what is required for the existence of an object.

5 Objects as Referents of Singular Terms

Let's begin with the first step. One may wonder how an inquiry into the concept of an object can even be possible. For as Frege observes, this concept is "too simple to admit of logical analysis" (Frege (1891), p.140). Although Frege is no doubt right that a "proper definition" of the concept of an object is out of the question, I believe it is both possible and reasonable to ask for some further explication of the concept. Even if the concept cannot be *defined* in more basic terms, it can still be glossed or characterized, for instance by relating it to other concepts and by explaining the role that it plays in our thought and reasoning. Compare the notion of conjunction. Although this notion too is a primitive which cannot be defined in more basic terms, a lot can be said to gloss or characterize it. We can for instance describe its inferential properties and its possession conditions.

So let's examine the role that objects play in our semantic theories. There appear to be two different but related roles: objects serve as referents of singular terms and as values of bound first-order variables. Frege's explication of the notion of object focusses on the former role of objects as the referents of singular terms. He takes objects to be the kinds of item that singular terms refer to. Quine, on the other hand, focuses on the latter role of objects as values of bound variables, as encapsulated in his famous slogan that "to be is to be the value of a bound variable". Since the referent of any singular term can also serve as the value of a bound variable, it follows that everything that is an object in Frege's sense is also an object in Quine's sense. I will return to the question of whether the converse holds.

Which explication is better? My own preference is for Frege's explication over Quine's. For I take singular terms and their reference to be more fundamental than quantifiers and their ranges. Quantification is explained in terms of its relation to its instances, which involve singular terms. I therefore propose the following Fregean alternative to Quine's slogan: To be is to be a possible referent of a singular term.

6 Reference as Possession of Semantic Value

The second step of my argument relates the notion of a referent of a singular term to that of a semantic value. In semantics and the philosophy of language it is widely assumed that each component of a complex expression makes some definite contribution to the meaning of the complex expression. This contribution is known as its *semantic value*. I will write [[**E**]] for the semantic value of an expression **E**. For instance, Frege held that the semantic value of a sentence is its truth-value and that the semantic values of other expressions are their contributions to the truth-values of sentences in which they occur. In particular, the semantic values of singular terms are just their referents.

It is also widely assumed that the meaning of a complex expression is functionally determined by the semantic values of its components and their syntactic mode of combination. This assumption is known as *compositionality*. For instance, according to Frege the semantic value of a simple sentence such as 'John runs' is determined by the equation:

(1) [[John runs]] = [[runs]] ([[John]])

That is, the semantic value of the sentence 'John runs' is the result of applying the function which is the semantic value of the predicate 'runs' to the argument which is the semantic value of the subject 'John'. More generally, let C be some syntactic operation applicable to syntactic expressions $E_1,\ldots, E_n$. Then there is some semantic operation C^* corresponding to C such that the semantic value of the result of applying the syntactic operation C to the expressions $E_1,\ldots, E_n$ is identical to the result of applying the semantic operation C^* to the expressions' semantic values:

(2) $[[C(E_1,\ldots, E_n)]] = C^*([[E_1]],\ldots, [[E_n]])$.

Why should the ordinary notion of reference be explicated in terms of the technical notion of semantic value? My main reason for doing so is that the notion of semantic value carries with it much less intuitive baggage than that of a referent. The ordinary notion of a referent is naturally understood in a thick way. A referent is naturally taken to be something that one can somehow encounter, that plays an ineliminable role in the truth of predications, and that is completely independent of us and our representational devices. The technical notion of a semantic value carries no such baggage.

However, provided that these thick connotations are set aside, I have no objection to continued talk about the semantic values of singular terms as their referents.

7 When Does a Singular Term Have a Semantic Value?

The third step consists of a sufficient condition for a singular term to have a semantic value. This is the most important and distinctive step of the argument.

Let SV(t, a) be the relation that holds between a singular term t and its semantic value a. What makes it the case that t has a as its semantic value? Since the relation can hardly be a primitive one, there must be something that is responsible for its obtaining. Compare the relation of ownership, which also isn't a primitive one. So when I bear the ownership relation to my bicycle, there must be something responsible for the obtaining of this relation. The study of what it is in virtue of which expressions have semantic values is sometimes called *meta-semantics*.

The sufficient condition for possession of semantic value that I wish to defend is inspired by the abstractionist approach outlined above. I will thus be concerned with languages whose singular terms are associated with an item α and a relation ~ defined on such items. For instance, in the language of directions, each singular term is associated with a line l and the equivalence relation of parallelism. The item associated with a singular term must not be confused with the term's referent. The role of these items is rather to present the referents. For instance, a line serves to present the direction that it has. Let's refer to the items that play this role of presenting the proper referents as *presentations*. The role of the relation ~ is to specify when two presentations determine the same referent. Let's refer to relations that play this role as *unity relations*.

Recall that the abstractionist view claims that it suffices for an abstraction term to refer that it has sense and that its presentation stands in the relevant unity relation to itself. Since a singular term is guaranteed to have sense already by the fact that it is associated with a presentation and a unity relation, the abstractionist view is simply that it suffices for a term to refer that it has been assigned a presentation and a unity relation.

I will illustrate this view by means of the example of directions. Let D be a domain of lines and other directed items. Assume $\mathcal{L}$ is a first-order language with identity such that:

(i) the variables of $\mathcal{L}$ range over D,
(ii) each singular term of $\mathcal{L}$ has been assigned an element of D,
(iii) each atomic predicate of $\mathcal{L}$ is defined on each element of D,
(iv) for any two singular terms t_1 and t_2 that have been assigned l_1 and l_2 we have: $\ulcorner t_1 = t_2 \urcorner$ is true iff $l_1 \parallel l_2$.

When these assumptions are met, I say that $\mathcal{L}$ has a *pre-interpretation*. A pre-interpretation is much like a proper interpretation except that it is based on a domain of presentations (in this case lines) instead of proper referents (in this case directions). A consequence of this is that the identity predicate is interpreted non-standardly: the identity predicate can be true of two non-identical lines provided they are parallel. Note that the standard laws of identity require that every predicate **P** of $\mathcal{L}$ be a congruence with respect to parallelism, in the sense that the following holds:

> Assume **P** is n-adic and that $l_i \parallel l_i'$ for each i from 1 to n. Then **P** holds of $l_1,\ldots, l_n$ iff **P** holds of $l_1',\ldots, l_n'$.

My sufficient condition says roughly that a pre-interpretation suffices for a proper interpretation. But let's be more precise. Assume $\mathcal{L}$ has a pre-interpretation and that t_i are singular terms of $\mathcal{L}$ which have been assigned lines l_i respectively. Then the sufficient condition says that expressions from $\mathcal{L}$ can be assigned semantic values such that:

(a) singular terms have the same semantic value iff the lines they have been assigned lines are parallel, that is: $[\![t_1]\!] = [\![t_2]\!] \leftrightarrow l_1 \parallel l_2$

(b) the principle of compositionality holds for simple predications, that is: $[\![\mathbf{P}(t_1,\ldots, t_n)]\!] = [\![\mathbf{P}]\!]([\![t_1]\!],\ldots, [\![t_n]\!])$

(c) the semantic values are *sui generis*.

Some explanations are in order. The first claim, (a), says that the semantic value assigned to a singular term t_i depends only on the assigned line l_i up to parallelism. There is thus a function d such that the semantic values of the terms t_i are given as

$$[\![t_i]\!] = d(l_i)$$

and such that

$$d(l_i) = d(l_j) \leftrightarrow l_i \parallel l_j.$$

Note that this latter formula is just an abstraction principle. So we now have an explanation of why such principles are so important: they play an important role in the meta-semantic account of reference.

To understand the second claim, (b), let's consider an example. Assume $\mathcal{L}$ contains a two-place predicate **P** which is true of t_i and t_j iff l_i and l_j are orthogonal. Then (b) says that **P** has a semantic value **P** which is true of the directions $d(l_i)$ and $d(l_j)$ iff the lines l_i and l_j are orthogonal. This means that **P** is an orthogonality predicate on directions. We are here relying on the fact that two directions are orthogonal iff any two lines whose directions they are, are orthogonal.

Claims (a) and (b) are highly plausible. To see this, recall what semantic values are supposed to do. The semantic value of an expression was explained as the contribution that the expression makes to the truth-value of sentences in which it occurs. Assume that $\mathcal{L}$ has a pre-interpretation and that the singular term t has been assigned a line l as its presentation. What is the semantic contribution that t makes to the truth-values of sentences in which it occurs? Clearly t makes *some* contribution: for all sentences involving the term have truth-values which typically depend on the presentation l. But the contribution cannot be anything as specific as the line l. For we know that any parallel line l' would make precisely the same contribution. Rather, the semantic contribution of t must be something which is shared by all lines l' that are parallel with l. But this is exactly the sort of contribution that claim (a) ascribes to t. An analogous motivation can be provided for (b). Clearly the atomic predicates make some semantic contribution. But this contribution does not discriminate between singular terms with the same semantic value, as asserted by (b).

In pure mathematics it would be natural to represent the semantic contribution of the singular term t as the equivalence class of l under the equivalence relation of parallelism, that is, as the set of all lines l' that are parallel to l. This too would ensure that claim (a) holds. Moreover, we could let the semantic value of the predicate for orthogonality be the function that maps two such equivalence classes to the true iff any two lines from each of the two classes are orthogonal, and to the false otherwise. This is easily seen to ensure that claim (b) holds. So this provides a useful model of the desired assignment of semantic values.

However, a model of an assignment is not the same as the intended assignment itself. Although the semantic values of the language of directions can be *represented by* equivalence classes of lines, they should not be *identified with* such equivalence classes. Doing so would ascribe to the semantic values properties that go beyond the contribution that the relevant terms make to the truth-values of sentences in which they occur. For instance, equivalence classes have set theoretic elements, which is a notion that is completely foreign to the geometrical language in question. Accordingly the third claim of the sufficient condition, (c), says that it is permissible to assign to the expressions in question semantic values that are primitive and *sui generis* and not just set theoretic constructs. These semantic values are nothing more than the contributions made by the relevant singular terms. This provides a good beginning of a response to what I called the problem of existence. In other work I argue that this version of minimalism allows for good responses to the other three problems as well.

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