Showing Space,
or:
Can there be Sciences
of the Non-Discursive?

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Introduction:
Space and Wittgenstein

The aim of this paper is to explain a theory of space, and how it features, conceptually and practically, in everyday life. The theory, which is known as space syntax, bears on the work of Wittgenstein in three ways. First, from the point of view of language, space seems to the kind of problem that Wittgenstein identified for logic in the *Tractatus*, that what is expressed *through* language cannot be represented *in* language (TLP 4.12, 4.121). Practically speaking, this means that although natural language, like life, is built on a scaffolding of space and spatial relations, so much so that 'language and space' is now a key theme in cognitive neuroscience (Bloom et al. 1996), the patterns of space in which we live our lives are not representable in linguistic terms. The spatial relations and structures of our lives are in effect *non-discursive*: we live them but don't know how to talk about them. The non-discursivity of space is a particular problem in architecture where the primary task is to create a pattern of space—a 'layout'—adapted for functional purposes, yet there are no linguistic means to describe the differences between one layout and another, forcing architects to proceed by intuition, precedent or metaphor. Yet architects must predict function from space, and so need a consistent language, if not a science of space. This poses the question: can there ever be a science of space when space is non-discursive in this sense? Space syntax then addresses the architectural problem of space, but anticipates that in doing so it will address the problem of space in general.

The second point of contact with Wittgenstein is that graphical representations played a critical role in the creation of the theory, and still play a critical role in its applications. These representations were able to *show* features of spatial patterns that language was not able to describe. But more unexpectedly, perhaps, from a Wittgensteinian point of view, these representations led directly to seeing how we could quantify structural features of space, and so seek to render the non-discursive discursive. We can make the point in a Wittgensteinian way. In the *Untersuchungen* he says:

> Our language can be regarded as an ancient city: a maze of little streets and squares, of old and new houses, of houses with extensions from various periods […] surrounded by a multitude of new suburbs [such as the symbolism of chemistry] with straight and regular streets and uniform houses. (PI § 18)

We can say that we sought to add a new suburb to language, based on a science of space, which will allow the spatial scaffolding of life to be described. But this also means that in Wittgensteinian terms we are trying to say the unsayable. The question is: do we succeed and what are the implications?

The third point of contact concerns what Wittgenstein would call the 'formal unity' of the 'language of space' at which we arrive. Once the structural features of spatial relations are brought to light, we seem to discover that space is subject to something like laws, not in the form of universal behaviours, but of the form 'if we intervene physically in space in this way rather than that—say by placing an object in the centre of a room rather than a corner, or making the object rectangular rather than square—then the structural consequences for the ambient space are these.' These 'laws' seem to be known intuitively to human beings, in the same sense that when we throw a ball of paper so that its parabola leads it to land in the waste paper basket, we 'know' the laws of mathematical physics—what cognitive science has called 'intuitive physics.' It seems to be these lawful potentials that human beings exploit in culturally differentiated 'spatial games.' We also find that the emergent forms of complexity, such as the spatial structures of cities, that accumulate from human spatial behaviours, manifest astonishing cross-cultural invariants. All this suggests that the human language of space, in spite of its power to generate differentiated cultural expression, is in some sense a universal language. Should we then conclude that space has the *formal unity* that the later Wittgenstein doubted for natural language?

In this paper then, we aim to clarify space by seeing it through key Wittgensteinian ideas. We do not aim to clarify Wittgenstein! But we end with a Wittgensteinian twist, which may suggest other areas where Wittgenstein's thought might be explored, namely in the sciences of complexity. Although the emergent structures that arise in large and complex spatial systems can be represented graphically, and so shown clearly, and can even be tested against functional evidence for their verisimilitude to reality, they cannot so far be described within a formal language. They can, it seems, only be *shown*. As we will argue, this is a general problem in the sciences of complexity, so we seem to be back to square one, and the *Tractatus*, but with a more generalised view of the problem of *showing*!

In what follows, we deal with these questions in turn. Let me first try to show that space is a seriously Wittgensteinian problem.

Space as a Wittgensteinian Problem

If we are to talk about space and language, we should begin by addressing a fundamental theme in Wittgenstein's writing, that language prescribed the limits to thought. It is hard to defend this point of view in the early twenty first century in the light of cognitive research, and the field where this can be said most emphatically is space. For example, according to Bowman:

> If any domain has plausible claim to strong language-independent perceptual and cognitive organisation, it is space. Our mental representations of space are constrained not only by our biology but also by their fit to the world out there. Little wonder it has seemed likely to many investigators that the language of space closely mirrors the contours of non-linguistic spatial understanding. Several kinds of empirical evidence support the assumption that children know a great deal about space before they can talk about it, and that they draw on this knowledge in acquiring spatial words. (Bowman, 1996: 387)

Johnson-Laird goes further: 'Human reasoners use functionally spatial models to think about space, but they also appear to use such models in order to think in general' (Johnson-Laird, 1996: 460). How language handles space is then critical for both language and space.

The problem of space in human thought and language, then, seems almost to involve a contradiction. Thought is profoundly spatial, yet at the same time language lacks any means of describing the everyday spatial complexity in which we live. How can this be? We can begin by looking more carefully at how language is spatial. A key part of the spatial structuring of many languages, including English, comes from the system of prepositions. In general, prepositions specify different spatial relations with considerable precision, but only between two or three entities. For example, 'inside' implies an 'outside' and something which distinguishes one from the other, 'through' implies a kind of origin and a destination as well as an entity passed through, 'beyond' specifies an origin and a place the other side of an intervening place, and so on. Where more entities are specified, as in the English 'among', then there is less precision in the spatial relations, and, in the case of 'among', the set of entities into which the 'among' entity is set is treated as a single entity, a grouping without further spatial form. Prepositions also differ in the kinds of relation they specify. For example, 'next to' is a symmetrical relation, in that if *a* is next to *b*, then *b* is next to *a*, while 'above' or 'under' are asymmetric relations, in that if a is above *b* then *b* cannot be above *a*. 'Between' then can be seen to specify both symmetrical relations between the two outside entities, and an asymmetrical relation between them and the inner entity. Prepositions also specify the numbers of objects in particular places in a scheme of relations. For example, you cannot be between or among one entity or inside more than one, unless one is inside the other.

There seems, in effect, to be something like a formal structure underlying the set of spatial prepositions. Each preposition specifies a scheme of spatial relations, with both abstract and concrete properties, and comes as a kind of irreducible bundle, so that it is quite hard to specify what each means without using the word itself. For example, it is quite hard to say exactly what 'between' means without at some stage wanting to say 'between'. We could perhaps try to say that there is an object with another object at one side and another on the opposite side, but this describes a line of three objects without pointing to the central one as being in a special relation to the other two as 'between' does. Or imagine a line of houses. Each, except the end two, is 'between' a pair of others, but we would not say this if we were looking at the line, which appears to us as a series of 'next to' relations. If we were living at number 10, though, we might well see ourselves as being between number 9 and number 11. So in effect betweenness needs to be pointed to, and is hard to indicate in
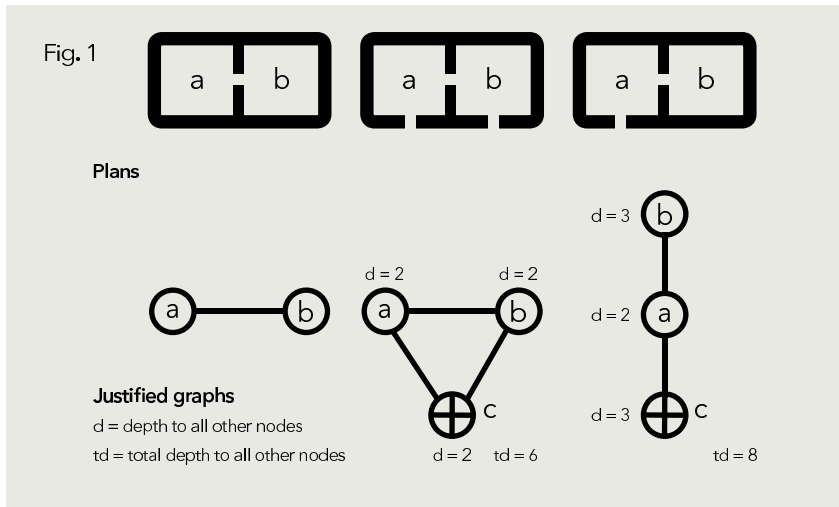
any other way. This seems a very clear case of Wittgensteinian 'showing' and what is shown is the scheme of relations bundled up in the word. The word, we might say, is a form of showing.

It is because prepositions bundle up relations in words that they seem to offer a kind of bridge between the perceptual and the conceptual. We see them, and show them, all at once, and what is shown are irreducible schemes of spatial relations. It is this that makes them suitable vehicles for abstract as well as concrete thought. But more significantly, it is perhaps possible at this stage of our evolution for this kind of bundling to be possible with up to three objects. Perhaps superior intelligences in the future will be able to do this with four, five or many more entities. But for the time being, we are in a situation in which our languages are pervasively spatial, yet lack any kind of terminology to describe even the simplest kind of everyday spatial complexity. Space then seems a very clear case of what is transmitted through language not being expressible in language.

Defining Spatial Configuration

So how do we proceed with the problem of spatial non-discursivity? (For a more extended treatment see Hillier & Hanson 1984 and Hillier 1996) Prepositions, as little schemes of spatial relations with both abstract and concrete properties, offer a clue. Can we somehow extend this mix of abstract and concrete to more complex patterns? We suggest this is possible by defining spatial configuration formally as spatial relations which take into account other relations. We must begin by saying exactly what we mean by this. How can relations affect other relations?

In Figure 1 (overleaf) we show top left a two cell plan with an opening between two spaces, *a* and *b*. The relation of *a* and *b* is *symmetrical*, meaning that *a* is to *b* as *b* is to *a*—just as if *a* is *b*'s neighbour then *b* is *a*'s neighbour. We then introduce a third space, *c*—in fact the outside—and in the middle case link both *a* and *b* to *c*, but in the right case link *a* but not *b*, to *c*, so we must pass through *a* to get to *b* from *c*. The relation between *a* and *b* has not changed intrinsically, and remains exactly the same as it was, but if we consider the third space, in the middle case the relation between *a* and *b* with respect to *c* is still symmetrical, but in the right case is has become *asymmetrical*, in that we must pass through *a* to get to *b* from *c*, but we do not need to pass through *b*

Fig. 1

Plans

Justified graphs
d = depth to all other nodes
td = total depth to all other nodes

to get to *a* from *c*. So the presence of a third element has changed the relation between *a* and *b* with respect to that element. This is what we mean by spatial configuration, in contrast to the simpler concept of spatial relations. Elements acquire values by virtue of their relation to all other elements in the system.

We can show this more clearly through a simple but powerful representation, and one which played a critical role in creating space syntax: the 'justified' graph, as in the bottom line in Figure 1. This means that you select a node as a 'root', and align all those connected to it one layer above, all those two steps from it two layers above, and so on, with the effect that you can *see* the configuration. With this representation, we can easily see what turn out to be the two key configurational properties of space. The first addresses the question 'do you have to pass through intervening nodes to get from one node to another?' We can call this the *depth* or, in space syntax parlance, the *integration* property—the less spaces are deep from each other the more integrated they are, and the more accessible they are as destinations from all other nodes. The second is: are there different routes from one node to another, which we can call the *choice* property, and how likely is each node to be used on a route between any pair of nodes. Any choice of routes implies rings or cycles in the graphs, since in any graph without rings there will be exactly one route from any node to any other node. These will turn out to be

the critical social properties of space, and because we can see how to detect their presence in simple cases we can learn to measure them in complex cases. How much integration and choice is there in a particular graph? The resulting values will turn out to have social meanings and even social effects.

Showing Configuration

We can also show how the justified graph can make accessible more complex configurational properties to intuition, to *show* them, in Wittgenstein's terms. The top line in Figure 2 shows the plans of notional 8-cell houses. As plans it is difficult to say much about them apart from making a list of local relations. Of course, by living in the houses we would quickly intuit the configurational differences, and functional possibilities and inhibitions. But they would remain non-discursive. In the bottom line we see in the j-graphs from the outside immediately the configuration properties of depth and rings, so we can see how shallow or deep, ringy or tree like each graph is, more or less at a glance.

But we are still in the realm of seeing more clearly what can not be expressed in language. To take the next step, we must learn to measure the presence of these properties. This will lead us to the most fundamental property of graphs from a spatial point of view: graphs differ in the properties of integration and choice when seen from the points of view of different nodes
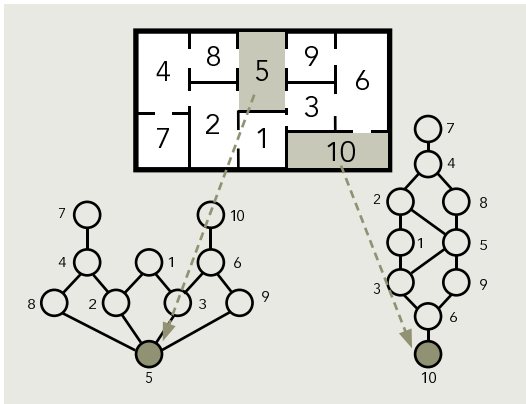

Fig. 2   Plans and j-graphs of 8-cell houses
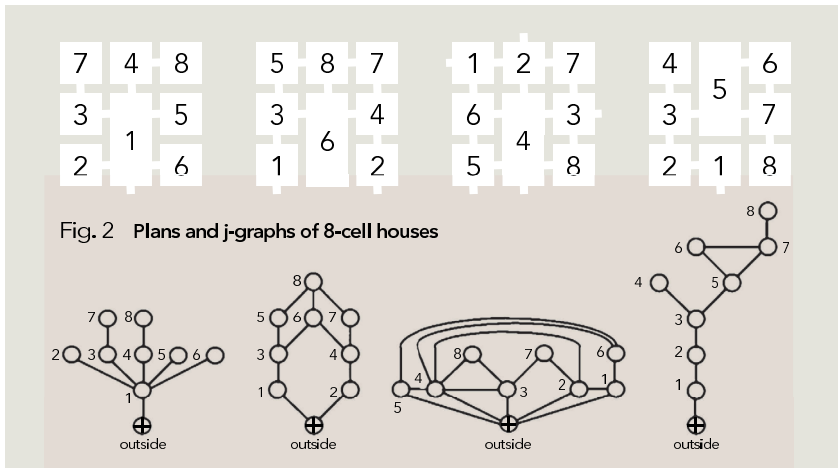
Fig. 3

**Two j-graphs illustrating different points of view.**

within the graph. Consider the two graphs in Figure 3. The two seem quite different in terms of the configurational properties we have described, but in fact the two are the same graph seen from two different points of view, namely spaces 5 (the shallow graph on the left) and 10 (the deep graph on the right). So we can say the graph is integrated from the point of view of space 5, and segregated from the point of view of space 10. This is the fundamental property of graphs that is exploited by buildings and cities, from the scale of the domestic dwelling to that of metropolitan Tokyo: graphs are different from different points of view.

To capture these differences means measuring the depth and choice values of the system when seen from the point of view of each space in the complex. These are essentially the familiar mathematical measures of *closeness* (depth) and *betweenness* (choice) (they were not familiar when we first developed them!), though with certain syntactic normalizations to allow comparisons of systems of different sizes. Once we have this we can give a spatial meaning to the pattern of functions in a building. For example, as in Figure 4, which represents the real case of a house in Normandy in France (Hillier et al 1987), we commonly find that a 'living room' or a 'kitchen' is not just a space with certain furnishings and implements, but also a certain configurational position in the house, and a certain way of relating to all other spaces. These differences can be clarified and demonstrated by assigning values to spaces which index how it is related to all the other spaces in the house. In this way, we can find a clear and culturally variable *spatial meaning* to the idea of *function*.

Fig. 4

Spatial aspects of a house in Normandy.
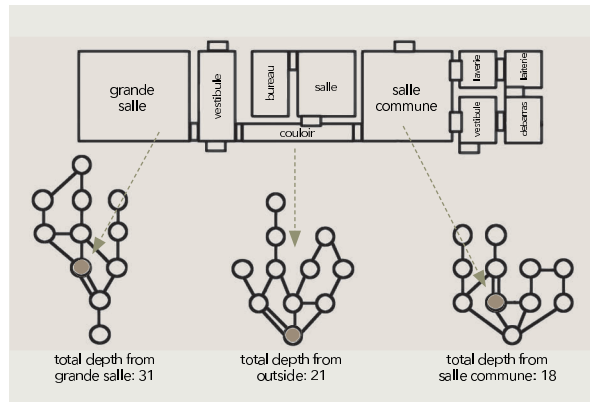
A *form-function* relation exists because function has been *realised spatially* through the positioning of the function in the layout as a whole. So ideas are seen to be objectively present in the layout itself, and its pattern of assigned functions, as well as in minds.

These values give us the configurational properties of individual spaces with respect to the whole, and this is sufficient to give a picture of functional patterns. If we may be permitted a little Wittgensteinian immodesty, this formulation solves a key theoretical problem in architecture. Although a leading element of the architect's trade is to match the spatial form of a building to its functioning, there is no theory to inform how this can be done other that of the most generalised kind. There is no design level theory capable of informing decisions about the relations between spatial form and functioning. A key reason for this is that most buildings are made up of the same kinds of spaces—room, corridors, courts, and so on—varying in size but not much else, so most functioning happens in a similar range of spaces. It was only by showing that the key properties of spaces were those which linked it to the pattern of space as a whole, that each space acquired properties which were distinctive to its function, as for example in the French house in Figure 4, a *salle commune* must be integrated into the plan to work as an everyday gathering space, while a *grande salle* needs to be segregated from everyday activity to preserve its identity as a special space. Space syntax shows in effect that it is the *extrinsic* properties of spaces which relate to function in a non-trivial way, because this is the way spaces acquire distinct identities in the plan.

But what we still lack is a picture of any kind of the pattern of the whole. Configurational value gives a picture of the whole from a certain point of view, but no discursive picture of the whole that we see when we look at the layout. At this level we have not translated the non-discursive into the discursive. However, we can take an important step in this direction by using another syntactic device: the translation of numbers into colours. By representing bands of numerical values as colours, and always from red (or 'hot) for strong values on any variable through to blue (or 'cold') for weak, the picture of the whole can be *shown* in Wittgensteinian terms, but still not of course described in any kind of language. Here then is a strong, simple and clear example of something that is an expression of the spatial relations that are to be found throughout language not being sayable *in* language, but only *shown*—in this case by a graphical trick!
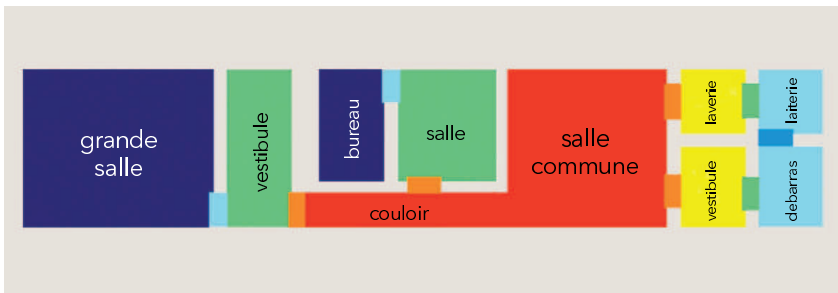

Fig. 5   **Warm and cold colours representing the total depth value.**

More Complex Representations

But so far we have looked only at the first, simple steps in space syntax. To take the next steps we need to consider how to represent space in order to make configurational calculations. So far we have used rooms as spatial elements. But cities, and indeed many buildings, do not have rooms, or anything that can be easily recognised as well-defined spatial elements. So how, in such cases, can space be represented to allow configurational computations?

To answer this we need a little philosophy! Through our education, we acquire the habit of seeing space in a Cartesian way as 'extension without
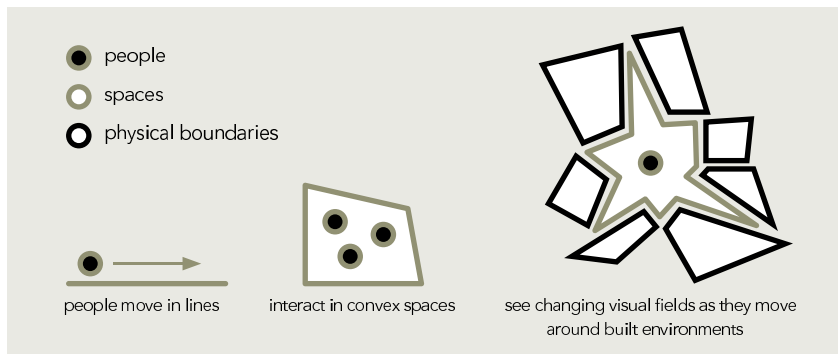
Fig. 6   **Shapes of human activity and experience.**

the object', and so as the *background* to objects, and in architecture we extend this to seeing space as the *background* to human activity. But as soon as we try to translate this idea into human space, all is lost. We are condemned not to understand it. The reason is simple. Space is *intrinsic* to human activity, not a background to it. Movement is fundamentally linear, whether we are leaving a room or crossing a city. Interaction is convex, because all interactors must be co-present. Our experience of space as we move about in buildings or cities is of strange jagged shapes we call *isovists*, and somehow we use these to put together an intelligible picture of where we are. Once we understand this, we see that we shape space in ways which reflect the different types of human activity and experience, and through this the space we make becomes humanised. This is where we have to begin if we want to understand space analytically.

We can now define space syntax as a set of methodologies for making configurational analyses of space represented in different ways as points, lines, convex spaces and isovists, and at different radii from each home element. Let us show you the power—including the predictive power—of these analyses. Visual integration analysis means taking a layout as a set of points at an arbitrarily fine scale, and calculating how many visual steps are needed to see all points in the layout from each point. In effect it analyses the whole pattern of visual fields in a layout and assigns integration values to the roots of each visual field. We then colour up from red for strong through the blue for weak as usual and so extract a *visual integration structure* from the layout.
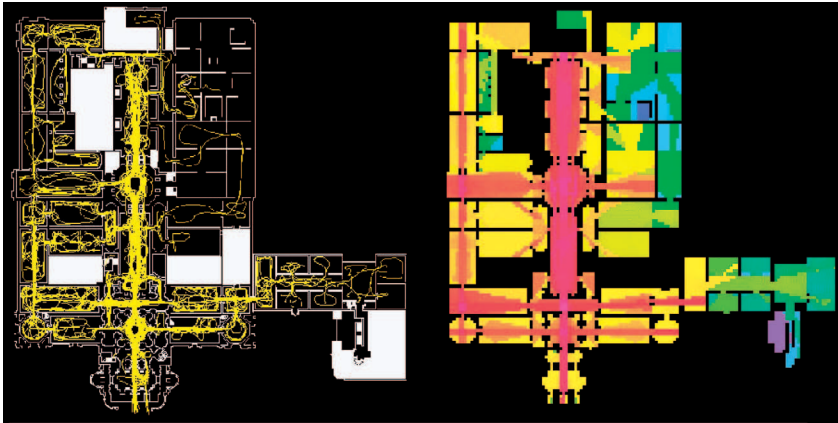
Fig. 7   Left: **Trace of 100 people entering**   Right: **Visual integration structure.**
        Tate Britain and moving for 10 minutes.

For example, Figure 7 shows the visual integration structure for the Tate Britain gallery, a structure which links galleries together and links the whole pattern to the entrance (we call it a shallow core). On the right are traces of visitors entering the gallery and moving about for ten minutes. Each visitor takes an individual path through the layout, but when we look at the aggregate of traces, we see that the pattern of traces and the structure of visual integration resemble each other strongly. The similarity of the two patterns can be checked statistically, and in fact something like 70 % of the differences in individual movement patterns can be accounted for by the visual integration structure of the layout. This leaves no doubt that visitors are using the spatial layout of the gallery, consciously or unconsciously, as their main navigational aid. The spatial form and functional pattern of the building ressemble each other, not through design intention but as an emergent effect.

Now reflecting on this case and that of the French house we can begin to see that space works in two ways. A spatial layout can *reflect* and *embody* a social pattern, as in the case of the French house, where space was laid out and categorised to give reality to a culturally given pattern of activity, and so reinforce and reproduce it. So we can perpetuate things about ourselves and our cultures by building them into space, and so making them seem inevitable and natural. We can call this the *conservative* use of space, since space is

being use to reflect and so reproduce a given social pattern by the control of copresence. But space can also *shape* a social pattern, as in the case of the Tate Britain movement study, since by shaping movement, space also creates a pattern of natural co-presence in space, and so potential encounter and potential social relation arising from co-presence. We can call this the *generative* use of space, since we are using space to create the potentials for new co-presence and potentially for social patterns. We will see in due course that this dual potential of space is one of the keys to understanding space in cities.

The Discovery of Spatial Laws

Once we have the concept of measuring configuration as the relations between all the spatial elements in the system and all others, then whatever representation we use, we can easily discover spatial laws. These laws do not take the form of universal behaviours, but govern the ways in which different types of spatial configuration arise from the placing and shaping of objects in space—which is what architects do.

For example, as Figure 8 shows, if we take a square object and place it within a bounded square space, and move the object from corner to centre edge and then to centre, with each step we *decrease* the degree of visual integration in the ambient space, as well as changing its pattern.

Fig. 8

Decrease of visual integration due to changing positions of a square object within a bounded square space.
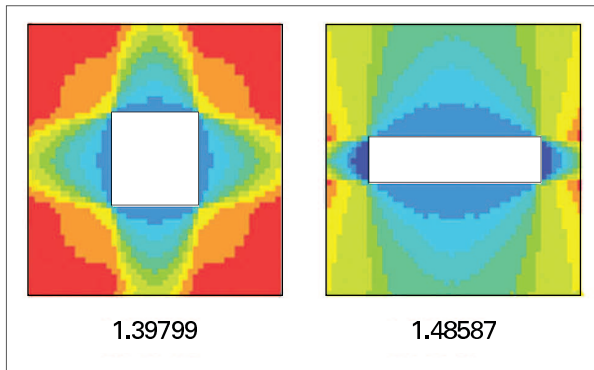
1.39799   1.48587
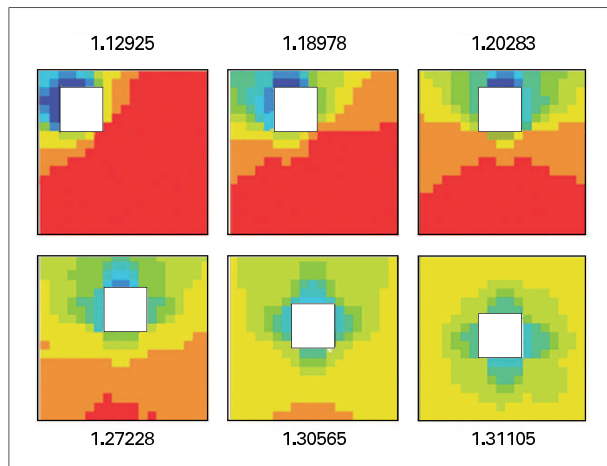
Fig. 9

Decrease of visual integration due to changing the shape of an object placed within a bounded square space.

We find the same with metric integration. The mean length of trips in the ambient space increases as we move the object from corner to centre. If we change the shape of the object from square to rectangular while conserving the area, as in Figure 9, we find again that visual integration decreases and mean trip length increases, and again the pattern changes:

These effects can be explained easily by means of a diagram—Figure 10—and some simple calculations. As we move a partition in a line of cells from centre to edge, the total *inter-visibility* from each cell to all others increases, though of course the total area remains constant. So both this and *inter-accessibility* effect arise from the simple fact that to measure either we need to square the numbers of points on either side of the blockage (Hillier 2009). All we need to know is that twice the square of a number, n, will be a smaller number than $(n\text{-}1)^2 + (n+1)^2$:

$$2n^2 < (n\text{-}1)^2 + (n+1)^2 \qquad\qquad (1)$$

We call these all-points-to-all-others-measures *configurational metrics*. We use them when instead of being interested in, say, the distance from *a* to *b*, we are interested in the distance, metric, visual or topological, from each point or element in the system to all others. So we see that the *configurational metrics* we use in real human space are not the same as geometry and in fact behave in a quite different, though still lawful, way.

These simple laws have some highly interesting effects. They mean that a large space and a small space are, from the point of view of intervisibility
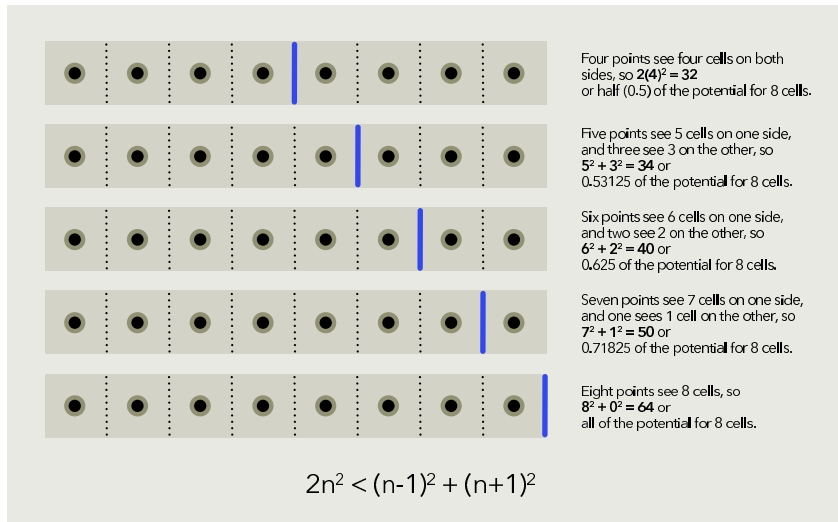
Four points see four cells on both sides, so $2(4)^2 = 32$ or half (0.5) of the potential for 8 cells.

Five points see 5 cells on one side, and three see 3 on the other, so $5^2 + 3^2 = 34$ or 0.53125 of the potential for 8 cells.

Six points see 6 cells on one side, and two see 2 on the other, so $6^2 + 2^2 = 40$ or 0.625 of the potential for 8 cells.

Seven points see 7 cells on one side, and one sees 1 cell on the other, so $7^2 + 1^2 = 50$ or 0.71825 of the potential for 8 cells.

Eight points see 8 cells, so $8^2 + 0^2 = 64$ or all of the potential for 8 cells.

$$2n^2 < (n-1)^2 + (n+1)^2$$

Fig. 10   **Increasing intervisibility from all points to all others as a partition is moved from centre to edge.**

within the spaces, 'larger' than two similarly sized spaces of equal total area, and that a long line and a short line are 'longer' than two lines of similar length and equal total length. They also mean that small blocks placed in the centre of a system are more integrating than small blocks placed on the edge of the system. All these lawful phenomena are found pervasively in cities. The laws are, we believe, intuitively known to human beings in the sense, as we said, of 'intuitive physics'. We feel the laws of physics in our minds—even in our arms—when we manipulate objects by throwing them or moving them. This could be shown by examining human spatial behaviour and by spatial experimentation (Hillier 2009). But the most powerful evidence that these laws are known to human beings comes from the degree to which evidence for them is found in the largest and most complex spatial objects that human beings make: cities. The fact is that in spite of the cultural differences we find in the forms of cities in different parts of the world, and in spite of the differences in the circumstances in which they are created, there is an underlying *universal city* made up of an astonishing range of invariants common to all cities.
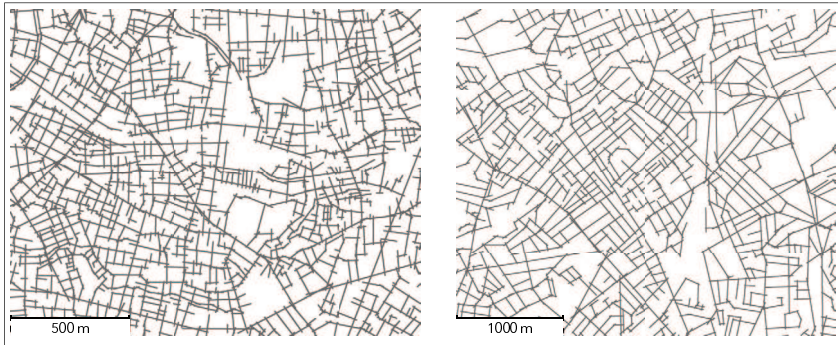
Fig. 11   **Least line maps of areas of Tokyo** (left) **and London** (right).

Cities as Spatial Configurations

To analyse a city spatially in syntactic terms means analysing its street network. But before we do so, simply by measuring and counting we can discover invariants. To illustrate this we can use arbitrarily selected areas of London and Toyko, both largely 'organic' cities, and about as far from each other in distance and socio-cultural background as it is possible for cities to be. By representing both as *least line maps* (the fewest lines that cover the network and make all connections) we find:

- that at all scales, from local areas to whole cities, cities are made up of a very small number of long lines and a very large number of short lines (Hillier 2002), so much so that in terms of the line length distributions in their least line maps cities have been argued to have scale-free properties (Carvalho & Penn 2004). This is just as true of more geometric cities such as Chicago and Athens, as it is for more 'organic' (meaning lacking obvious geometry) such as Tokyo or London,

- that in 'organic' cities (as defined above), the longer the line the more likely it is to be end-connected to another by a nearly straight connection (between about 5 and 25 degrees), creating sequences of such lines, which the eye instinctively identifies when looking at the map, and the shorter the line the more likely it is to intersect with others at near right angles, creating local clusters of such lines. In more geometrical cities,

a similar pattern can be found but with more often straight rather than nearly straight long lines,

- that through these metric and geometric regularities, cities' street networks acquire a *dual* structure, made up of a dominant *foreground network*, marked by linear continuity (and so in effect *route continuity*) and a background network, whose more localised character is formed through shorter lines and less linear continuity.

Applying syntactic measures to least line maps of cities—in this case treating the street segment between junctions as the spatial element and using least angle change, rather than simple metric distance as the measures of distance (since this has been shown to be how people read and move about in cities—see Hillier & Iida 2005), we bring to light further regularities. For example:

- by measuring least angle integration (normalised mathematical 'closeness'— or more simply the relative accessibility of each street segment from all others) analysis without radius restriction (so the most 'global' form of the analysis), a dominant structure is identified approximating the form of what we call a *deformed wheel*, meaning a 'hub' of lines in the syntactic centres, strong 'spokes' linking centre to edge and strong 'rim' lines. Figure 12, for example, shows the underlying deformed wheel pattern in both metropolitan Tokyo (with multiple rims) and London within the M25.
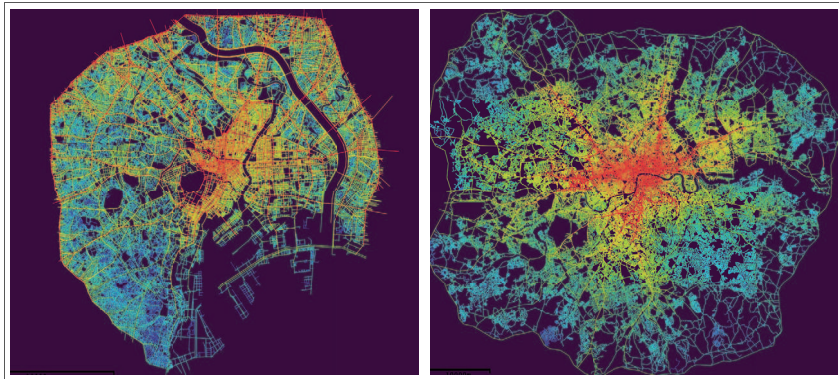

Fig. 12   **Least angle integration** (normalised closeness) **for metropolitan Tokyo** (left) **and London within the M25** (right) **in each case showing a variant of the 'deformed wheel' structure, with multiple rims in the case of Tokyo.**

Fig. 13   **Least angle choice** (betweenness) **analysis of Tokyo** (left) **and London** (right) **showing the network pattern in both cases.**

The syntactic measure of least angle choice (mathematical 'betweenness', or the potential of each segment for through movement on routes from all segments to all others) then commonly identifies a network spread through the system, though strongest in the more syntactically central locations (see Figure 13). In other words, in spite of the differences in socio-economic and temporal circumstances in which cities grow, they seem to converge on common generic forms which have metric, geometric and configurational properties. However the similarities between cities does not stop there. On close examination, for example:

- all cities seem to exhibit a property we call *pervasive centrality*, meaning that 'central' functions such as retail and catering concentrations diffuse throughout the network at all scales, from the city as a whole to the local network of streets. For example, Figure 14 is Mike Batty's image of the 168 largest centres in London within the M25. By comparing Figure 14 to Figure 13 we find a strong 'eyeball' correspondence. However, the image also makes clear that the global properties shown in the map are not sufficient in themselves to identify the location of centres. We typically find for example that along the length of a high global movement potential alignment we find the centre occurring only in certain locations. For example, if we take the Edgeware Road between the North Circular Road and Oxford street, there are three high streets with the rest fairly free of shops. In each case, the centre occur where local *grid intensification* (a dense and smaller scale local grid) co-incides

Fig. 14

**Mike Batty's map
of the 168 main centres
and sub-centres in London
within the M25.**
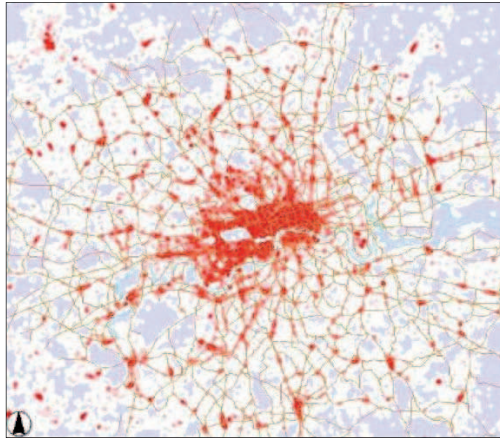
with the globally strong alignment. The pattern is far more complex than en-
visaged in theories of *polycentrality*. It is notable also that pervasive centrality
seems spatially sustainable because it means that wherever you are you are
close to a small centre and not far from a much larger one. (Hillier 2009)
If we reduce the radius of the measures we then find the—much more nu-
merous—smaller scale centres. For example, at radius 750 metres, all of the
'urban villages' in a section of north west London are picked out in red.

Fig. 15

**Least angle through
movement potential at a
radius of 750 metres
in an area of north west
London with the
dark red lines picking out
the urban villages.**
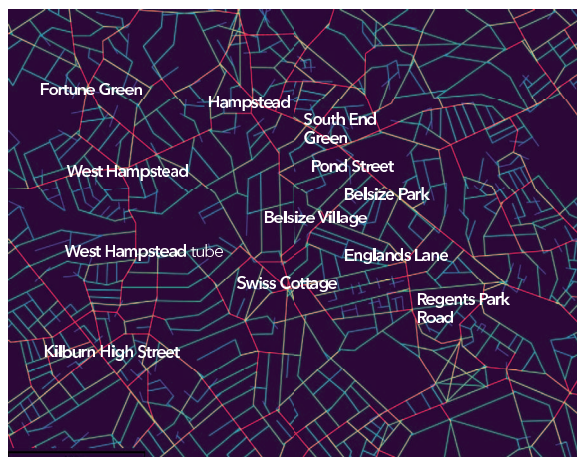
The regularities that we find in cities with least angle analysis suggest a new definition of the city. Cities of all kinds, however they originate, seem to evolve into *a foreground network of linked centres at all scales, from a couple of shops and a café through to whole sub-cities, set into a background network of largely residential space*. The *foreground* network is made up of a relatively small number of longer lines, connected at their ends by open angles, and forming a super-ordinate structure within which we find the *background* network, made up of much larger numbers of shorter lines, which tend to intersect each other and be connected at their ends by near right angles, and form local grid like clusters. We suggest this is the proper generic definition of what a city is as a large object.

The Dual City of Economic and Social Forces

So what forces give the city this shape. We believe the answer lies in two key new phenomena which research using space syntax has brought to light. The first we call *spatial emergence*: the network of space that links the buildings together into a single system acquires emergent *structure* from the ways in which objects are placed and shaped within it. As we have seen, this process is law-governed, and without an understanding of these laws the spatial *form* of cities cannot really be deciphered. The second phenomenon is *spatial agency*: the emergent spatial structure *in itself* has lawful effects on the functional patterns of the city by, in the first instance, shaping movement flows, and, through this, emergent land use patterns, since these in their nature either seek or avoid movement flows. Through its influence on movement, the urban grid turns a collection of building into a living city. Movement is literally the lifeblood of the city. It is these two linked processes of *spatial emergence* and *spatial agency* that set in train the self-organising processes through which cities acquire their more or less universal spatial form.

What then drives these processes? In fact, the universal functional structure associated with the dual network suggests the answer. Within the envelope created by cognitive constraints—the need for the city to be intelligible in order to be usable at all (see Hillier 2009)—we can now see how economic and social forces put their different imprints on the city. The foreground structure, the network of linked centres, has emerged to maximise grid-induced movement, driven by micro-economic activity. Micro-economic activity takes

Fig. 16

The old city of Nicosia showing the deformed wheel core in spite of culturally differentiated residential space.
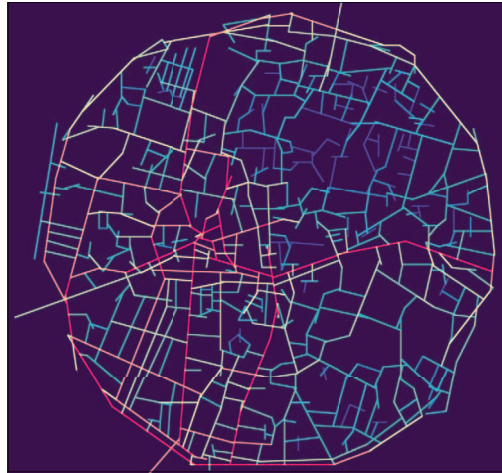
a universal spatial form and this type of foreground pattern is a near-universal in self-organised cities. The residential background network is configured to restrain and structure movement in the image of a particular culture, and so tends to be culturally idiosyncratic, often expressed through a different geometry which makes the city as a whole look spatially different. The first is an instance of the *generative* use of space, since it aims to generate co-presence and make new things happen, and the second *conservative* since it aims to use space to reinforce existing features of society. In effect, the dual structure has arisen through different effects of the same laws governing the emergence of grid structure and its functional effects. In the foreground space is more random, in the background more rule-governed, so with more conceptual intervention.

We can illustrate this most clearly in a city with more than one culture (now unfortunately separated): Nicosia (Figure 16). Top right is the Turkish quarter, bottom left the Greek quarter. Their line geometry is different. In the Turkish quarter, lines are shorter, their angles of incidence have a different range, and there is much less tendency for lines to pass through each other. Syntactically, the Turkish area is much less integrated than the Greek area. We can also show that it is less intelligible, and has less synergy between the local and global aspects of space. Yet in spite of these strong cultural differences in the tissue of space, we still find Nicosia as a whole is held together by a clear deformed

wheel structure. This shows how micro-economic activity spatialises itself in a universal way to maximise movement and co-presence, while residence tends to reflect the spatial dimension of a particular culture, and the expression is in the first instance geometrical. Since residence is most of what cities are, this 'cultural geometry' tends to dominate our spatial impressions of cities.

Two Wittgensteinian Remarks

So we seem to have a theory of the city which links form to function, micro-structure to macrostructure, and differentiates between socio-cultural and micro-economic factors in creating the spatial form of the city. Have we then succeeded in our aim of creating a science of the non-discursive for space, and so emancipated ourselves from Wittgenstein's strictures? There are two key issues. Does space have a formal unity (since there cannot be a science of space if it does not)? And have we described structure and function in spatial phenomena (since without this we cannot make predictions from theory)? Two Wittgensteinian remarks seem in order.

The first concerns the relation between the microeconomic and cultural aspects of urban form, as found in the foreground and background networks. The former seems to reflect the universals of human micro-economic behavior, the latter the relativity of human cultural behavior. Should we then see the 'language of space' as being composed of spatial games, or does it have 'formal unity' (to use Wittgenstein's own expression in the *Untersuchungen*)?

In our view, the evidence indicates strongly that there is a single language of space and that it has formal unity because it is based on the underlying laws we have described. We would go farther and suggest that it is because the language of space is a formally unified language that differentiated cultural expression is possible. For example, the spatial and functional differences between the foreground and background grids arise because the laws governing the emergence of spatial patterns through the placing and shaping of buildings, and those linking patterns of space to patterns of movement and co-presence, are being used in different parts of the network to create different functional outcomes. Both are in this sense the expression of the same laws, but used in one case to maximize movement and co-presence and in the other to restrict and structure it.

The same applies to the difference between background grids that we find in different cities, and even in the same city (as in the case of Nicosia). The

same spatial means are used to give different degrees and patterns of relative segregation. To put it at its simplest, the spatial language which permits the creation of patterns of integration also permits the creation of patterns of segregation. And because the language is unified in fact, we can use the same methodology to give parametric characterisations of different spatial cultures (Hillier 2002). Different cultures, in effect, are using the same language to say different things, rather than using different languages. We must conclude then that space as a language does have the formal unity that the later Wittgenstein denied for natural language.

The second Wittgensteinian remark concerns the near-invariant structures for cities which space syntax has brought to light. In what sense are these scientific entities? Here we do after all encounter a severe Wittgensteinian problem. Although the structures that we see by 'colouring up' maps undoubtedly exist, in that they are in reality patterns of mathematical values whose correspondence with reality can be tested by observing and correlating them with function, there is no scientific way to describe the pattern itself. What we can do mathematically, of course, is to take the set of values and examine their distribution *statistically*. In this way we can discover, for example, that the distribution of the length of lines of sight and access in all cities is scale-free, meaning that at whatever scale we examine space, from a small area to the whole city, we will always find a fractal pattern with few long lines and many short lines. But analyses of this kind are statistical in nature and so operate in an abstract space, and make no mention of the actual patterns of connections of different kinds of line which give the city its structure. We can only *show* these patterns and wave our hand at them, and hope our audience will agree with the—usually metaphorical, such as 'deformed wheel'—way we describe them. Remarkably, we are in effect in the same kind of mental position we were in with the word 'between'. We can point to it and show it, but we cannot describe it within the formal language we have developed for space.

This is in fact a fundamental problem in the 'sciences of complexity'. Everywhere in nature we find complex processes which generate simple emergent structures which then 'forget' the complexity of their creation and operate as autonomous relatively systems at the emergent level. As Cohen and Stewart (Cohen & Stewart 1993) argue: 'We must … explain why, on every level of existence, we can deal with the world as though it were simple.' In addressing complexity, they argue, science is asking the less interesting question. 'The interesting question is precisely the opposite … where does the simplicity

come from?' These simple (in comparison to the processes that create them) emergent structures can be generated and identified, and their interactions studied, but their structures cannot yet be described in a rigorous scientific way.

The spatial structure of cities is a singularly clear example of this. The city structure evolves through a step by step bottom up process which we can more or less describe. But because the pattern of movement flows the structure generates reflects the global patterns of interconnectedness of spaces, not their local or intrinsic properties, we are compelled to acknowledge that the structure of the city works top down to set in train the functional processes which turn collections of buildings into living cities. Like so many emergent phenomena in nature, then, the structure of the city emerges bottom up, but works top-down. We can if we wish describe this structure by listing all the elements and connections, but this only re-describes the problem. It does not characterise the structure in such a way as to show what it is about this structure that *works*. So the structure that emerges and creates the living city we can only point to and say 'look'. We are back, it seems, to the *Tractatus*!

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