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Mathematics and Forms of Life

Abstract

According to Wittgenstein, mathematics is embedded in, and partly constituting, a form of life. Hence, to imagine different, alternative forms of elementary mathematics, we should have to imagine different practices, different forms of life in which they could play a role. If we tried to imagine a *radically* different arithmetic we should think either of a strange world (in which objects unaccountably vanish or appear) or of people acting and responding in very peculiar ways. If such was their practice, a calculus expressing the norms of representation they applied could not be called false. Rather, our criticism could only be to dismiss such a practice as foolish and to dismiss their norms as too different from ours to be called 'mathematics'.

1. Mathematics as Grammar

What is the meaning of an arithmetical equation, such as '7 + 5 = 12'? Does it describe relations between abstract objects, numbers, as Platonists believe? Or is it rather, as John Stuart Mill held, a well-confirmed empirical hypothesis to the effect that adding five objects to seven objects will always produce a total of twelve objects? Neither, according to Wittgenstein, who rejects the very assumption on which both opposing schools are agreed, namely that arithmetical equations, like all other declarative sentences, must be *descriptions* or *statements of fact* of some sort. Wittgenstein suggests that it is more appropriate to regard them as norms of representation, something like grammatical rules, not describing anything, but merely fixing the meaning of certain words that are then usefully applied in empirical sentences (*LFM* 33):

> If you know a mathematical proposition, that's not to say you yet know *anything*. I.e., the mathematical proposition is only supposed to supply a framework for a description. [*RFM* 356f]

In that respect mathematics would be just like logic, which is also devoid of any factual content. As Wittgenstein puts it in the *Tractatus*: 'the propositions of logic say nothing' (*TLP* 6.11), they 'are not pictures of reality' (*TLP* 4.462).

Equations, like propositions of logic, are rules of inference. Thus, '7 + 5 = 12' licences an inference from 'There are seven boys and five girls in the class' to: 'There are twelve children in the class'. Inference rules are not descriptions, they are conceptual tools we use when giving descriptions. They can be compared to a tape measure, which does not itself make an assertion, but defines concepts (e.g. cm) that we use in descriptions (cf. *RFM* I §5).

While logic expresses very general and ubiquitous norms of our language, arithmetic regulates only a certain part of our language: it fixes and develops the grammar of number words and their use in determining quantities of various kinds.[1] But in as much as using a language constitutes a form of life (*PI* §§19, 23), elementary mathematics, as part of our everyday language, constitutes an aspect of our form of life.

2. Conventional Truths

But, one may object, do logical and mathematical propositions not also express truths? Is it not true that 'p & (p → q)' entails 'q'; and is it not true that 7 + 5 = 12? – Certainly, we call such sentences 'true', but on Wittgenstein's view they are merely *conventional* truths, comparable to:

(1) One metre is 100 centimetres.

[1] For further discussion of Wittgenstein's key idea that mathematics is grammar see Schroeder 2014.

Or, with respect to the game of chess:

> (2) A bishop moves only diagonally.

Yet what is merely conventional could have been otherwise. One metre could have been subdivided into 120 centimetres. Or, as Wittgenstein suggests, our footrules could have been made of very soft rubber, so that the results of our measurements would be much more variable (*RFM* I §5). But is it really conceivable that we might use a different logic or arithmetic? One may be inclined to object that whereas different units of measurement are of course always possible, there can be no alternative to our logic and arithmetic. A logical calculus that allowed one to infer 'p' from 'p v q' would just be flawed; likewise an arithmetic containing the equation '7 + 5 = 13'. Yet on Wittgenstein's view such different logical or mathematical systems could not be rejected as *false*, although they may well be impractical. Is that a tenable position? Are drastically different rules of logic or arithmetic even conceivable?

3. Logical Inference and Measurement

Wittgenstein repeatedly compares logical inference to a translation from one unit of measurement to another.

> Compare saying that one thing follows from another with changing the unit of measurement … "It has 30 cm, therefore it has so-and-so many inches." [*LFM* 200f.]

In fact, at one point he presents transforming the unit of a measurement as an actual example of a logical inference:

> What we call 'logical inference' is a transformation of our expression. For example, the translation of one measure into another. One edge of a ruler is marked in inches, the other in centimetres. I measure the table in inches and go over to centimetres *on the ruler*. [*RFM* I §9]

This is surprising, as the translation is not an a priori one: it does not appear to be based on the meanings of the terms alone (as when I make a calculation based on the definition that 1 inch = 2.54 cm). It is like working out that 17 + 56 = 73 by counting buttons (first, 17, then another 56, then the lot), instead of doing the sum. It wouldn't be an arithmetical calculation, but an experiment (cf. *RFM* I §37). Perhaps what Wittgenstein has in mind here is that the double-edged ruler serves as a canonical sample: providing a kind of ostensive definition or paradigm of the relation between the two units (cf. *RFM* 430c). Thus, if both 'inch' and 'cm' are taken to be defined by this particular ruler, the correlation of, say, 12 inches and 30.48 cm read off it, can be said to be derived from the definition of those terms.

In any case, deriving an object's length in cm from its length in inches is certainly an inference of one statement from another. The result is not really a new piece of information, but only a different way of presenting the same piece of information (say, the length of the window). Just as '~(p & q)' is the same piece of information as its logical implication '~p v ~q', only expressed differently. This is the point of Wittgenstein's remark: Logic is not a source of new knowledge, but merely a technique of expressing (parts of) the same claims in different ways.

However, in *RFM* I §5 logical inference seems to be compared not with transformation of the expression of a measurement, but with measuring itself. Inferring according to different rules of logic would be like measuring with different kinds of rulers:

> … What would happen if we made a different inference – how should we get into conflict with truth?
>
> How should we get into conflict with truth, if our foot rules were made of very soft rubber instead of wood and steel? – …[*RFM* I §5]

In what way is measuring like an inference? After all, measurement is not the transformation of an expression, but the production of an expression, a certain description of reality (e.g., 'The book is 7 inches long'), from scratch. Perhaps it could be seen as an inference of the following form:

> The distance between two successive lines on this ruler is 1 inch.
>
> Laid alongside this ruler the book reaches from the first to the eighth line.
>
> Therefore, the book is 7 inches long.

Or, for short:

> This book reaches from the first to the eighth line on this ruler.
>
> Therefore, the book is 7 inches long.

If due to the material of the ruler the distance between the lines is different on different occasions (relative to our customary rulers), we are inclined to dismiss this result as unreliable.

Wittgenstein, however, questions whether we are indeed entitled to find the results of such unconventional measurements likely to be false:

> How should we get into conflict with truth, if our foot rules were made of very soft rubber instead of wood and steel? – "Well, we shouldn't get to know the correct measurement of the table." – You mean: we should not get, or could not be sure of getting, *that* measurement which we get with our rigid rulers. So if you had measured the table with the elastic rulers and said it measured five feet by our usual way of measuring, you would be wrong; but if you say that it measured five feet by your way of measuring, that is correct. – "But surely that isn't measuring at all!" – It is similar to our measuring and capable, in certain circumstances, of fulfilling 'practical purposes'. (A shopkeeper might use it to treat different customers differently.) [*RFM* I §5b]

In the cinema, Wittgenstein once saw Eddie Cantor in the film *Strike me Pink* (1936) play a crafty shopkeeper that uses an elastic yardstick, extending it when measuring out a piece of cloth for a customer, but shrinking it when cutting the cloth (thus delivering less than the customer is expecting to receive) (Rhees 1970: 121-2).

Or again, a ruler may be made of a material that dramatically changes in extension at different temperatures:

> If a ruler expanded to an extraordinary extent when slightly heated, we should say – in normal circumstances – that that made it *unusable*. But we could think of a situation in which this was just what was wanted. I am imagining that we perceive the expansion with the naked eye; and we ascribe the same numerical measure of length to bodies in rooms of different temperatures, if they measure the same by the ruler which to the eye is now longer, now shorter. [*RFM* I §5c]

Such a ruler could be useful if its changes were in sync with the changes of the objects to be measured (Rhees 1970: 121). This is comparable to the way we identify shades of colours by colour samples: The samples look very different in different illumination, but so do the surface colours with which we compare them, so that in spite of their changeability those samples serve us well. Thus, if we imagine having to identify aluminium sticks of the same length at drastically different temperatures, a tape measure with the same thermal expansion would conveniently allow us to ignore the changes in temperature.

Wittgenstein's point is that the measurements with such a changeable ruler should not be regarded as likely to be *false*. They appear (probably) false only if we misunderstand them as an incompetent attempt at our ordinary technique of measuring. As long as we bear in mind that the reading on the flexible ruler is not to be taken as an object's length according to our normal standards, those discrepant results will not be called false, but at worst uninteresting. Each kind of ruler defines a unit of measurement we may or may not find useful. In the latter scenario, the ruler defines a unit of length that is dependent on the temperature, whereas in the shopkeeper's scenario the unit must be understood to be to some extent variably determined by the person measuring.

4. Crispin Wright's Objection

Crispin Wright, in his discussion of *RFM* I §5, sides with the interlocutor's unsympathetic reaction: "But surely that isn't measuring at all!" He argues that we have a concept of length as a perceptible property that measurement is to determine more precisely. Hence our measurements, in order to deserve that name, must be roughly in agreement with our observational assessments, whereas the readings of such a soft ruler may differ wildly from our visual impression of an object's length. Moreover:

> It is a feature of the concept of measuring that an accurately measured object will yield distinct readings at distinct times only if *it* changes; so much is implicit in the notion that measuring is to ascertain a property of the object measured. [Wright 1980: 58]

Yet, the shopkeeper wielding a soft ruler is supposed to produce different 'measurements' for different customers (or at different points during the sales process), knowing full well that the goods themselves have not changed. Indeed, such a shopkeeper's cunning use of a soft ruler would presuppose "some concept of sameness of length other than is determined by soft-ruler measurement": otherwise he couldn't know that he treated different customers differently. So the soft ruler would not define a new concept of length, but only be an unsuitable means for ascertaining length (Wright 1980: 60).

But is it really true that acceptable measurements of an object must never vary unless the object itself has changed? Frequently in everyday life we only need a rough idea of an object's length, so that measuring procedures that we know to be imprecise (such a using one's stretched out index finger and thumb as a unit) are quite sufficient. But then it's easily conceivable that some unsophisticated people may rely entirely on such makeshift means of measurement. It is also worth remembering that in the past units of measurement were often defined in terms of parts of the human body regardless of differences among individuals. Thus an ell was defined as the length of a man's arm from the elbow to the tip of the middle finger, which would obviously vary considerably from man to man.

As for the correlation between measurement and visual assessment, two things can be said in defence of Wittgenstein's examples: First, a flexible ruler would not yield entirely random results. It can be stretched only to a certain extent, and we can imagine it to be used in a way that when two comparable objects are to be measured one is not supposed to handle it in a very different manner. Secondly, it is quite a common thing for our measurements to contradict and revise our previous visual impressions. So we don't expect a rigorous correlation between the two anyway.

Finally, is the shopkeeper's use of a flexible ruler logically parasitic on a more rigorous concept of measurement? That depends on how we imagine the case. We could of course imagine that the ruler in question could be used in two distinct ways: either like an ordinary tape measure or stretched out to give smaller measurements at will. In that case, it would be more natural to say that the shopkeeper had an ordinary, tolerably accurate ruler – only that it could also be misapplied in a deceitful way by stretching it. But imagine the scenario as follows: The ruler is such that one cannot use it at all without stretching it; in its unstretched state it is all rolled up and so compressed that its markings cannot be distinguished. So even an unbiased use – without any intention of influencing the result in one direction – depends on the force exerted by the user and so we "could not be sure of getting *that* measurement which we get with our rigid rulers". – It is true that where such a ruler is used in an intentionally biased manner the user must be aware of stretching it more or less than he would normally have done: so he must be aware of the distinction between, say, '10 inches' of cloth measured stingily, generously, or without any bias. But it doesn't follow that he must have a "concept of sameness of length other than is determined by soft-ruler measurement", since even his most fair-minded measurements are carried out with that soft and variable ruler.

Of course, an independent concept of the sameness of length is very likely to arise wherever it is possible to juxtapose different objects measured. Thus two pieces of cloth measured out as equal in length may appear very different in direct comparison, or vice versa. Hence, a more suitable example for Wittgenstein's purposes might be a device for measuring land, or a technique for measuring the length of time (e.g. by reciting a certain sequence of words).

In any case it should be noted that Wittgenstein doesn't altogether contradict his interlocutor's (and Crispin Wright's) outraged objection that "surely that isn't measuring at all!". He contents himself with saying that it is at least 'similar to our measuring' – even if it falls short of the more demanding concept of measuring spelt out by Crispin Wright. It is after all, like measuring, a systematic way of answering questions as to objects' length and we can imagine people having such a practice, call it what you will.

5. Deviant Logical Inference

Now we must consider the intended analogy between such deviant ways of measuring (or quasi-measuring) and different kinds of logical inference. Granted that a deviant measurement is not false if we understand it to be framed in an altogether different unit to be defined by that practice (not inches, but flexi-inches, so to speak), it is not clear that we can make a similar move with regard to an inference rule allowing the transition:

(3) p v q ⇨ p.

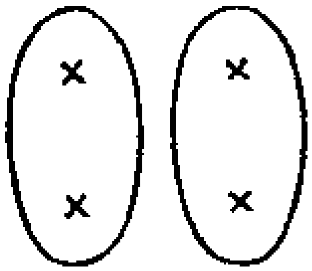
For what is to correspond here to the new unit of measurement? On an ordinary reading, this inference is invalid since it is possible for the premise (‘p v q’) to be true while the conclusion (‘p’) is false. Wittgenstein’s analogy suggests that we avoid this verdict by insisting that the conclusion ‘p’ must be taken in a different sense. But what sense could that be? – Presumably, the conclusion must be taken as a different kind of statement, perhaps one that does not commit itself to the truth of what is said, but puts it forward only as a likelihood. Given, that two possibilities have been envisaged (‘p v q’), it may be deemed licit to derive either *as a reasonable conjecture*. In that sense, ‘p’ can be said to ‘follow’ from ‘p v q’.

“But surely”, one may want to object, “that isn’t logical inference at all!” And Wittgenstein’s reply would be, as in the analogous case of measuring: It is similar to our logical inferences and capable, in certain circumstances, of fulfilling ‘practical purposes’. Of course, if we insist that a logical inference must be ‘truth-preserving’, such an alternative logic would not be acceptable; but we may not so insist. And as long as we know what we’re doing, adopting a logic of reasonable conjectures (some of which may well turn out to be false) does not get us into conflict with truth. After all, even writing fiction doesn’t get you into conflict with truth, as long as you’re not confused about the role and significance of your words.

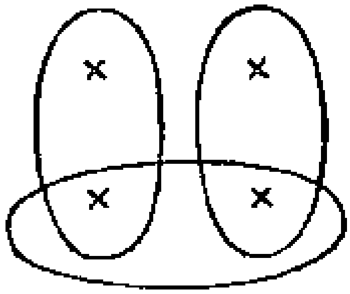
6. Deviant Mathematics

What about deviant bits of mathematics? Let us consider the following rather odd exchange:

“You only need to look at the figure



to see that 2 + 2 are 4." – Then I only need to look at the figure



to see that 2 + 2 + 2 are 4. [*RFM* I §38]

What's going on here? The case can be seen as a variant of the deviant pupil scenario from §185 of the *Philosophical Investigations*. Just as probably all of us will continue the series +2 after 1000: 1002, 1004, 1006; – and not: 1004, 1008, 1012 – we will naturally accept the first drawing as a demonstration that 2 + 2 = 4. Or to put it differently: when asked how many pairs there are in 4, we will answer 2 and the first diagram could serve us as a proof. The deviant pupil, however, may claim to have found three pairs in 4, as shown in the second diagram.

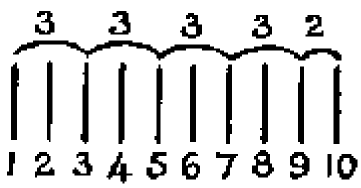
Of course we will remonstrate that one mustn't count the same thing twice: as belonging to more than one pair, but the deviant pupil will not be impressed. "I thought that was how I was meant to do it." Perhaps he'll argue that the pairs must after all be connected, like the links in a chain.

Wittgenstein's theme, here as in the familiar rule following scenario, is the idea of compulsion. Of course we accept the former figure as a proof. "But could I do otherwise? Don't I *have* to accept it?" – Well, we do accept it, quite emphatically, and so we use the emphatic expression "I have to admit this" (*RFM* I §33). – But are we not *forced* to accept it? – Are we? Not really, is Wittgenstein's reply (*RFM* I §51). After all, this is only a *picture*, and how can a mere picture contain an obligation (*RFM* I §55)?

7. Rule-Following and Mathematics

As in the case of rule following, we cannot compel people to play along if they stubbornly insist on a deviant understanding. However, whereas in a simple case of rule following (continuing a series of symbols) that is the end of the matter, in mathematics things would appear to be different. The natural response is that 2 + 2 + 2 = 4 is just false; and if our proof cannot convince the deviant pupil, nature will. For applications of this bizarre equation will constantly result in disappointment. (Buying 4 biscuits will not allow you to give Tom, Dick and Harry a couple each.)

For another example of such an aberrant calculation, Wittgenstein considers the possible results of an application:

> Imagine someone bewitched so that he calculated:
>
> 
>
>
> i.e. 4 × 3 + 2 = 10.
>
> Now he is to apply this calculation. He takes 3 nuts four times over, and then 2 more, and he divides them among ten people and each one gets *one* nut; for he shares them out corresponding to the loops of the calculation, and as often as he gives someone a second nut it disappears. [*RFM* I §137]

Under such mysterious circumstances – or, in such a strange world, where nuts disappear for no apparent reason – the bizarre equation '4 × 3 + 2 = 10' can be applied successfully.

Suppose – which is of course extremely unlikely – that all of us are bewitched in that way, and that due to the strange behaviour of nuts and other countable things we rarely get into trouble when applying the equation. That is the kind of situation that Wittgenstein envisages in the following remark:

> Imagine the following queer possibility: we have always gone wrong up to now in multiplying 12 × 12. True, it is unintelligible how this can have happened. So everything worked out in this way is wrong! – But what does it matter? It does not matter at all! – And in that case there must be something wrong in our idea of the truth and falsity of arithmetical propositions. [*RFM* I §135]

In what way do these far-fetched possibilities shed light on the truth or falsity of arithmetical propositions? – They are intended to rebut both the Platonic picture (according to which $12 \times 12 = 144$ is a super hard metaphysical truth) and the empiricist picture (according to which $4 \times 3 + 2 = 10$ is false, because nuts and things do not disappear like that).

First, the rule-following considerations undermine the idea of a Platonic realm in which all the steps are already taken. Following a rule, manipulating symbols according to a given recipe, is not spelling out what in some sense is already – and eternally – there. It is essentially a human activity: based on the stability of our attitudes and inclinations. The logical compulsion we experience in applying a rule this way and not that way is man-made: it is our own inexorable insistence on proceeding in one way rather than another (*RFM* I §118c). Socially speaking, it is compulsion in so far as we reject deviant behaviour: we shall not call that 'following the rule', 'adding', 'inferring' etc. (*RFM* I §§116, 131, 133). But then, where there is no deviancy: where we all agree to apply the rule in a certain way, there cannot be a mistake, for that would require the kind of Platonic standard of correctness Wittgenstein rejects.[2]

Hence, when we imagine the kind of weird behaviour of the deviant pupil to be constantly *our* behaviour – which in that case of course wouldn't appear weird to us – it can no longer be called erroneous.

[2] Note, however, that this does not mean that communal agreement is our criterion of correctness. Rather, beyond appeals to the rule – which in the envisaged cases of deviancy fail to settle the dispute – there *is* no further criterion of correctness. There are only brute facts about what we *take* to be correct. That is exactly one of the conclusions Wittgenstein draws from the case of the deviant pupil: that we cannot *prove* him to be wrong (cf. *RFM* I §§113, 115). For saying that we *all* do it like that – is not a proof.

So much for rule-following and the lack of Platonic underpinnings. But then, mathematics is more than producing formulae according to the rules of a calculus. Producing lines of patterns according to the rule 'Two strokes, one circle' (perhaps when producing wallpapers) is not mathematics (*LFM* 34). As Wittgenstein puts it:

> It is essential to mathematics that its signs are also employed in mufti. It is the use outside mathematics, and so the *meaning* [or significance] of the signs, that makes the sign-game into mathematics. [*RFM* 257]

Now this may be in need of qualification. It is certainly not true that every mathematical *proposition* or formula has an application outside mathematics. For one thing, the link with applications may only be indirect. A line *in the middle* of a lengthy mathematical proof is obviously not meant to be applied, but only a step on the way to something that may be applicable. And even the end result of a proof may be intended for further inner-mathematical work, rather than an application in engineering, say. For another thing, even an indirect outer-mathematical usefulness of a formula may only be a vague hope, rather than a clear intention. Even so: even if the link between a given piece of mathematics is typically only indirect and often just a vague future possibility, it can be argued that if this practical dimension was removed entirely, it would be less clear that we would still be concerned with mathematics, rather than something more like a chess puzzle: the investigation of logical relations within a mere game.

Moreover, as the examples of *RFM* I make clear, Wittgenstein's concern (at least from 1937 onward) was for the most part with elementary mathematics, with what he once called 'the beginnings of mathematics' (MS 169,36v; 1949). That is, he aimed at clarifying an aspect of ordinary language: arithmetic and geometry, as we all learn it at school. And at least of this most common and important part of mathematics it is certainly true that its symbols and rules are essentially geared towards applications outside mathematics.

So, for an assumed mistake in the way we calculate 12×12 (or: $4 \times 3 + 2 = 10$) not to matter it's not enough that we (all) find the calculation convincing (even when checking it again and again); we also have to find it applicable. And the strange scenario of *RFM* I

§137 illustrates how that could be: In a world in which things vanish unaccountably, but regularly, it might be practical to have a rather odd system of arithmetic, or even of counting (cf. *RFM* I §140).

8. Arithmetic and Experience

But note the difference between Wittgenstein's account and an empiricist one:

The empiricist view of arithmetic is that equations are general claims about the results of adding or taking away physical objects:

> Put two apples on a bare table, see that no one comes near them and nothing shakes the table; now put another two apples on the table; now count the apples that are there. You have made an experiment; the result of the counting is probably 4. [*RFM* I §37]

On the empiricist view, this and similar experiences lead us to the general claim that 2 objects added to 2 objects always result in 4 objects, which is expressed by the equation: 2 + 2 = 4. Thus, an equation is a universal claim about countable objects that would be refuted if we ever encountered an exception: a case of adding 2 things to 2 things resulting in only 3 things, say.

Wittgenstein, however, realised that that is not the way arithmetic relates to experience:

> If 2 and 2 apples add up to only 3 apples, i.e. if there are 3 apples there after I have put down two and again two, I don't say: "So after all 2 + 2 are not always 4"; but "Somehow one must have gone". [*RFM* I §157]

That is to say, when on the basis of my experience with nuts and apples etc. I put forward the sum '2 + 2 = 4' it is not presented as an empirical generalisation, but as a rule or norm of representation, which means that it is made immune from empirical falsification. We will insist on it even where experience seems to contradict it: in that case we declare our experience as inaccurate. We say that something else must have happened that we didn't see.

However, even though no conflicting experience can *falsify* a mathematical proposition, repeated conflicting experiences can undermine its usefulness. If putting together pairs of apples frequently resulted in more or fewer than 4 apples, we would have to say that our arithmetic was not applicable to apples; as, in fact, it is not applicable to drops of water (one and one make one) or measures of different liquids (one quart of alcohol and one quart of water yield only 1.8 quarts of vodka).

> … But if the same thing happened with sticks, fingers, lines and most other things, that would be the end of all sums. "But shouldn't we then still have 2 + 2 = 4?" – This sentence would have become unusable. [*RFM* I §37]

It would not be false; it might still be regarded as a theorem in a certain calculus; but that calculus would, at least for the time being, have no useful application.

And that is the case of the bizarre piece of maths Wittgenstein appeared to prove in *RFM* I §38: 2 + 2 + 2 = 4. The suggested pictorial proof *might* convince people, so that they adopt it as mathematical proposition. One could not object to that by urging that they would adopt a falsehood. For in their (admittedly odd) calculus it would then be a theorem. Of course, what we naturally take to be the corresponding empirical statements would be false: If you put three pairs of apples or nuts on a table, and take care that nothing is removed or added, you will certainly count a total of 6 apples, not 4. But the falsity of such an *empirical* statement doesn't make '2 + 2 + 2 = 4' false. It only makes it impractical.

Moreover, we could imagine, in the manner of the example in *RFM* I §137, curious circumstances under which countable objects vanish in a way that makes that queer sum usefully applicable. Or, less fancifully, we might hit upon things or chemical substances that add up in just the way suggested by the second 'proof' in *RFM* I §38: so that pairs 'overlap' in one unit ('2 + 2 = 3'). (Thus this calculus might be used for calculating something like the length of a bicycle chain.)

9. Unreasonable Wood Merchants

So, mathematics is not forced upon us as true to the facts; it is to be assessed as more or less practical or useful, relative to our circumstances and purposes. A deviant and bizarre piece of mathematics (such as a method of proof that leads to '4 × 3 + 2 = 10') is not false, but only extremely unlikely to be practical. In *RFM* I Wittgenstein imagines that the circumstances are so strange that the equation in question might actually be practical. In a lecture (1937) he imagines the more likely case that it doesn't work (*LFM* 202f.). The example is slightly different, but of the same kind: using a diagram in which groups of three strokes are taken together, but in a way that those triples overlap in one stroke, it is calculated that 9 = 4 × 3.

> Suppose people … calculated this way when they wanted to distribute sticks. If nine sticks are to be distributed among three people, they start to distribute four to each. Then one can imagine various things happening. They may be greatly astonished when it doesn't work out. Or they may show no signs of astonishment at all. What should we then say? "We cannot understand them." [*LFM* 203]

If bad mathematics is not false, but impractical, people that hold on to it in spite of its apparent uselessness are difficult to understand. It's not that they got their facts wrong, it's just that they seem so unreasonable. As unreasonable as those whose mathematical calculations are impeccable, but who calculate what we regard as the wrong things: who apply their calculations in an oddly impractical way. This is the case of the firewood merchants Wittgenstein presents in *RFM* I §§149-50: They pile their wood up in heaps of arbitrary, varying height and then sell it at a price proportionate to the area covered by the piles.

> How could I shew them that – as I should say – you don't really buy more wood if you buy a pile covering a bigger area? – I should, for instance, take a pile which was small by their ideas and, by laying the logs around, change it into a 'big' one. This might convince them – but perhaps they would say: "Yes, now it's a lot of wood and costs more" – and that would be the end of the matter. – We should presumably say in this case: they simply do not mean the same by "a lot of wood" and "a little wood" as we do; and they have a quite different system of payment from us. [*RFM* I §150]

They may just be stupid (*RFM* I §151; *LFM* 202), but then again, they may have different priorities from ours, so that they are not interested in maximising their profits or savings. By 'a lot of wood' they mean wood covering a lot of ground, knowing full well that the same quantity of wood can be made into 'a lot of wood' or 'a little wood'. Perhaps they appreciate the seller's skill of spreading the logs in a way that they cover a lot of ground and are quite happy to pay more when the article is so skilfully arranged (as many of us are happy to pay more for nice packaging or for some affable smiles from the sales staff).

10. Different Practices

Crispin Wright is as critical of Wittgenstein's discussion of those curious wood sellers as of his example of elastic rulers. His objection is that Wittgenstein fails to make it plausible that the imagined tribe has a different concept of measuring (in one case) or of quantity (in the other) (Wright 1980: 68). He thinks that Wittgenstein faces a dilemma: either these people's purposes in the described practice are like ours (e.g. determining how much wood they need to buy to last the week), or they are not. In the former case, their practices must be criticised as based on the same concepts of length or quantity, but extremely inept. In the latter case, however, in which we cannot criticize them as inept, we also have no good grounds for attributing to them a concept of *length* or *quantity*. So Wittgenstein fails to show that there could be alternative concepts of measuring or quantity (Wright 1980: 71-2).

However, Wright's criticism is curiously unfocused: the result of attributing to Wittgenstein an agenda that is alien to his remarks. Does Wittgenstein aim to describe people with an alternative concept of *quantity*? That is not what he says. Rather, we are told that those people use the expressions 'more wood' and 'less wood' altogether differently: *not* to describe the *quantity* of wood, but to denote an aspect of its arrangement. The strange thing is that they take that aspect as crucial for the pricing of the wood, even though they seem to know that it can easily be changed. The point of the story is not to show that there can be different concepts of quantity, but that we can imagine a social life that is not as focussed on the concept of quantity as ours is, not even when it comes to exchanging goods. We take it for granted that the price of wood must be proportional to its quantity. But different forms of exchange are conceivable, for example that:

> each buyer pays the same however much he takes (they have found it possible to live like that). And is there anything to be said against simply giving the wood away? (*RFM* I §148)

Strange as those people behave, they don't get their facts wrong. They apply their calculations in a curiously selective manner (only to surface, not to volume), but clearly that doesn't make their calculations incorrect. And that is the way, I take it, Wittgenstein wants us to regard cases of strangely different forms of mathematics too: not as erroneous, not as false claims about reality, but as tools that are used in an odd and probably impractical way. In the wood merchants' case, the tools are like ours, but applied in an odd way which strikes us as stupid; in the earlier example of distributing nuts, the tool itself appeared unsuitable (unless strange things happened). But in neither case are the tools, the calculations as such, to be assessed as false.

If a mathematical equation were a statement of fact, it could be true or false independently of people's beliefs. We could then imagine a whole community to believe erroneously that $2 \times 2 = 5$. But can we really?

> But what would *this* mean: "Even though everybody believed that 2×2 were 5, it would still be 4"? – For what would it be like for everybody to believe that? – Well, I could imagine, for instance, that people had a different calculus, or a technique which we wouldn't call "calculating". But would it be *wrong*? (Is a coronation *wrong*? To beings different from ourselves it might look extremely odd.) [*PPF* §348; *LW I* §934]

The crucial point is that in order to attribute to a community a considered mathematical belief that $2 \times 2 = 5$ we'd have to imagine them as having a corresponding mathematical practice: a calculus and its applications. What they *believe* in mathematics is a reflection of what they *do* in calculating and applying mathematics. Hence *if* we can indeed imagine them having such a practice, the corresponding belief would be true as a matter of course. So our criticism cannot be that their belief is untrue (it isn't if it correctly reflects their practice), it can only be that their practice is utterly impractical or even ludicrous (cf. *RFM* I §§152-3). Moreover, we should probably not regard it as 'calculating' or a form of what we like to call 'mathematics'.

The same applies to the case of traders that do not insist that the price be proportional to the quantity of the goods sold. We can imagine such a practice, although we find it rather incomprehensible, if not insane. But here, as in the 2 × 2 = 5 example above, Wittgenstein reminds us of familiar elements in our own culture that to rational outsiders may appear bizarre, such as the ceremony of a coronation (*RFM* I §153). And as he notes that we may find '2 × 2 = 5' too odd to be called 'mathematics', just as we may refuse to call a deviant transition from one sentence to another (e.g. from 'p v q' to 'p') 'logical' or 'inferring' (*RFM* I §116), – similarly, Wittgenstein would obviously agree with what Crispin Wright meant to present as an objection, namely that those wood-sellers' use of 'more' or 'less wood' does not describe something that we should call quantity.

11. Conclusion

"Of course", writes Wittgenstein, "in one sense, mathematics is a body of knowledge, but still it is also an *activity*" (*PPF* §349; *LW I* §935) – embedded in, and partly constituting, a form of life. Hence, to imagine different, alternative forms of elementary mathematics, we should have to imagine different practices, different forms of life in which they could play a role. If we tried to imagine a *radically* different arithmetic we should think either of a strange world (in which objects unaccountably vanish or appear) or of people acting and responding in very peculiar ways. If such was their practice, a calculus expressing the norms of representation they applied could not be called false. Rather, our criticism could only be to dismiss such a practice as foolish and to dismiss their norms as too different from ours to be called 'mathematics'.

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