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On a Philosophical Motivation for Mutilating Truth Tables

Abstract

One of the reasons colours, or better the conceptual organisation of the colour system, could be relevant to the philosophy of logic is that they necessitate some mutilation of truth tables by restricting truth functionality. This paper argues that the so-called 'Colour Exclusion Problem', the first great challenge for Wittgenstein's *Tractatus*, is a legitimate philosophical motivation for a systematic mutilation of truth tables. It shows how one can express, through these mutilations, some intensional logical relations usually expressed by the Aristotelian Square of Oppositions, as contrariety and subcontrariety.[1]

Introduction

Colours are trivially irrelevant to logic, but their conceptual organisation can be very challenging for the philosophy of logic. For instance, the colour system imposes some difficulties for truth functionality and for any image of logic exclusively based on extensional notions, i.e., based solely on the fixed meaning of some logical connectives. In other words, the colour system shows some intensional (or modal) conceptual arrangements that have to be

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explicitly introduced in formalism in order to nicely capture certain fine logical relations.

Consider these two sentences: (i) "if a point in the visual field is blue, it is not red" or (ii) "a point in the visual field is blue and red". They are not *just* true or false sentences. The problem here is not with the truth value of those complex sentences, but rather with the very *impossibility* of some truth conditions or combinations of meaningful sentences. In order to grasp their peculiar logical status, those sentences should be rephrased, respectively as: (iii) "if a point in the visual field is blue, then it *cannot be* red" or (iv) "It is *impossible* that a point is both blue and red".

As a result, (iii) and (iv) show some commitment to what we could call logical necessities, a commitment which seems to be grounded on conceptual incompatibilities. But what is the logical status of these sentences? In which sense could (iii) or (iv) be held as logical truths or just tautologies? Interestingly, Wittgenstein was *inter alia* also engaged in this kind of philosophical puzzle upon his return to philosophy in 1929. He mysteriously calls those sentences a "certain kind of tautology" (Wittgenstein 1929: 167).[2]

In 1929, Wittgenstein begins to call (iii) and (iv) *rules* and explicitly investigates how to block some lines of truth tables in order to capture a set of logical relations which could not be rendered by the help of extensional connectives in the way that they were usually presented by for instance the kind of (realist) semantics of truth conditions presented in his *Tractatus* (4.41, 4.431, 4.442, 4.45-4.461 and 4.463).[3]

This paper is divided into four sections. In Section I, we discuss how the early Wittgenstein searched for an adequate notational means to show his ideal of logic as a completely neutral and truth-

[2] Goddard also acknowledges the peculiar logical status of these kinds of sentences by instructively considering their necessity a case of "an intensional tautology" (1960: 105). Goddard explicitly contrasts it with a "formal tautology", whose necessity is based on a combination of (full) truth-functional logical connectives. Goddard seems to identify intensional with material – see, for example, his short discussion on material contrarieties (1960: 99). In his contribution, Goddard neither discusses nor mentions the Aristotelian Square of Oppositions, even when, in a short final footnote in his paper, he suggests some connections between contrarieties and subcontrarieties (1960: 105).

[3] All decimal numbers here come from the usual *Tractatus* numeration.

functional enterprise. Section II explores some inevitable limits of this image of logic imposed by a full recognition of the Colour Exclusion Problem. This problem imposes severe restrictions on truth functionality. Section III argues that this philosophical problem motivates some systematic mutilation[4] of truth tables in order to capture logical relations usually expressed by the Aristotelian Square of Oppositions. Section IV draws some consequences concerning these logical restrictions within Wittgenstein's investigation about the nature of language in his return to philosophy in 1929.

I. Truth table as a notational means

To work with truth tables[5] is to operate with many tenets of the Tractarian philosophy of logic, as Hacker (1971), Dreben & Floyd (1991) and von Wright (1996) show. A natural result of this approach is that common attacks against this notational means should also expose direct problems in the Tractarian image of logic and vice versa.

In a sense, if one is familiar with truth tables, one is familiar with the peculiar Tractarian metaphysics[6] (and its limitations). This notation – a form of tabular representation of truth conditions and truth values of propositions, which is conventionally advanced today in the manuals for propositional calculus – was inaugurated in the Tractarian period as *WF Notation*.[7] In 4.442, for example, Wittgenstein called it *WF Schemata*. This passage can be viewed as

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[4] I borrow the term "mutilation" from von Wright (1996). He used the term to address a formal strategy of expressing some modal relations in the 60's and used it again, in his paper from 1996, to examine Wittgenstein's truth tables from 1929, but without mentioning the paradigm of exclusion by contrarieties. Here, by using the term 'mutilation', I would like to both refer to a strategy of blocking some truth-conditions in truth tables and emphasise Wittgenstein's dramatic need to restrict the Tractarian logical space, early thought of as absolute and eternal.

[5] We refer here to the classical usage of truth tables outside a multi-valued background.

[6] I will not engage here in a discussion about resolute and traditional reading of the *Tractatus*, which (unfortunately) seems to have dominated the last decade of scholarship on Wittgenstein's first masterpiece. I agree with Engelmann (2013) when he criticises this distinction as a false dilemma.

[7] As we will see, Wittgenstein consistently continues approaching truth table as *his* notation in texts from 1929 and 1930.

the birth certificate of truth tables.[8] This special notation provides a means by which, in one movement, we could (i) mirror the alleged deep (and hidden) syntax of our language[9] and (ii) calculate and fully determine the radical difference between empirical (meaningful) and logical propositions, since logic for him at that time was reducible to tautologies (6.1). The idea underpinning an adequate symbolism should be: if it is indeed a correct way to express the domain which we are dealing with, we can manipulate the adequate notational means using clear rules and then avoid philosophical misunderstandings.

Ramsey, in his historical 1923 review of the *Tractatus*, had already recognised the truth table notation as an improvement in certain aspects of the expression of dependence between propositions and their operators in comparison with the notation of *Principia*, stating:

> It may, of course, be doubted whether it is possible to formulate this rule [on dependence of propositions] as it seems to presuppose the whole of symbolic logic; but in any perfect notation it might be possible; for example in Mr. Wittgenstein's notation with T's and F's there would be no difficulty. (Ramsey, 1923: 472)

In spite of this appraisal, Wittgenstein sometimes externalises consternation because Ramsey could not really understand the crucial (philosophical) stress that he put on symbolism:

> Ramsey versteht den Wert den ich auf eine bestimmte Notation lege ebenso wenig, wie den Wert den ich auf ein bestimmtes Wort lege, weil er nicht sieht, dass darin eine ganze Anschauungsweise des Gegenstandes ausgedrückt ist, der Winkel von dem ich die Sache jetzt betrachte. *Die Notation ist der letzte Ausdruck der Philosophischen Anschauung*. (1994: 165, our italics)

8 Although I will not discuss historical priority here, it must be said that some comparisons, both with a philosophical and a technical approach, between Post's truth tables (1921) and the *Tractatus* truth tables developed at the same time but independently, would be a great contribution in this historical field, since, for example, Anellis (2004, 2011) defends Peirce as *the real* father of truth tables.

9 This is not to be held in a Chomskyan sense, but in the context of the early analytic philosophy tradition. (See, for example, Frege (1918), Russell (1918) and 4.002 from the *Tractatus*)

In fact, truth table notation shows an appeal to logic as an exclusively neutral field, completely combinatorial, where no possibility is excluded and no hierarchies are expected (5.556).[10] Truth tables very nicely incorporate the Tractarian paradigm of truth-functionality (5 and 5.1) and neutrality of logic (5.551), or in a sense, the view that the sense of complex sentences should be *completely* reducible to the sense of the elementary sentences that compose them. As a result, any connective which reveals itself as not being extensional would just not be part of logic.

However, on his return to philosophy in 1929, Wittgenstein revealingly maintains that "a is blue and a is red" is not to be held as a formal contradiction, yet the two propositions which compose the former conjunction do exclude each other (Wittgenstein 1929: 168). This implies that he, at that time, thought that it was no longer possible to hold all exclusions as formal contradictions and all logical connectives as (full) truth-functional. There should be more exclusions than formal contradictions[11] and this should, as a direct result, affect his Tractarian truth tables.[12] As shown in the clear *mea culpa* of the last paragraph of *Some Remarks on Logical Form:*

> It is, of course, a deficiency of our notation that it does not prevent the formation of such nonsensical constructions [such as "a is red and a is blue"], and a perfect notation will have to exclude such structures by definite rules of syntax. *These will have to tell us that in the case of certain kinds of atomic propositions described in terms of definite symbolic features, certain*

[10] See also 2.0121, where Wittgenstein states that logic should deal with *all* possibilities and that *all* possibilities are its facts.

[11] Also consistent with this sort of problem and in the same period, Wittgenstein in conversations with some members of the Vienna Circle seems to recognise the paradigm of contrarieties in the kind of exclusion among colours (WWK: 127, 148).

[12] As Wittgenstein states in his essay from 1929: "the top line [of a truth table] must disappear" or "certain combinations of the T's and F's must be left out" (Wittgenstein 1929: 170-1). Some passages of his *Nachlass* from the same period are consistent with this discussion: "Wegfall der ersten Linie" (Wittgenstein 1994, p. 58) "eine Reihe einfach durchstreichen, d.h. als unmöglich betrachten" (*ibid.*), "ich muss die ganze obere Reihe durchstreichen" (*ibid.*), "die ganze Linie ausstreichen" (p. 59), "die obere Linie streichen" (*ibid.*). One of the main points in Goddard (1960) is to show that the exclusive "or" necessitates what he calls "abbreviation of truth tables" or "quasi-truth-conditional connectives". Goddard mentions Wittgenstein's truth table from 1929, but without examining the sort of exclusion to be found in the colour system that was challenging *Tractatus's* author around 1929.

> *combinations of the T's and F's must be left out.* Such rules, however, cannot be laid down until we have actually reached the ultimate analysis of the phenomena in question. This, as we all know, has not yet been achieved. (Wittgenstein 1929: 171, my emphasis)

It seems unproblematic to assume that "our notation" in this quotation is the truth table notation, as the diagrammatic notations in this article's last pages show us. It is also interesting to note that the ideal of a suitable notation in this period is replaced by the ideal of a notation that could be combined and supplemented by the syntactical or grammatical rules of a particular system. Here Wittgenstein's emblematic example for a particular system with particular grammatical rules which challenges "our notation" is the colour system.

The natural question to be raised is: why should colours be considered as triggering this change of mind and how did their conceptual organisation impact a peculiar view of logic centred in the notion of neutrality (5.551)?[13] To answer this, we have to access some philosophical problems connected with logic, although there are some valuable connections here with phenomenological issues, as pointed out by Spiegelberg (1981), Prado Neto (2004) and Engelmann (2012). Our general leitmotif for evaluating the kind of logical challenge Wittgenstein faced in 1929 is the following: if the logical organisation of colours represents a problem for his Tractarian logic, it should represent a problem for his notation too.

II. The Colour Exclusion Problem: beyond the colour system

Hacker already very vividly maintained that "Wittgenstein's first philosophy collapsed over its inability to solve one problem – colour exclusion" (Hacker 1971: 86). In fact, in the *Tractatus*, Wittgenstein states that the conjunction of "a is blue" and "a is red", with "a" being a point in the visual field, is a contradiction (6.3751), i.e., an always false complex proposition. Accommodating

[13] It is relevant to emphasize here that I do not intend to solve the Colour Exclusion Problem, as Moss (2012) and Young (2005) have tried to do, because, among other reasons, this kind of approach very often neglects some rich discussions of Wittgenstein's (early) Middle Period.

this sort of exclusion in the *Tractatus* as a contradiction is completely in line with Wittgenstein's account of denial at that time – in the same way that for those who only have a hammer, every problem looks like a nail. As argued in Silva (2014), since the only negation expected in the *Tractatus* is one which works as a switcher of truth conditions, the only exclusion expected should be one due to contradiction (5.1241 and 5.5513).

However, some exclusions, different from a contradiction, are ubiquitous. Exclusions such as finding that a table cannot measure both 3 metres and 4 metres in length, or finding that a refrigerator cannot be set at both 15ºC and 16ºC, or that a bottle cannot hold exactly 2 litres and exactly 3 litres of some liquid at once, or that a point in the sky cannot be both blue and red are all common exclusions. In this context, it is crucial to note that these incompatibilities are not the results of a formal contradiction, because the two alternatives are not exhaustive, although they are exclusive ones. The two articulated sentences that express the former impossibilities, as "point *a* is red" and "(the same) point *a* is blue" cannot be true together, *but can be false together*. Here we have a classic case of contrariety and not of contradiction, though we still have a case of exclusion or incompatibility between alternatives. It is a logical feature of these alternatives that they are embedded in a *dense* system of relations with numerous alternatives, much more than two. The so-called Colour Exclusion Problem is more than a problem with the colour system; it is a logical puzzle based on the distinction between forms of exclusion.

Moreover, in a sense, the logical pattern of exclusions by contrarieties is always *holistic*. For instance, one colour brings with it the *whole* system of colours with its oppositions and complementarities or "the table is 2 metres long" brings the *whole* metric system (since not being 2 metres long implies all other possible lengths). What marks the contrariety is the idea of a degree or gradation, and consequently, an explosion of numerous or, at least in some cases, infinite number of alternatives.[14] Sentences of

[14] Horn & Wansing (2015), discussing forms of negation and exclusion, defend an interesting relation between contrarieties and the Law of Excluded Middle (but without mentioning Wittgenstein's colour exclusion problem): "Contrary terms (*enantia*) come in

gradation are clearly mutually exclusive, but they are not contradictory because they can be false together. For example, it is possible that, in the case of a table, it is neither 3 metres nor 4 metres long or that a patch of the visual field is neither blue nor red.

The same holds for the other examples: not only length measurements, but also temperatures, volumes and colour gradations allow themselves to be mapped by numerical indexes and some applications of arithmetic. Something cannot be (all over) green and yellow, even though green could be created using yellow. And a meteorologist who maintains that the temperature tomorrow afternoon will be 30ºC *and* 37ºC will not be taken seriously, even if s/he says something more enigmatic like "it will rain and it will not rain". In each case, the members of the conjunction are mutually exclusive, but this is different from the exclusion involved in a contradiction, which has the property of being exhaustive. The exclusion in "it rains and it does not rain" seems to be more "radical" than the exclusion in "the temperature is 30ºC and the temperature is 37ºC". The first is a contradiction as it involves exclusive and exhaustive alternatives that cannot be true together or false together, for it will either rain tomorrow afternoon or it will not. However, it is possible that the temperature tomorrow will be neither 30ºC nor 37ºC.

Discussing this point in conversations with the Vienna Circle in early 1930, Wittgenstein stated that "die Tautologie ist ja ganz nebensächlich", since only in the truth table notations can tautologies show syntactical rules (Waismann 1984: 91-2). The assertion that tautologies are indeed irrelevant is blatantly at odds with the philosophy of logic presented in the *Tractatus*, which fully reduced logic to tautologies (6.1) and exclusions to contradictions.

two varieties. In immediate or logical contraries (*odd/even*, *sick/well*), a true middle—an entity satisfying the range of the two opposed terms but falling under neither of them – is excluded, e.g., an integer neither odd nor even. But mediate contrary pairs (*black/white*, *good/bad*) allow for a middle – a shirt between black and white, a man or an act neither good nor bad. Neither mediate nor immediate contraries fall under the Law of Excluded Middle [LEM] (*tertium non datur*)".

Not by accident, this fact is shown in the failure of (full) truth tables to block nonsensical articulation of signs.

We should only know the rules that underlie a system (colour, length, volume, sound, temperature, etc.) and try to express them clearly in a notation, case by case. It then seems natural to think that some notations might be more appropriate for some systems of propositions than others. For instance, it makes no sense to express the kind of logical oppositions between colours by a truth functional notation as a full truth table, since some lines have to be ruled out, as Wittgenstein also recognises in §79 of *Philosophical Remarks* (PR). This is a very pervasive problem, since virtually every empirical sentence deals with some property which admits gradation.[15]

III. Motivating mutilations of truth tables to capture some intensional notions

Brandom (2008), addressing incompatibility semantics, also recognises non-reducible modalities in certain domains such as colour, shape, quantities and biological taxonomy without really treating them as systems or pointing out some resemblance to Wittgenstein's *Satzsysteme*. This irreducibility is the reason why Brandom calls them "persistent incompatibilities". As he states:

> Aiming at maximal generality, I will impose only two conditions on the incompatibility relations whose suitability as semantic primitives I will be exploring here. First, I will only consider symmetric incompatibility relations. This is an intuitive condition because it is satisfied by familiar families of incompatible properties: colours, shapes, quantities, biological classifications, and so on. Second, if one set of claims is incompatible with another, so too is any larger set containing it. That is, one cannot remove or repair an incompatibility by throwing in some further claims. I call this the 'persistence' of incompatibility. If the fact that the monochromatic patch is blue is incompatible with its being red, then it is incompatible with its being

[15] Goddard (1960) states something similar about the mandatory dependency of sentences in some domains: "In some way, therefore, the range of the variables has to be restricted; or, equivalently, some relation between them has to be formulated so that the choice of a value for one determines the choice for the other" (p. 102).

> red and triangular, or its being red and grass being green. (Brandom 2008: 123)

Because Brandom does not connect these primitive incompatibilities with Wittgenstein's difficulties in expressing colour exclusion in the early 1930s, we should bring this discussion back to some of the most fundamental tenets in the *Tractatus*. There, if p and q are elementary propositions, they should be (logically) independent, because the concatenation of them does not generate contradictions, since there is no negation in elementary propositions. But if p and q belong to the same system, such as a system for measuring lengths or for the ascription of colour to visual points, we may have exclusions without repetition or negation, that is, exclusions without the form of contradiction (p.~p).

This kind of subtle exclusion in incompatible properties appears to be utterly beyond the power of analysis required by bipolarity and carried out by the prominence of the truth-functionality. If the base is meaningful, this meaningfulness does not guarantee the meaningfulness of the complex strictly generated from this base. For instance, given p and q as elementary propositions, their truth conditions may not be enough to determine the semantics of "p and q", as in non-truth-functional logics. But even more problematic than this, such truth conditions are not enough to determine if the conjunction of "p and q" is possible. After all, it should be impossible to judge nonsense.

Ramsey (1923) had already seen the problem with the exclusivity of thinking of logic as comprised of tautologies and contradictions, but without associating it with measurement problems, or numbers, or exclusions by contrariety, or even with biological taxonomies, as Brandom does (2008, p. 138). As Ramsey pointed out (p. 473), there would be other necessities that could be called logical, but that could not be reduced to tautologies, as the necessary properties of space and time,[16] which seem to bring

[16] Interestingly, the first example of what Goddard calls material contrariety is given using different locations (cities of Sidney and Amidable) of the same thing at the same time.

semantical aspects into the neutral and combinatorial Tractarian logic.

From Ramsey's visits to Wittgenstein in Lower Austria, and the content of his historical review from 1923, as well as discussions on problems and obscure points in the *Tractatus*, we can speculate that Ramsey, was the first to notice the so-called Colour Exclusion Problem, i.e. the issue of the promissory note in passage 6.3751 that could not be paid in Tractarian terms.[17] To resolve this logical incompatibility, we should review the Tractarian conceptual geography which was so certain for Wittgenstein at that time. The Tractarian logic could not take care of itself or the logical necessity could not just be rendered by its tautologies, contradictions and extensional connectives. As Wittgenstein affirms in §76 of PR, there is a logical construction inside an elementary proposition, which does not appeal to truth-functions and so cannot be explored using by his early notation.

What is evident here is the ineptitude of the truth table or of any scheme of (full) truth-functionality to explain the exclusion of degrees. For example, the logical product and logical sum do not have sufficient sensitivity to express the exclusion of non-exhaustive colours. If we take "this is white" as p and "this is black" as q, the logical product cannot be TFFF (p, q), precisely because the conjunction's parts *cannot be true* together. Moreover, if p is the case, we find that q cannot be the case and vice versa. So there is a picture of exclusion and implications in the mosaic of colours. The result for the *Tractatus* seems to be trivial: if elements of a proposition are mutually exclusive, they are not elementary, so

[17] See, for instance, the Preface to *Philosophical Investigations*: "For since beginning to occupy myself with philosophy again, sixteen years ago, I have been forced to recognise grave mistakes in what I wrote in that first book. I was helped to realise these mistakes—to a degree which I myself am hardly able to estimate—by the criticism which my ideas encountered from Frank Ramsey, with whom I discussed them in innumerable conversations during the last two years of his life." Ramsey drew Wittgenstein's attention to this structural problem, so one might highlight this change or development in Wittgenstein's philosophy as a kind of Ramsey effect, triggered by his critical insight. Ramsey was the first to point out, even if incidentally, in his review already in the reception of the *Tractatus*, the nerve problem that led to the subsequent abandonment of the work. (For further discussion, see Jacquette, 1990.)

one must keep on analysing to "sublimate" the operational complexity and display the elementary propositions at its base.

It is important to note what happens with truth tables in 1929. This is not a big deal from a technical point of view, but it is philosophically momentous. Wittgenstein retains the Russellian idea of full analysis, but talks about *laying down some rules* (Wittgenstein 1929: 171). At this time, the problem is not with the truth value in the last column. It is not about falsehood, but the representation of colour exclusion with a full truth table is a nonsensical construction. Here it is important to note that the exclusion itself is not nonsensical, but Wittgenstein's former representation of it is. We would need a truth table like the one below for making the colour exclusion a contradiction:

a is red	*a* is blue	*a* is red and *a* is blue
T	T	F
T	F	F
F	T	F
F	F	F

However, as Wittgenstein acknowledges in 1929, to write the contradiction using the above truth table "is nonsense, as the top line, 'T T F' gives the proposition a greater logical multiplicity than that of the actual possibilities" (p. 170). The problem is the scheme of truth conditions itself; in other words, the problem is with the free distribution of truth values. The combinatorial procedure has to follow some *rules*. It has to be contextually sensitive and respect some intensional constraints. As shown in this case about colours, the first line is not just false, but rather, it is *impossible* due to the colour system. As a result, a more adequate representation should be this one:

a is red	*a* is blue	*a* is red and *a* is blue
~~T~~	~~T~~	~~F~~
T	F	F
F	T	F
F	F	F

We could also have the mutilated truth table below in which some other "phenomenological" systems are shown:

a is red *a* is 3m long Now it's 28°C (the same with degrees of volume, hardness, etc.)	*a* is blue *a* is 4m long Now it's 29°C (the same with degrees of volume, hardness, etc.)	*a* is red and *a* is blue *a* is 4m long and *a* is 3m long Now it's 28°C and 29°C (the same with degrees of volume, hardness, etc.)
~~T~~	~~T~~	~~T~~
T	F	F
F	T	F
F	F	F

In either case, the first line has to be ruled out, taken away, blocked or, in a word, "mutilated", as von Wright (1996) argues. Some combinations must be blocked *ad hoc*, as Goddard (1960) suggests. For Wittgenstein (1929), this naturally means a dramatic (philosophical) turn, since to impose restrictions on truth tables means to impose restrictions on truth functionality, extensionality and other typical (classical) Tractarian features. We must have some intensional notions to grasp those logical relations, at least, in some *Satzsysteme*.

In this sense, we have a clear philosophical motivation for systematically mutilating lines of truth tables, because we have to *add up* some rules to restrict a combinatorial and neutral logical space. This follows an intuitive notion of rules as constraints, that is, rules meaning restrictions of a *Spielraum*. As a result, systematic mutilations may aptly capture some other logical patterns usually presented in the Aristotelian Square of Oppositions, which is often presented as a means for representing some special conceptual linkage, including (but by no means being reduced to) generalities (Beziau & Jacquette 2012).

Here we have it in mind to express contrariety, subcontrariety and contradiction by mutilating respectively the first line, the second line and then both the first and last lines of a full truth table.

p	q
~~T~~	~~T~~
T	F
F	T
F	F

In this first case, by removing the first line of a truth table, we are showing that p and q *cannot* be both true together, but *can* be false together.[18]

p	q

[18] See Wittgenstein's truth table for the ascription of two different colours to a same visual point (1929, p. 170). There he does not acknowledge the paradigm of contrarieties, but his truth table, just like ours here, is precisely expressing exclusion by contrariety through the mutilation of a row in the truth table.

T	T
T	F
F	T
~~F~~	~~F~~

In this second schema, by removing the last line of a truth table, we are showing that p and q *cannot* be both false together, but *can* be true together.

p	q
~~T~~	~~T~~
T	F
F	T
~~F~~	~~F~~

In this third case, by removing the first and last line of a truth table, we are expressing that p and q *cannot* be both true together and *cannot* be both false together. In this account, a contradiction can be nicely taken as an intuitive combination of both contrariety and subcontrariety.

Another natural question to be answered in this context is whether this is a problem with propositional logic. Wittgenstein discussing Ramsey's objection in some entries from his *Nachlass* points out:

> … if 'f(x)' says that x is in a certain place, then 'f(a).f(b)' is a contradiction. But what do I call 'f(a).f(b)' a contradiction when 'p.~p' is the form of the contradiction? Does it mean that the signs 'f(a).f(b)' are not a proposition in the sense that 'ffaa' isn't? Our difficulty is that

> we have, nonetheless, the feeling that here there is a sense, even if a degenerate one (Ramsey). (Wittgenstein 2000, MS112)[19]

This passage from 1929 clearly shows that both Wittgenstein and Ramsey discussed the Colour Exclusion Problem and the challenges for formal logic at that time. This problem would not be a problem with propositional logic; predicates interpreted as simple extensions (or subsets of a fixed domain) would not do the work of expressing some intensional logical relations. Some concepts are linked in a way that full truth functionality must be blocked, restricted.

IV. Developing some philosophical consequences in mutilating truth tables

Although Wittgenstein in 1929 continues with the general Tractarian project of fully analysing language and bringing it to its atomic level, the accent in his early middle period should be on the search for a greater expressiveness to capture the multiplicity of phenomena. We may lose the decidability of truth tables, but not expressiveness with regard to various logical multiplicities (e.g. colours, temperature, sounds, length etc.).[20] Wittgenstein, in §83 of PR, a text from the same period, offers a very instructive kind of *mea culpa*, articulated with problems with measurements and spatial intuitions, the possibility of non-truth-functional connectives and the limitations of expressiveness of the Tractarian notation: "The concept of an 'elementary proposition' now loses all of its earlier significance. The rules for 'and', 'or', 'not' etc., which I represented by means of the T-F notation, are a *part* of the grammar of these words, but not the *whole*. (...) In my old conception of an elementary proposition there was no determination of the value of a co-ordinate; although my remark that a coloured body is in a colour-space, etc. should have put me straight on to this" (1975: 111).

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[19] Engelmann's translation (2012: 273)

[20] This is to be found throughout Chapter VIII of PR, where Wittgenstein returns to re-evaluate 6.5731 in terms of many systems with the same kind of exclusion as found in the colour system.

If we read this passage carefully, we may speculate that, in an important sense, his Colour Exclusion Problem, which is traditionally handled by the secondary literature in 6.3751, can already be observed when we conjugate passage 2.0131, about the necessary belonging of objects in a space of possibilities, with passage 2.061, also from the ontological part, which maintains that states of affairs must be independent of each other. Here it is not directly about a ban on metaphysical forms as in "an object cannot be in different places simultaneously", but *prima facie* a ban on logic: "a point in the visual field must have a colour, and only one, that is, if it is green, *it cannot be* red, blue, etc." If a tangible object has a hardness, then other hardness values are (necessarily) excluded. A musical note must have a pitch, so other pitches must be logically excluded, when a pitch is ascribed to it. In all these cases, there is no room for another value.

As defended in Silva (2014), negation should be held as an equivocal term, since we must have *at least* two, a formal one based on the notion of contradiction and a non-classical one based on the notion of contrarieties, the very logical pattern which arises in every (phenomenological) *Satzsysteme* in Wittgenstein's return to Philosophy. In a sense, the paradigm of contrarieties is logically much more (conceptually) sophisticated than the paradigm of contradictions. (Assuming that sophistication is *not* a matter of degrees of abstraction.) For to correctly operate with contrarieties, we need to understand a whole system of logical relations. Conceptual systems governed by contrarieties are inferentially thick or robust. As a result, one may defend that contrarieties impose "semantics", meaningful conceptual articulations and full recognition of primitive material linkage between notions and not only manipulation of signs, as an instance of any kind of neutral syntax or Boolean algebra. It is not sufficient to manipulate with signs to understand the negation of a proposition which we use to ascribe a degree for an empirical quality or a colour to a visual point.

As we saw, the expression of contrarieties is a problem for truth tables, in particular, and for truth functionality, in general. In our former section, it was shown how to generalize a notational procedure to systematically express other logical relations to be found within the (classical) Aristotelian square of oppositions. The basic philosophical idea is that truth functionality and truth tables have to be limited depending on the context of investigation and application, since, for instance, if *p* is meaningful and also *q*, then it may be that "p and q" is not meaningful at all depending on the system wherein those propositions are embedded. This example shows that a neutral and universal application of classical conjunction should be under attack.

It is important to acknowledge here that some terms are primitively incompatible due to its conceptual relations, as Wittgenstein draws our attention in the collapse of his logical atomism. If we do have elementary propositions, they should be all inserted in several different systems, which are logically organized through exclusions by contrariety. The negation in systems governed by contrarieties "explodes" in several, if not in infinite alternatives. To operate propositions within these systems we should know the whole system in which the proposition is inserted to find out which combinations are allowed or not. Only with formal contradictions, it is hard to see how we could express intricate relations among phenomena, for instance. Phenomena are logically and conceptually organized, but not through tautologies, formal logic, and truth-functionality (extensional connectives). We need some other operators and modalities, or better, intensional notions.

Our discussions so far direct us also towards a better understanding of Wittgenstein's remark in December 1929:

> I once wrote: 'A proposition is laid like a yardstick against reality. Only the outermost tips of the graduation marks touch the object to be measured.' I should now prefer to say: a system of propositions is laid like a yardstick against reality. What I mean by this is: when I lay a yardstick against a spatial object, I apply all the graduation marks simultaneously. It's not the individual graduation marks that are applied, it's the whole scale. If I know that the object reaches up to the tenth graduation mark, I also know immediately that it doesn't reach the eleventh, twelfth, etc. The assertions telling me the length of an object form a system, a system of propositions. It's such a whole

> system which is compared with reality, not a single proposition[21]. (1975: 317)

If my interpretation is correct, connecting the mutilation of truth tables with the need for expressing intensional relations of incompatibility, provides us with another way of understanding a shift in Wittgenstein's philosophical development from his early logical atomism to a special kind of logical holism: No proposition in systems governed by *contrary relations* can be logically isolated. You may fully analyse some propositions within those systems; but, this end will not have (logically) independent propositions, as defended in the *Tractatus*. Propositions in *Satzsysteme* are actually dense in relations, as they have numerous (in some cases, infinite) implications and exclusions (by contrariety). As we saw, they are "inferentially thick".

Conclusion

My contribution here was to elucidate how the so-called Colour Exclusion Problem serves as a motivation, both for philosophers (in general) and for Wittgenstein (as a matter of his own philosophical development), to impose *restrictions* on truth-functionality and extensionality (and *not to ban* those notions from the analysis of language). I do not know any previous work on this issue which connects directly this discussion on Colour Exclusion with the logical relations which are usually presented in the Aristotelian Square of Opposition.

In a horizon which only allows tautological consequences as logical consequences and contradictions for all kinds of exclusions, we may see how this image of logic can undergenerate what we hold as being logical. We have many more logical consequences than tautologies and many more "necessary exclusions" than contradictions. It is as if, after the Colour Exclusion Problem,

[21] I am grateful to an anonymous reviewer for calling my attention to the suitability of this last quote to my overall view in this paper. I agree with him that the "Maβstab" invoked at the Tractarian passage 2.1512 anticipates the transitional move from the *Tractatus* to the later philosophy. This transition was in part motivated by the full recognition that graded concepts have a deep (logical) connection with a phenomenal and grade-sensitive language.

Wittgenstein actually dropped out of the Tractarian view of a purist all-encompassing logic, as he has to deal with material articulations (based on exclusions by contrariety) between concepts and not just formal relations between propositions.

Here we clearly see how the limitation of the truth table points out a limitation in the conceptual framework of the *Tractatus*, and vice versa. The (classical) conjunction of sentences for the ascription of colours to the same point is not only false; it is a nonsense that his early notational system could not prevent. And this inability to prevent nonsense is a serious problem throughout the Tractarian project.

As a result, we have investigated how mutilations of truth tables can capture some logical relations not recognised in the *Tractatus*. We have explored a philosophical motivation for removing some lines of truth tables in order to express intensional logical relations presented in the Aristotelian Square of Oppositions, such as contrarieties, subcontrarities and contradictions. More has to be done to fully systematise these mutilations with a proper formalism. What we have done here may contribute to the implementation of an alternative semantics to approach some modal logics, although Wittgenstein never thought of it in this vein.

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